Sloshing frequencies in a rectangular tank with a baffle

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A horizontal rectangular tank is partly filled with water. The natural frequencies of oscillation of the water are easily determined on the basis of the linearised theory of water waves by separation of variables. It is wished to decrease the dominant 'sloshing' frequency so as to avoid undesirable oscillations due to external forcing at that frequency. This may be achieved by inserting a plane rigid thin vertical baffle, completely spanning the tank parallel to one pair of sides, but extending over only part of the depth. The effects of introducing the baffle are similar to the changes in frequency that result from the appearance of a slit in a vibrating membrane.

The aim of the present work is to estimate the new natural frequencies as a function of the geometry of the tank, the depth of water, and the extent of the baffle. Two basic geometries will be considered, a surface-piercing baffle lowered from above and a bottom-mounted baffle inserted from below. The problem will be formulated for the former case only, the changes required to treat the latter case are trivial.

The motion is assumed to be two-dimensional. Cartesian coordinates are chosen with $y = 0$ the undisturbed free surface. The walls of the tank are at $x = b, -c$ and the water occupies $0 \le y \le h$. For the surface piercing case, the baffle occupies $x = 0,\ 0 \le y \le a\ (a < h)$.

On linear water wave theory we can introduce a velocity potential

$$\underline{\emptyset}\,(x,y,t) = R_e\,\{\emptyset(x,y)\,e^{-i\omega t}\}\,. \tag{1}$$

Then $\emptyset(x,y)$ satisfies

$$\nabla^2\emptyset = 0 \qquad 0 < y < h;\ -c < x < b,\ x \neq 0 \tag{2}$$

$$K\emptyset + \emptyset_y = 0 \qquad y = 0;\ -c < x < b,\ x \neq 0 \tag{3}$$

where $K = \omega^2/g$

$$\emptyset_y = 0 \qquad y = h;\ -c < x < b \tag{4}$$

$$\emptyset_x = 0 \qquad x = 0;\ 0 < y < a \tag{5}$$

$$\emptyset_x = 0 \qquad x = b,\ -c;\ 0 < y < h \tag{6}$$

This is an eigenvalue problem for the frequency parameter K.

Solutions may be posed in the form

$$\emptyset(x,y) = -\sum_{n=0}^{\infty} U_n \frac{\cosh k_n(b-x)}{k_n \sinh k_n b}\,\psi_n(y),\quad 0 < x < b \tag{7}$$

$$= \sum_{n=0}^{\infty} U_n \frac{\cosh k_n(c+x)}{k_n \sinh k_n c}\,\psi_n(y),\ -c < x < 0 \tag{8}$$

where

$$\psi_n(y) = N_n^{-1} \cos k_n (h-y) \qquad n = 0,1,2,\ldots \tag{9}$$

are orthogonal eigenfunctions in [0,h] with N_n a normalising factor. Here k_n (n=1,2,...) are the real positive roots of

$$K + k_n \tan k_n h = 0 \tag{10}$$

while $k_0 = ik$ $(k > 0)$ is one of the two imaginary roots of (10). The U_n are are the Fourier coefficients in the expansion of the horizontal velocity U(y) across x = 0, $0 \le y \le h$, so that

$$U(y) = \sum_{n=0}^{\infty} U_n\psi_n(y), \; U_n = \int_L U(y)\psi_n(y)\,dy \tag{11}$$

where L is the interval [a,h] and U(y) = 0, 0 < y < a. The forms (7) and (8), with U_n given by (11), satisfy equations (2)-(6) and ensure continuity of horizontal velocity across L. Continuity of the potential across L gives, after some manipulations, the integral equation

$$\int_L u(t)\, K_1(y,t)\, dt = \psi_0(y), \; y\epsilon L \tag{12}$$

for
$$u(y) = AU_0^{-1}U(y) \tag{13}$$

where
$$\frac{\sin kb \; \sin kc}{\sin kd} \equiv A = \int_L u(y)\psi_0(y)\, dy \equiv \langle u,\psi_0\rangle \tag{14}$$

