CHANGES OF THE WAVE ELEVATION CAUSED BY UNDER-WATER RIDGES.

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INTRODUCTION

When water-waves, at relatively constant water depth, pass over under-water ridges, the waves are modified. The modifications are due to two effects:

* Reflection of wave energy from the ridges and transmision of wave energy across the ridges.
* A phase shift of the transmitted and reflected waves compared with that of the incoming wave.

The first effect (in its two-dimension analogy) produces a decrease of the wave amplitude on the lee side of the ridges. The second one creates a non-uniformity in the wave field, causing partial cancelation of the waves in some domains and a strengthening in others. The sum of these effects cause a wave pattern in the area around the ridges where there are domains with decreasing and with increasing wave height.

The present study is based on the following simplifying assumptions:

* Linearized, long wave length theory.
* The ridges are assumed to be slender

To solve the problem the method of matched asymptotic expansions is applied.

The objective of the project is to determine a configuration of under-water structures which is capable of reducing the wave height in a specified area of the Ekofisk field in the North Sea.

PROBLEM FORMULATION.

The problem is, at this stage, formulated for one single under-water ridge with an incoming, long crested wave. If we regard the problem from a view point situated "far" from the ridge (i.e. at a distance of the order of the length of the ridge), it appears in the picture as a line with a finite length. If we, on the other hand, are closing in on the ridge and regard the problem from a view point which is situated close to the ridge (i.e. at a distance of the order of the crossectional dimensions of the ridge), it appears as an extremely long cylinder with finite crossectional dimensions. The solution to this problem is found by applying the method of matched asymptotic expansions and the development of the solution to leading order basically follows Tuck /1/.

The near field solution.

In this domain the characteristic length scale is of the order of the crossectional dimensions of the ridge, and accordingly much smaller than the wave length. This means that the free surface condition is given from the long wave-length limit, i.e. as $\frac{\partial\Phi}{\partial n} = 0$, or in other terms: as an impermeable lid. Further more, the ridge is assumed to be slender. This implies that the derivatives in the axial direction (i.e. the y-direction) are much smaller than in the crossectional directions, which in turn means that Laplace's equation reduces to its two-dimensional version in the near field:

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 .$$

Consistent with the boundary conditions the velocity field far away from the body (in the inner domain) is given in terms of a parallel flow, with the asymptotic expansion of the velocity potential given as:

$$\Phi(x,z;y,t) \rightarrow U(y,t)\cdot x + U(y,t)\cdot\frac{\Delta}{2}\cdot\frac{x}{|x|} + C(y,t)$$

as $|x| \to \infty$. $\Delta(y)$ is the "blockage length" which has to be determined from the detailed solution of the inner problem and C(y) an additional constant (determined from the matching to the far field, as will be shown later). The solution of the inner problem, and hence $\Delta(y)$, can be found by means of different numerical techniques. For the special case of a rectangular crossection the values of Δ are found by Flagg and Newman /2/. To compute the forces and overturning moments acting on the ridge, the details of the near field solution has to be inspected more closely.

The far field solution.

In this domain the solution consists of an incoming wave plus a diffracted wave system. Both are assumed to progress as *waves* in this domain, and thus having a dependency on the vertical coordinate of the common form:

$$\Phi = \phi(x,y;t)\cdot\frac{\cosh(kz)}{\cosh(kH)}$$

where k is the wave number, corresponding to the actual frequency, and H is the water depth. z is the vertical coordinate, pointing upwards and being zero at the bottom and (x,y) are the horizontal coordinates.

This means that the function $\phi(x,y;t)$ is governed by the equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + k^2\phi = 0\,.$$

The incoming wave is given in the form:

$$\phi_I = exp(-ik_x x - ik_y y)exp(i\omega t)$$

where $k = \sqrt{(k_x^2 + k_y^2)}$. In addition to this we have the diffracted wave-system, determined from a distribution of singularities along the slit formed by the slender ridge. From the outer expansion of the inner solution:

$$\phi \to U(y;t)x + U(y;t)\frac{\Delta}{2}\frac{x}{|x|} + C(y;t)$$

it is obvious that the singularity should have a continuous normal velocity across the slit, but must have a discontinuous potential. Thus a dipole is chosen, with the Green function:

$$G(x,y;\xi,\eta;k) = \frac{\partial}{\partial\xi}H_0^{(2)}(k\rho) = -\frac{\partial}{\partial x}H_0^{(2)}(k\rho)$$

where $H_0^{(2)}$ is the Hankel function of zeroth order and second kind, satisfying the proper radiation conditions, and ρ is the radius (in the horizontal plane) from the source point to the field point:

$$\rho = \sqrt{(x-\xi)^2 + (y-\eta)^2}\,.$$

Accordingly the potential can be written as:

$$\phi(x,y;t) = exp(i\omega t)\cdot(exp(-ik_x x - ik_y y) + \int_L m(\eta)G(k(y-\eta),kx)d\eta)$$

where $m(\eta)$ is the dipole-strength along the slit. The coordinates (x,y), (ξ,η) are defined so that ξ and x are zero along the slit.

