TRAPPED WAVE AND NON-LINEAR RESONANCE IN A SEMI-SUBMERSIBLE

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1. INTRODUCTION

In this work some properties of trapped waves over submerged cylinders are reviewed and, for bodies not too close to the free surface, an analyticall asymptotic expression for the lowest mode is derived. The excitation of these modes and the ensuing non-linear resonance is also analysed and the response can be analytically determined with help of the asymptotic expression for the lowest mode. In this way it is shown here that the excitation of trapped waves can be of importance in the analysis of semi-submersible plataforms.

2. EXISTENCE AND SOME PROPERTIES OF TRAPPED WAVES

In this work we will assume that the cross section of the slender body is in the plane (y,z), where z is the vertical axis pointing upwards.

A trapped wave is a solution of the form

$$\overline{T}(x,y,z) = T(y,z) \cdot e^{i(K_T x - \Omega t)} \qquad (2\text{-}1)$$

that decays exponentially with y.

For this last condition it is necessary that $K_T > K_0 = \Omega^2/g$, where we used the deep water dispersion relation.

In fact, $T(y,z) \sim e^{-\lambda_0 |y|} . f_0(z)$ when $|y| \to \infty$ where

$$f_0(z) = (2K_0)^{1/2} . e^{K_0 z}$$

$$\lambda_0 = (K_T^2 - K_0^2)^{1/2} \qquad (2\text{-}2)$$

The difficulty of the associated eingenvalue problem is the infinite size of the fluid region. Working with the formulation of the Hybrid Element Method this problem is transformed to a standard eigenvalue problem in a finite domain and we can easily show then that:

i) For an arbitrary submerged body and any frequency there exists at least one trapped wave $\{K_T(\Omega); T_0(y,z;\Omega)\}$.

ii) $\dfrac{K_T(\Omega)}{K_0(\Omega)} \to 1$ when $\Omega \to 0$ or $\Omega \to \infty$

If ∂B is the contour line of the cross section and B its width we define the integral

$$I(K_0; \partial B) = -2K_0^2 . \int_{\partial B} e^{2K_0 z} . n_z . d\partial B \qquad (2\text{-}3)$$

In the limits $\Omega \to 0$; $\Omega \to \infty$, $\lambda_0(\Omega)$ is the root of the equation

$$B . \lambda_0^2(\Omega) + 2\lambda_0(\Omega) - I(K_0; \partial B) = 0 \qquad (2\text{-}4)$$

where $I(K_0; \partial B) \to 0$ if $\Omega \to 0$ or $\Omega \to \infty$. In these limits also $\lambda_0(\Omega) \to 0$ and

$$T_0(y,z) = f_0(z) \,.\, e^{-\lambda_0 |y|} \tag{2-5}$$

In reality the "approximations" (2-4), (2-5) are asymptotically correct when $I(K_0;\partial B) \to 0$. So they are good for all frequencies if the body is not too close to the free surface — as it is the case, for instance, of a semi-submersible. We have used them to study these structures.

3. EXCITATION AND NON-LINEAR RESPONSE

Trapped waves can be excited only by non-linear interaction of incoming waves. Once excited the response can be determined by multiple scales. It is given by:

$$\Phi(x,y,z,t) = \delta^{2/3} \{ \frac{1}{2} A(X,T) \,.\, T_0(y,z) \,.\, e^{i(\overline{K}_T x - \overline{\Omega} t)} + (*)\} + O(\delta) \tag{3-1}$$

where

δ = (wave amplitude)/B

$(X;T) = \delta^{4/3}(x;t)$

$(\overline{K}_T;\overline{\Omega}) = (K_T + \delta^{4/3}\Delta K_T;\ \Omega + \delta^{4/3}.\ \Delta\Omega)$

$K_T = K_T(\Omega)$ = wave number of excited trapped mode.

and A(X,T) is solution of the equation:

$$i \cdot \left(\frac{\partial A}{\partial T} + c \cdot \frac{\partial A}{\partial X}\right) + (\sigma + i \cdot \mu_v) \cdot A + (n + i\mu_r) \cdot |A|^2 A = P \qquad (3\text{-}2)$$

In the above expression

$c = \frac{d\Omega}{dK_T}$ = "group velocity" of trapped wave

$\sigma = \Delta\Omega - c \cdot \Delta K_T$ = detuning

μ_v = viscous damping coefficient

n = non-linear coefficient

μ_r = non-linear radiation damping coefficient

P = exciting term.

The coefficient μ_v has been estimated using traditional oscillatory boundary layer theory and it is a small factor. The trapped mode, in fact, leaks energy to infinite at order $\delta^{4/3}$ and μ_r is associated with this. It can be shown that $\mu_r/n \sim O((\lambda_0/K_0)^4)$ and so the radiated energy is very small when $\lambda_0/K_0 << 1$ — an usual situation for a body not too close to the free surface. The coefficients $\{c;\ \mu_v;\ n;\ \mu_r\}$ depend solely on $T_0(y,z)$, the trapped mode, but P, the exciting term, depends also on the quadratic interaction of the linear potentials associated with the incoming waves.

The stationary solution of (3-2) satisfies the algebraic equation $(\sigma + i\mu_v)\, A_0 + (n + i\mu_r) \cdot |A_0|^2 A_0 = P$. This equation is identical to the one associated with resonance of a non-linear oscillator. Using the asymptotic approximations (2-4), (2-5) and Froude-Krilov approximation for linear diffraction we can analytically determine the response for a semi-submersible. It can be shown that the excitation of trapped waves can be of importance in this case. A full report of this work has been submited to the Journal of Fluid Mechanics.

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Discussion

Evans: This relates to work done by Jones (1952). The difficulty in this problem is in extending the theorems of Courant-Hilbert from a finite domain to an infinite domain. Presumably, you have done something similar here. Jones showed that for symmetric bodies there is always at least one trapped wave.

Aranha: I express the outer solution in terms of eigenfunctions and then match the inner and outer solution by requiring continuity in the potential and normal velocity.

Mei: You mention the application to semi-submersibles as a motivation of this study. However, the slow length scale in the longitudinal direction is much longer than a semisubmersible. Perhaps this formulation is more appropriate for a submerged ridge.

Aranha: Two-dimensional effects are accounted for and the solution is x-dependent. The solution is not wavelike.