Here
$$K_1(y,t) = \sum_{n=1}^{\infty} s_n\psi_n(y)\; \psi_n(t) \tag{15}$$

and
$$s_n = kk_n^{-1} (\coth k_nb + \coth k_nc) \; . \tag{16}$$

The forms (12) and (14) suggest a variational form for A and it follows that

$$A = \langle u,\psi_0\rangle^2 \,/\, \sum_{n=1}^{\infty} (\langle u,\psi_n\rangle^2 s_n) \; . \tag{17}$$

From this it may be shown that A is stationary with respect to first-order variations of u(y) about the exact solution of (12) and the resulting approximation is never greater than the true value of A. We now assume a form

$$u(y) = \sum_{m=0}^{M} u_m\psi_m(y) \quad , \tag{18}$$

substitute into (17) and apply the conditions for stationarity , $\partial A/\partial u_i = 0$ (i = 0,1,2,...M) (see, for example, Mei and Black [1]). The determinant condition for non-trivial solutions for the u_i gives an eigenvalue relation which may be solved numerically.

An approximate eigenvalue relation for an arbitrarily shaped baffle may be derived on the assumption of 'wide-spacing' (see Martin [2]. The general result is

$$T^2 e^{ikd} = (e^{-ikb} - R_2 e^{ikb})(e^{-ikc} - R_1 e^{ikc}) \tag{19}$$

where R_1 (R_2) is the reflection coefficient for waves incident from infinity upon the baffle from the left (right) and T is the transmission coefficient, known to be independent of the direction of the incident wave. The total width of the tank is denoted by $d(=b+c)$.

For a thin vertical barrier occupying any part, or parts, of the line $x = 0$, it is known that $R_1 = R_2 = R$ and $R + T = 1$ enabling (19) to be reduced to

$$\sin kb \ \sin kc / \sin kd = (1-R)/2iR. \tag{20}$$

For surface-piercing and bottom-mounted barriers in deep water simple expressions for R are given by Ursell [3].

Results are given for the full linear theory and the approximate relation (20) applied in deep water. All results are given in terms of k rather than K as for a tank without baffle the natural modes of oscillation occur for $kd = n\pi$, n an integer. In Fig. 1, results are given for a surface-piercing barrier at the centre of the tank. The symmetric modes are unaffected by the barrier as the horizontal velocity is then zero on the centre-line. Agreement between the full theory and the approximate solution is excellent, the comparison is made for the lowest mode only as for other modes the results are graphically indistinguishable. The full theory results are for a finite depth $h/d = 1$ and as a/d tends to one kd must go to zero for the lowest mode. When the barrier does not divide the tank equally, as in Fig 2, more modes are affected by the presence of the barrier. Results for bottom-mounted barriers are given in Fig.'s 3 and 4. Here a is the submergence of the end of the barrier. It is apparent that, for the geometries considered, the barrier must be quite close to the surface before the natural frequencies are significantly changed.

References

1. C C Mei and J L Black (1969) Scattering of surface waves by rectangular obstacles in waters of finite depth. J.Fluid.Mech., 38, 499-511.