The inner expansion of the outer solution is now found as the limit of ϕ as $x \to 0$. This is simply written as:

$$\phi(x,y;t) \to exp(i\omega t)\cdot\{(1-ik_x x)exp(-ik_y y) - \frac{\partial}{\partial x}\int_L d\eta\, m(\eta)H_0^{(2)}(k\rho)|_{\xi=x\to 0}$$

$$-x\cdot\frac{\partial^2}{\partial x^2}\int_L d\eta\, m(\eta)H_0^{(2)}(k\rho)|_{\xi=x\to 0}\} + O(x^2).$$

Bringing into mind that $H_0^{(2)}(k\rho)$ satisfies the equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)\cdot H_0^{(2)}(k\rho) = 0$$

and that the contribution to the integral over $\frac{\partial}{\partial x} H_0^{(2)}$ only will give a contribution from the point $y = \eta$ we get:

$$\phi(x,y;t) \to exp(i\omega t)\cdot\{exp(-ik_y y) + 2im(y)\frac{x}{|x|}$$
$$+ x[-ik_x exp(-ik_y y) + (\frac{\partial^2}{\partial y^2} + k^2)\int_L m(\eta)H_0^{(2)}(k|y-\eta|)d\eta]\}.$$

The matching.

The problem is now formulated in the two domains and the proper asymptotic expansions of the two solutions in their respective extremes are found (i.e. the inner expansion of the outer solution and the outer expansion of the inner solution). They both are in the same form.

We still have some unknowns to determine, namely: U(y;t) and C(y;t) from the inner problem and m(y) from the outer problem. If we now apply the method of matched asymptotic expansions, requiring that the two asymptotic expansions should be identical (for all values of y and t), we get the following three equations:

$$C(y;t) = exp(i\omega t)\cdot exp(-ik_y y)$$

$$U(y;t) = exp(i\omega t)\cdot\{-ik_x \exp(-ik_y y) + (\frac{\partial^2}{\partial y^2} + k^2)\int_L m(\eta)H_0^{(2)}(k|y-\eta|)d\eta\}$$

and

$$U(y;t)\cdot\frac{\Delta}{2} = 2im(y)\cdot exp(i\omega t).$$

By eliminating U(y;t) from these equations and neglecting the common factor $exp(i\omega t)$ the following integro-differential equation appears:

$$(\frac{\partial^2}{\partial y^2} + k^2)\int_L m(\eta)H_0^{(2)}(k|y-\eta|)d\eta = \frac{4i}{\Delta}\cdot m(y) + ik\cdot exp(-ik_y y).$$

This equation can be viewed upon as a second order differential equation in the integral given on the left hand side, and the problem is thus reformulated (following Tuck /1/) as a mixed Volterra/Fredholm integral equation:

$$\int_L m(\eta)H_0^{(2)}(k|y-\eta|)d\eta = f(y) + A\ cos(ky) + B\ sin(ky)$$

where

$$f(y) = \frac{4i}{k\Delta}\cdot\int_{-\frac{L}{2}}^{y} m(\eta)sin(k(y-\eta))d\eta + \frac{i}{k_x}exp(-ik_y y)$$

where the ridge is assumed to be situated between y=-L/2 and y=+L/2. The two integration constants, A and B, are determined from the condition that the dipole strength vanishes at both ends of the ridge.

Since the integral equation is linear we can solve it for the three different right hand sides and constructs the general solution as:

$$m(y) = m_1(y) + Am_2(y) + Bm_3(y)$$

where A and B are determined from the boundary conditions:

$$m(-\frac{L}{2}) = m(\frac{L}{2}) = 0.$$

Once m(y) is determined the values of U(y;t) and C(y;t) are found, and the forces and moments acting on the ridge can be found in addition to the wave elevation in the far field as:

$$\zeta(x,y;t) = \text{Re}\{-\frac{i\omega}{g}\cdot exp(i\omega t)\cdot[exp(-ik_x x - ik_y y) + \int_L m(\eta)G(k(y-\eta),kx)d\eta]\}.$$

THE NUMERICAL SOLUTION OF THE INTEGRAL EQUATION.

