SECOND-ORDER DOUBLE FREQUENCY LOADS AND MOTIONS FOR THREE-DIMENSIONAL BODIES

Bernard MOLIN

Institut Français du Pétrole
BP 311
92506 Rueil-Malmaison Cedex
France

ABSTRACT

The calculation of second-order loads and motions for floating bodies in regular waves has been a topic of intense investigation in the recent past. The main difficulty one is confronted with in this problem stems from the non-homogeneous free-surface condition that arises in the second-order diffraction problem. As a result the classical methods that have been used to solve for the first-order problem may not be straightforwardly employed, and some controversies have been going on the correctness of the results that have been published so far.

The work that is reported here is a continuation of previous work carried out at IFP on the subject. In a previous paper /1/, an original method, based on Haskind's theorem, was introduced to simply calculate the second-order horizontal loads upon a fixed body. In another paper /2/, the problem of describing the body motion to second-order, and deriving the resulting hydrostatics, was considered. Applications were then only focused on the calculation of drift forces in heave and pitch.

Since then some new developments have taken place, and a complete numerical model has been built up that permits to derive to second order the loads and motions for vertical axisymmetric bodies in regular waves. Some features of the underlying theory and some results are described below.

A preliminary step is to develop Newton's equations to second-order, and to formulate the associated hydrodynamic problem. The first-order flow can be derived using standard numerical methods (that is solving one diffraction and six radiation problems, calculating the body response, and constructing the total first-order potential in the fluid domain). Similarly it is shown that the second-order flow (at the double frequency) can be split up into an incident component, a diffraction component (that takes account of the first-order motion), and six elementary radiation components. Only the diffraction component obeys a non-homogeneous free-surface equation. The radiation ones are identical with those of the linear potential theory, but take place at the double frequency.

The second-order diffraction loads are obtained in a manner similar to the one presented in /1/. That is, combining the second-order diffraction problem with the elementary radiation potentials at the double frequency, each component of the loads due to the second-order diffraction potential is expressed as a sum of integrals over the body surface and over the free-surface at rest. The theoretical and numerical convergence of the free-surface integral has been carefully investigated. This required to gain some insight of the asymptotic behavior of the second-order diffraction potential far away from the body.

<u>Behavior of the second-order diffraction potential at large radial distances</u>.

It has been shown here that it consists of two components. The first one ensures that the free-surface equation be fulfilled to second order, and describes waves that are "locked" to the first-order wave system. The second one consists of waves that obey the homogeneous free-surface equation, and therefore travel independently of the first-order wave system. They asymptotically behave as radially diverging waves with wave number related to the double frequency and the waterdepth through the classical dispersion equation.

The analytical expression of the locked component may be simply obtained by considering that the first-order diffracted and radiated waves can be asymptotically described as plane progressive waves, traveling in the radial direction. Thus at some point ($R \cos \theta$, $R \sin \theta$) far away from the body the first-order wave system can be regarded as the superposition of two sets of plane progressive waves, at the same frequency but traveling in different directions: the incident waves with wave number vector $\underline{kI}$ $(-k, 0)$, and the diffracted/radiated waves with wave number vector $\underline{k\theta}$ $(k \cos \theta, k \sin \theta)$ and amplitude = $O(R^{-1/2})$. It is a known result that in such a case the associated second-order wave system consists of three sets of waves, at wave number vectors $2\ \underline{kI}$, $2\ \underline{k\theta}$, and $\underline{kI} + \underline{k\theta}$. The first ones are the incident second-order waves, the second ones are of order $O(R^{-1})$, and thus of reduced interest, but the third ones are of the order $O(R^{-1/2})$. An interesting feature is that they travel (locally) in the median direction and therefore do not propagate radially.

It has appeared that the existence of these second-order locked waves causes the free-surface integral to be a highly oscillatory function of the upper radial bound to which it is computed, but to eventually converge toward the desired value (in the numerical applications some tricks are to be employed to filter out these oscillations and to speed up convergence).