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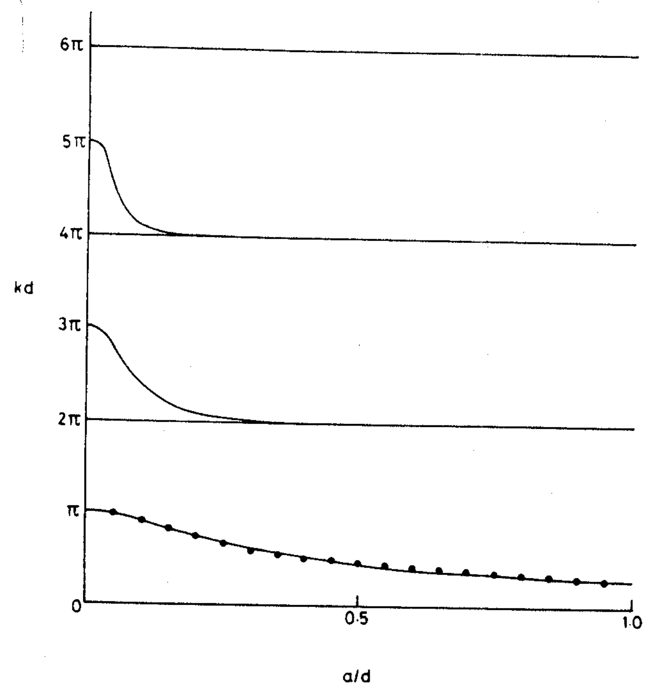
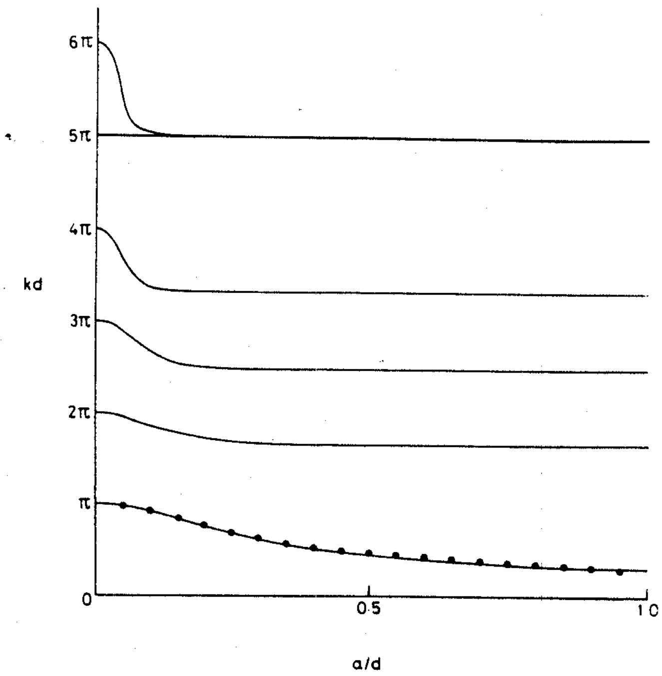


Surface-piercing barrier, b/d = 0.5

(—— w-s approx.

• • • full linear, d/h = 1)

Figure 1

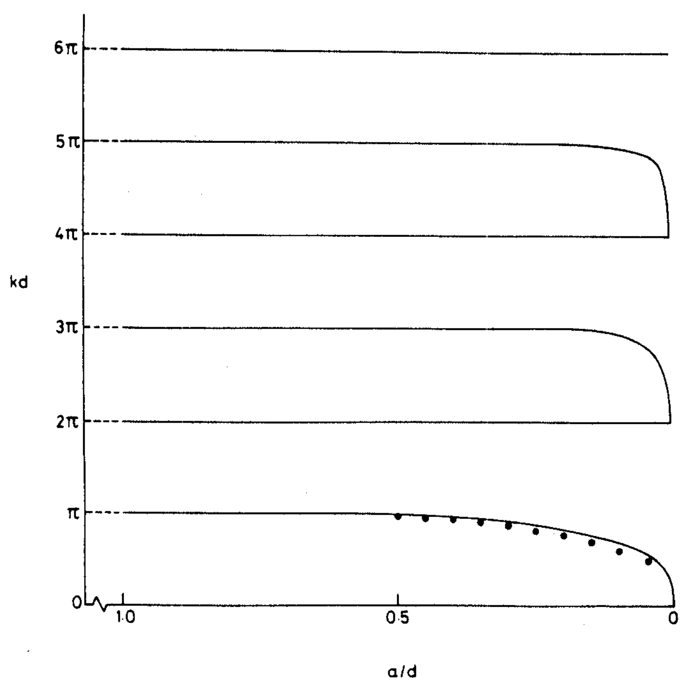


Surface-piercing barrier, b/d = 0.4

(—— w-s approx.