The integral equation is solved by assuming the dipole strength to be piecewise constant. The length of the ridge is divided into N segments, $s_j = [\eta_j, \eta_{j+1}]$ where $\eta_1 = -\frac{L}{2}$ and $\eta_{N+1} = \frac{L}{2}$. The dipole strength is assumed constant over each segment and the integral equation is collocated at the midpoint of each segment.

The integrals involved are partly integrated analytically and by help of Gauss quadrature (after a subtraction of the singular part of the Bessel function). The numerical method was tested against the problem leading to the solution: m(y)=y. The comparison for N=20 is shown in table 1. The very good agreement is a justification for using the particular numerical scheme.

COMPARISON BETWEEN THE NUMERICAL SOLUTION OF THE INTEGRAL EQUATION AND THE EXACT SOLUTION M(Y) = Y.

EXACT	NUMERICAL	
M(Y)=Y	RE(M)	IM(M)
-0.9938	-1.0004	0.00071
-0.9694	-0.9734	0.00023
-0.9210	-0.9247	0.00014
-0.8500	-0.8534	0.00010
-0.7581	-0.7611	0.00008
-0.6475	-0.6501	0.00007
-0.5209	-0.5230	0.00006
-0.3815	-0.3831	0.00006
-0.2327	-0.2337	0.00005
-0.0782	-0.0786	0.00003
0.0782	-0.0785	0.00001
0.2327	0.2337	-0.00001
0.3815	0.3830	-0.00004
0.5209	0.5230	-0.00008
0.6474	0.6500	-0.00012
0.7581	0.7610	-0.00018
0.8500	0.8533	-0.00027
0.9210	0.9246	-0.00045
0.9694	0.9732	-0.00085
0.9938	0.9999	-0.00274

Table 1.

NUMERICAL RESULTS.

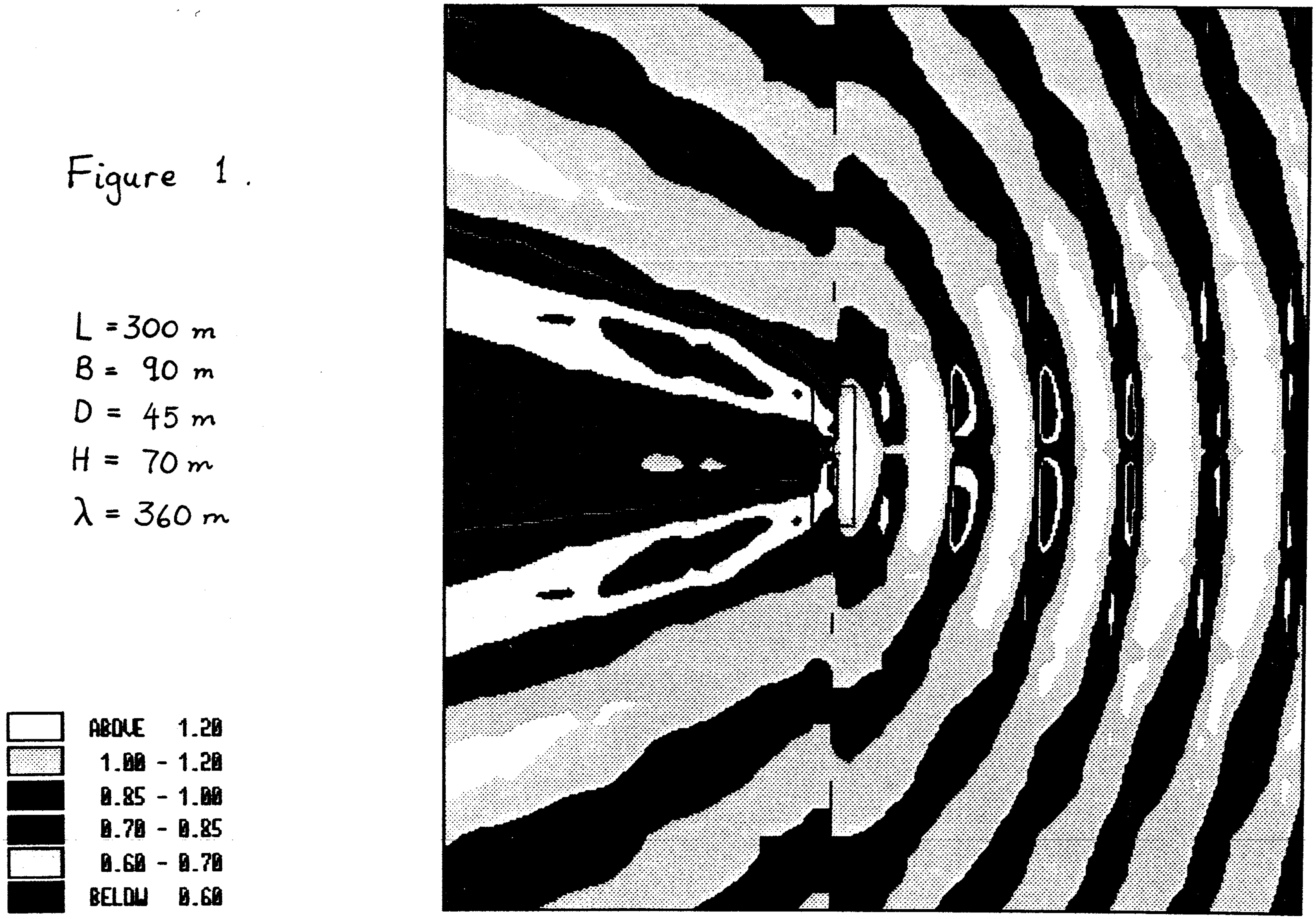
On figure 1 the wave elevation calculated from the far-field solution for incoming waves with normal incidence on a rectangular ridge is shown. The (color) plot shows a typical reduction in wave amplitude on the lee-side with the smallest amplitude in the lobes originating from the ends of the ridge. For normal incidence the relative wave amplitude is identically equal to 1 along x=0. Since there is a color change at this amplitude, the plot depicts an apparent discontinuity due to small numerical errors.