Numerical results.

They have been first obtained for the classical case of a circular cylinder standing on the sea-floor. Analytical expressions have been derived for the different components of the second-order horizontal force, except for the free-surface integral which requires a final numerical evaluation. An interesting feature that has been obtained is that the free surface integral, at a given wave number, does not reach an asymptotic value when the waterdepth exceeds half a wavelength of the incoming wave system. This peculiarity is to be linked to a standing wave effect that takes place on the weather side of the cylinder and that induces second-order pressures slowly decreasing with the submergence. That such a phenomenon was to take place could easily be foreseen from the previous analysis on the second-order locked waves.

Next a diffraction/radiation code developed at IFP was extended in order to derive the second-order loads and responses for three-dimensional bodies in regular waves. This code assumes vertical axisymmetry, which allows to develop the velocity potentials as Fourier series of the polar angle θ, and to reduce the three-dimensional problem to a suite of two-dimensional ones in the plane (R,z). In this code the fluid domain is divided into two zones: a zone close to the body where the solution is constructed through fluid finite elements, and an infinite external domain where elementary analytical expressions are used.

The obtained numerical results were first checked against the case of the circular cylinder and a good agreement was obtained. Applications were then made to the case of a buoy, for which experimental results were obtained at ENSM wave tank. A good agreement was observed between numerical and experimental values for the second-order heave force (the buoy being held fixed in waves). Radiation tests in heave were also carried out but the agreement in this second case appeared to be less good. Viscous effects could be inferred, but also theoretical and numerical problems related to the evaluation of the body surface integral, at the hull edges. Further investigation is required on this point.

REFERENCES

/1/ B. MOLIN: "Second-order diffraction loads upon three-dimensional bodies", Applied Ocean Research, vol. 1, n°4, 1979.

/2/ B. MOLIN, J. P. HAIRAULT: "On second-order motion and vertical drift forces for three-dimensional bodies in regular waves", Proceedings of the International Workshop on Ship and Platform Motions, Berkeley, 1983.

Discussion

Papanikolaou: In the second-order free oscillation problem, you define a second-order diffraction potential which seems to contain radiation effects. I miss the interaction of the radiation effects with themselves. You could have a more clear formulation for the velocity potential. How are the second-order forced motion effects taken into account?

Molin: I do not consider the free-oscillation problem, but directly the body response to an incoming wave system. In this case, it appears more convenient to gather all the inhomogenous terms together (due to first-order diffraction and radiation), and solve only one second-order problem. There are other ways to proceed, but they require solving more than one second-order problem.

Kleinman: I did not see how the solution depended on the inhomogeneity in the free surface condition. Also what radiation condition did you assume and how does the solution behave in the far field?

Molin: I agree that my derivation of the asymptotic behavior of the second-order diffraction potential may not be mathematically correct. Rather, I rely on physical and intuitive arguments.

Mei: The radiation condition is usually stated that we must have outgoing waves or as a differential equation. These are local statements. But we can formulate an integral form via Green's theorem (see e.g. Mei 1983 Applied Dynamics Ocean Surface Waves Eq. 6.3, p. 319) and demand that this integral vanishes at infinity. I think it can be proven that this condition is satisfied by this formulation.

P.F. Wang: I wonder what the radiation condition will be if we have two incident waves propagating in the same direction but at different frequencies. Then the second-order waves may propagate in the same or the opposite direction.

Molin: The same procedure as I used here can be applied to the case of a bichromatic wave system, it just makes the derivations somewhat more difficult.

Kim: I agree with Mei on the point that vanishing of the integral at infinity is sufficient as a radiation condition when using Molin's approach even though it is much weaker than the direct form. Specifying the radiation condition of a second-order potential without proof leads to erroneous results if we try to solve the boundary value problem directly. Even the condition of outgoing waves is not obvious especially for the case of bichromatic waves.