• • • full linear, d/h = 1)

Figure 2

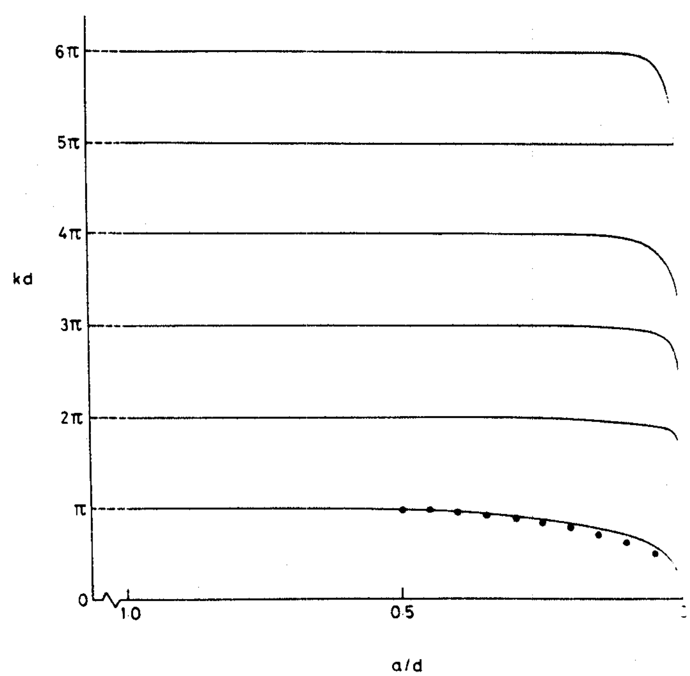


Bottom-mounted barrier, b/d = 0.5

(—— w-s approx.

• • • full linear, d/h = 1)

Figure 3



Bottom-mounted barrier, b/d = 0.4

(—— w-s approx

• • • full linear, d/h = 1)

Figure 4

Discussion

Van Hooff: J.J. van den Bosch from Delft University did experiments on an anti-rolling tank with a curved bottom, corresponding to your case with a submerged barrier. These experiments may be useful for confirming your results.

Tuck: Regarding practical applications, there is a major problem with sloshing in tank trucks of which there are many thousands in the world, and many (perhaps hundreds) of accidents per year related to sloshing.

Agnon: You will get more information if the integral equation is solved by a Galerkin method.

McIver: The integral equation/variational formulation and the Galerkin formulation are equivalent. The Galerkin formulation readily gives explicit equations to determine the eigen functions, however the main interest for the linear problem is in determining the eigen-frequencies.

X. J. Wu: It is an interesting mathematical derivation relating the determination of resonant frequencies in a simple rectangular tank. In realistic marine applications, such a resonant wave phenomenon is well recognized and studied in two aspects, i.e., the anti-rolling tank and the liquid sloshing problem. In the former case, the resonant water motion in the anti-rolling tank is utilized to reduce the peak roll response of a ship. For tanks of more complicated geometries, approximate formulas have been derived in our work (for example, see X.J. Wu et al. J. Shanghai Jiao Tong Univ., No. 2, 1983 and X.J. Wu and W. G. Price, Int. Conf. Vibration Problems, Xi'an, June 1986). The latter is of special significance to the sea transportation of oil, LNG, etc. Tanks should be specially designed to avoid resonant sloshing motion of liquid. A more sophisticated numerical procedure has been developed by Mikelis et al at Lloyd's Register of Shipping (Ref. Mikelis et al, read at the Autumn Meeting, Soc. Naval Arch., Japan, Nov. 1985) for predicting sloshing motion in arbitrarily shaped tanks with internal structures. I am particularly interested in their conclusion that "viscosity does not affect the liquid's sloshing response and therefore simplification is possible."