ON THE SOLUTION OF THE RADIATION PROBLEM IN THE TIME DOMAIN

by F. T. Korsmeyer
Department of Ocean Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Until recently, predicting wave-body interactions has been an exercise carried out in the frequency domain. In the past, computational limitations have necessitated treating three-dimensional problems as a series of two-dimensional problems by invoking scaling arguments based on ratios of body dimensions to the wave length of the excitation. These are arguments which are ambiguous in the time domain where all scales are present at any time. Now however, the computational power which is readily available to most researchers has removed the reduction to two dimensions as a requisite part of the model. Consequently, some of the attractiveness of the frequency-domain solution has faded, and several authors have recently reported results for wave-body interaction problems based on computation in the time domain.

For the linear problem with no current or forward speed, it can be argued that the time-domain and frequency-domain solutions are competitive. Both usually involve a discrete approximation to the body geometry and the use of Green's theorem to construct a Fredholm integral equation to be solved for the unknown potential on the body surface. However, the computational effort required for the solution of the systems of algebraic equations so generated is different for the two cases. In the frequency-domain solution, the full matrix of influence coefficients is frequency dependent and so must be inverted for every frequency considered. In the time-domain solution, the full matrix of influence coefficients is not time dependent and therefore is inverted only at the outset. However, this solution is a convolution which at any time step consists of a series of matrix multiplications proceeding through all previous time steps. For large numbers of panels, the time domain solution probably requires less computational effort, but will require considerably more storage. Non-linear problems, or those involving current or forward speed, benefit from the straightforwardness of the time-domain formulation and will enjoy some computational advantages as well. Therefore, the unsteady forward speed problem for ships, and the second order force problem for arbitrary bodies at zero speed are problems which are now being attacked in the time domain.

A computer code which solves the radiation problem in the time domain has been written for the VAX 750 at the Computational Hydrodynamics Facility at MIT. The solution technique is like that described above, and uses the time dependent Green function, $G(x,t)$, associated with impulsive body acceleration. This Green function has been known for some time, but unlike its frequency domain counterpart, its behavior has not been thoroughly investigated. The impulsive motion problem requires the evaluation and integration of $G(x,t)$ and its second derivative with respect to time and space (in the panel normal direction), $G_{nt}(x,t)$, N x N x NT times (where N = number of panels and NT = number of time steps). Clearly, it is important to make this evaluation and integration as efficient as possible while maintaining suitable accuracy. The former task is handled by a subroutine which uses different algorithms as dictated by the magnitudes of the space and time arguments. The latter task is one of the subjects of this investigation. Ideally, one would hope to approximate the integrals of $G(x,t)$ and $G_{nt}(x,t)$ over a panel by the product of these functions evaluated at the panel centroid and the panel area. It is expected that for deeply submerged panels this technique is acceptable. However, for large times and when field point and source point are both close to the free surface, $G(x,t)$ and all of its derivatives are highly oscillatory in space. This oscillatory behavior increases in frequency with time, but the depth over which it is of concern decreases exponentially with time. This may necessitate a more careful integration method for certain arguments of $G(x,t)$ and $G_{nt}(x,t)$, and may be crucial in second order and forward speed problems which employ a line integration of the potential around the body at the free surface.

That $G(x,t)$ and $G_{nt}(x,t)$ are highly oscillatory is demonstrated as a function of time and field point and source point distance to the free surface. The effect of this behavior on the evaluation of the integrals of G(x,t) and $G_{nt}(x,t)$ over typical dimensions of a panel is demonstrated for several quadrature schemes. The importance of the implementation of these schemes is shown in the context of their effect on the calculation of hydrodynamic quantities for a three-dimensional body.

Discussion

Beck: Are analytic solutions necessary? We think that there might be better approaches. The problem we are currently doing starts smoothly. In regard to the oscillations, we experience similiar phenomenon. The forward speed problem seems to be well suited to time-domain solutions. [Similiar opinions have been expressed by other authors.]