REFERENCES.

/1/ Tuck, E.O., "Ship Motions in Shallow Water", J. Ship Res., Dec. 1970.

/2/ Flagg and Newman, "Sway added-mass coefficients for rectangular profiles in shallow water", J. Ship Res., Vol 15, 1971.

Figure 1.



L = 300 m
B = 90 m
D = 45 m
H = 70 m
λ = 360 m
ABOVE 1.20
1.00 - 1.20
0.85 - 1.00
0.70 - 0.85
0.60 - 0.70
BELOW 0.60

Discussion

Tuck: You used cosine spacing to solve the integral equations. You might do better if you also used "Chebychev" midpoints as collocation points on the panel.

Vinje: I used cosine spacing for the segment end points and real midpoints as collocation points.

Tuck: Is the apparent discontinuity at $x = 0$ real or a numerical error?

Vinje: I attribute the discrepancies to small numerical errors causing an apparent discontinuity at $x=0$. This can be explained by the fact that the wave amplitude for normal incidence is unity at $x=0$ and the plot has a color change at this wave amplitude.

Troesch: What is the effect of oblique incidence?

Vinje: The orientation of the lobes changes with the angle of incidence.

Stiassnie: Is shallow-water theory justified? We did a similiar problem using elliptic coordinates. Our theory is not restricted to shallow water.

Vinje: I agree that the use of shallow-water theory is questionable. We were looking for a fast solution, not necessarily an accurate one.

Mehlum: Three theoretical methods have been used in parallel in addition to experiments. This has been done because it is of vital importance that the computations are correct.

Newman: Could you summarize the methods?

Mehlum: The methods are:

1) A variation of the so-called "mild slope equation",

2) A standard "panel" method,

3) and the method described in this lecture.

This third method is used to check the other two for simple geometries.

T. Wu: I am interested in the experiments because I have a colleague who is doing similiar problems (including a trench rather than the mount or wall you are using). In particular we have found that vortex formation and flow separation are important. I would appreciate it if you could comment on this effect.

Vinje: Flow separation is not considered to be important at least not for the far-field behavior. This is confirmed by comparing 3D inviscid calculations with experiments.

X.-J. Wu The inaccuracy mentioned by Nestegard is caused by end effects since the details of the ends of the ridge have been lost when applying the asymptotic matching method. Therefore, an alternative approach, called the 3D-strip method may be suggested to treat the ends of large curvature as 3D surfaces and the midbody of small or no curvature by a 2D strip-like discretization. These two concepts are combined to present a 3D-strip integral equation. Here, results for a submerged rectangular cylinder 117 X 45 X 15m are displayed together with experimental data and pure 3D computations, for three floating rectangular cylinders of length to width ratios 1.0, 2.0 and 3.0. Good agreement can be observed.

Ref. X.J. Wu "A hybrid 3D-strip method for evaluating surging coefficients of full-shaped ships" (7th Int. Conf. on Boundary Element Method, Como, Italy, 1985)