TRANSIENT SECOND-ORDER DIFFRACTION BY A VERTICAL CYLINDER USING THE WEBER TRANSFORM

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ABSTRACT

First and second order transient diffraction velocity potentials for a vertical cylinder are obtained by using the Weber transform.

Diffraction theories for a vertical cylinder have been the subject of research for nearly three decades. MacCamy and Fuchs (1954) solved explicitly for the linear diffraction velocity potential for a vertical cylinder in the frequency domain. Chakrabarti (1972) examined the nonlinear diffraction effect by using Stoke's fifth order wave and obtained an approximate expression for the wave forces. In his theory, the fifth order wave is treated as the sum of five linear independent waves. The diffraction potentials are obtained by satisfying the body boundary condition, but the nonlinear free surface condition is not satisfied exactly. Chen & Hudspeth (1982) used the eigen-function expansion method to obtain second order diffraction potentials for a single-frequency incident wave. All existing second-order diffraction theories possess three common properites : (i) apply in the frequency domain. (ii) employ an incident wave at a single frequency. (iii) a condition of outgoing waves at infinity is assumed without proof.

The present theory treats the first and second order diffraction potentials as initial boundary-value problem and solves them in the time domain aiming to avoid the three restrictions mentioned above. The

steady state frequency domain solution can be obtained by formally letting t approach infinity.

In the initial boundary-value problems for the first order ($\psi_D^{(1)}(\vec{r},t)$) and second order diffraction potential ($\psi_D^{(2)}(\vec{r},t)$), both $\psi_D^{(1)}(\vec{r},t)$ and $\psi_D^{(2)}(\vec{r},t)$ satisfy the (i) Laplace equation (ii) bottom boundary condition (iii) inhomogeneous body boundary condition (iv) homogeneous free surface condition for $\psi_D^{(1)}(\vec{r},t)$ and inhomogeneous free surface condition for $\psi_D^{(2)}(\vec{r},t)$, with the inhomogeneous terms being quadratic products of the linear solution. The initial conditions are assumed to be : $\psi_D^{(i)}(z=0,t=0) = \psi_{Dt}^{(i)}(z=0,t=0)=0$, i=1,2. In the above formulations the only difference between $\psi_D^{(1)}(\vec{r},t)$ and $\psi_D^{(2)}(\vec{r},t)$ is that $\psi_D^{(1)}(\vec{r},t)$ satisfies a homogeneous and $\psi_D^{(1)}(\vec{r},t)$ an inhomogeneous free surface condition. $\psi_D^{(2)}(\vec{r},t)$ can be linearly decomposed into two separate problems, each having only one inhomogeneous boundary condition.

The incident wave is decomposed into a series of cos$n\theta$ or sin$n\theta$ modes in the θ-direction , so are the diffraction potentials. For each mode n, the 3-D Laplace equation can then be reduced to a 2-D equation in the variables r and z, and mode number n as a constant parameter. The Green function for this equation and free surface condition are generated by using the Weber transform in the r-direction.

Weber Transform :

$$Z_n(Kr) = J_n(Kr)Y_n'(Ka) - Y_n(Kr)J_n'(Ka)$$

$$G(K) = \int_{\alpha}^{\infty} r\cdot Z_n(Kr)G(r)dr$$

$$G(r) = \int_{0}^{\infty} \frac{K\cdot G(K)\cdot Z_n(Kr)}{J_n'^2(Ka)+Y_n'^2(Ka)}\,dK$$

where a is the radius of the cylinder.

$\begin{pmatrix} J_n \\ Y_n \end{pmatrix}$ is Bessel function of $\begin{Bmatrix} \text{first kind} \\ \text{second kind} \end{Bmatrix}$

We take inverse Weber transform to obtain G(r,z,t). The mode number n appears as the order of Bessel functions. Both in the formulations for $\psi_D^{(1)}(\vec{r},t)$ and $\psi_D^{(2)}(\vec{r},t)$ there is no need to specify a radition condition as is the case in the steady state frequency domain problem. In the limit of $t \to \infty$, proper radiation condition for the second order problem is also studied.

Following the construction of the relevant Green function for the first and second-order problems, their solutions are written down as explicit integrals of them since both Green functions satisfy homogeneous boundary conditions on the body boundary and free surface.

A method is developed for first order and second order transient diffraction velocity potentials for a vertical cylinder. No radiation conditions are assumed. The first order and second order incident waves can be of arbitrary forms.

Discussion

Ursell: What is the meaning of the time-domain solution of the diffraction problem? You assume that there is no motion when $t < 0$ and also that there is a regular incident wave train. Are these not contradictory?

Wang: We are not interested in what happens for $t < 0$. For the diffraction problem, it is the two initial values at the free surface that affect the solution for $t > 0$.

Kleinman: In effect, you exchange the radiation condition in the time haromonic case for an assumption of limiting amplitude in the time dependent case. For the circular cylinder you can construct the time dependent solution and demonstrate explicitly that the steady state time harmonic solution (which satisfies the radiation condition) is approached for large time. For arbitrary bodies the limiting amplitude principle (approach to steady state) must be proven (which is difficult) or assumed which I think is equivalent to assuming a radiation condition.

Wang: The body shape does not matter. The behavior at infinity is the same.

Kleinman: You still need to make some assumptions about the steady-state limit in the time-dependent problem. Show that transient is decaying, etc.

Sclavounos: For the general problem, we need to discretize the free surface up to a large but finite radius and assume that the disturbance vanishes at larger distances. No radiation condition is necessary since at all finite times the transient disturbance vanishes as the radius tends to infinity more rapidly than in the steady-state case.

Papanikolaou: For an arbitrary body, you cannot avoid the necessity of deriving a suitable Green function, but there is no direct need to specify a radiation condition. In fact, using the Weber transform of Bessel functions actually implies a radiation condition.

Sclavounos: We cannot reach the steady state numerically.

Papanikolaou: Does your transform satisfy a radiation condition in the steady-state limit?

Wang: We do not know what the radiation condition should be for the steady-state case. That is why we start out with the transient problem.