Papanikolaou: I would like to make some comments on the competitiveness of frequency versus time-domain solutions raised in this paper. Comparing the two techniques with respect to their efficiency, one should consider, in both cases, the computational effort required to achieve a certain degree of accuracy for a certain amount of information when studying the motions of a body in waves. For example, one can study the complete linear and quasi-nonlinear (drift forces) motion of a simple rectangular barge discretized by N = 4 X 12 panel elements within 3 seconds CPU time per frequency on a CYBER 175 machine when using a typical panel method in the frequency domain [1]. The same case runs approximately 50 times slower on a small minicomputer like the PIXEL of the NTU of Athens. The results do not change significantly by doubling the number of elements N to 4 X 24. Of course for complicated body shapes the required number of elements has to be increased probably up to a total of approximately 400. However, taking symmetries of the body into account, as is usual in naval achitectural problems, one has to deal really only with 0.25 and 0.5 N elements respectively. Thus the inversion of the system of complex equations to be solved for the source strengths is not that serious a problem especially if the complex matrices are transformed to equivalent sets of larger but real matrices. Certainly the efficient evaluation of the elements of the matrix is a much more challenging task though it seems this has been simplified greatly through the recent work by J.N. Newman [2] and others on evaluation of the Green functions. In case of even larger numbers of N one could think of regridding the body shape, depending on the frequency of calculation, something not trivial in time-domain techniques.

As far as applications of the proposed method to the nonzero forward-speed case and true nonlinear ship motions one should consider the intricate radiation condition to be fulfilled at a "finite" infinity and the number of time steps required to establish a stable numerical solution.

As to R. Beck's remarks on the forward-speed problem and the proper evaluation of the line-integral term, I make reference to the paper in this workshop by J.N. Newman on the evaluation of the three-dimensional Green function with forward speed. The proposed procedure is expected to improve substantially the quality of numerical results in related three-dimensional computer codes. Of course, the line integral evaluation has to be addressed in a time-domain technique as well as being inherent to the hydrodynamics of the moving body.

[1] Papanikolaou, A. (1985): Proc. Ocean Space Util. Conf., Tokyo.
[2] Newman, J.N. (1985): Journal Engineering Mathematics.

Korsmeyer: There seem to be two separate points in question:

1) How many panels do we envision in the description of a complicated body in either the time or frequency domains?
2) Given these numbers, which technique is more efficient?

I would not try to sell the time domain on efficiency in the zero speed linear problem alone. The advantages of a time domain formulation for arbitrary mean body velocity or nonlinear effects at zero speed have been mentioned and are well accepted.

However, in answer to question one, for bodies of great structural complexity or where high frequency effects are critical, even exploiting symmetry, numbers of panels in excess of 400 have been used in practice and will continue to be used.

In light of this, the efficiency of the time domain formulation is clear. For the frequency domain, the effort is roughly:

$$(\text{number of panels} \times 2)^3 \times (\text{number of frequencies})$$

and for frequency-domain results from a time-domain formulation:

$$(\text{number of panels})^3 + (\text{number of panels})^2 \times (\text{number of time steps})$$

For reasonable values such as 30 to 40 frequencies for the former technique, and 200 time steps for the latter, it is easy to see where the crossover of effort will occur as the number of panels increases. This comparison is from the point of view that the solution of the system is the dominant task. Evaluation of the influence coefficients is not the more challenging task, as has been shown by Newman [Abstract Ref. 4.]

Ursell: From the transient response, you obtain by Fourier transformation the virtual mass and damping coefficients at all frequencies. However, the full heaving motion of a floating body is nearly damped harmonic and there is evidence that we can obtain virtual mass and damping only at the corresponding frequency. The present work is concerned with an impulsive start after which the body is held fixed (not with a free motion). It should perhaps be emphasized that the success of your work depends on this distinction.

Newman: The task of getting force coefficients in the frequency domain is quite easy, starting from the impulse response function (even for heave). The impulse-response function is very general and enables one to find a general solution in the time domain. What about more complicated bodies such as tension-leg platforms? As pointed out by Korsmeyer the problem will become more complex but certainly not impossible.

Yeung: There is a difference between obtaining added mass and damping by Fourier transforming a freely-floating heaving solution and transforming an impulsive-start force function. The freely-floating solution contains the information of added mass and damping centering mostly information around the "resonant frequency" of heave whereas the impulsive-start force function contains information over a much wider frequency range. I have discussed the possible forms of impulse response function associated with a source or a dipole type behavior in a survey paper given at the 1985 IUTAM Symposium on Wave Energy Utilization.