APPEARANCE AND DISAPPEARANCE OF IRREGULAR FREQUENCIES IN WAVE-STRUCTURE INTERACTION PROBLEMS

by

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1. Introduction

When a singularity distribution method analysis is applied to a free surface piercing body floating or fixed in a seaway, the integral equation governing the wave-structure interaction fails to produce correct solutions at an infinite number of irregular frequencies(1). This mathematical failure will be serious when hydrodynamic analysis in a higher frequency range is required, or a multi-hull or body system is investigated when the resonant wave effect(2) and the irregular frequency influence may occur together. Both the mathematical failure and physical resonant wave phenomenon induce rapid variation in the calculated hydrodynamic coefficients causing the integral formulation to be ill-conditioned. In a numerical analysis, it is difficult to distinguish which induced rapid variation in the numerical results is due to the physical resonance and which is due to the irregular frequency. Furthermore, when the irregular frequency bandwidth partially or totally overlaps that of a resonant wave mode, the latter will be affected.

Efforts have been expended to remove the irregular frequency effect, and some remedies have been proposed. These range from the imposition of a lid on the interior surface to suppress the interior resonance (3) to one which distributes an additional source on the interior free surface of the body, as in Ogilvie and Shin's(4) modified Green's function approach. By extending Ursell's(5) mathematical solution to describe the high frequency oscillatory problem, Ogilvie and Shin proposed a symmetric and an asymmetric Green's function to eliminate the irregular frequencies.

Their symmetric form is only effective for a symmetric section geometry oscillating in a symmetric mode of motion. This form was extended by Sayer(6) to the case of finite water depth, and then by Ursell(7) who presented a modified Green's function in a multipole expansion form. Martin(8) introduced a null-field equation method which was recently applied to a catamaran form(9). A comprehensive numerical investigation has been conducted by Takagi et al(10) who concluded that some elegant mathematical theories initially developed for simple shapes produce poor numerical results when applied to some more realistic or more complicated geometric structures, whereas Ogilvie and Shin's asymmetric Green's function form proved to be very effective.

Recently, Sclavounos(11) introduced a combined integral equation method to eliminate the irregular frequencies and applied this technique to 2D circular and rectangular sections.

In parallel to recent developments, extensive studies have been carried out by the authors focussing efforts on

(i) the prediction of the irregular frequencies for an arbitrarily shaped geometry since they are so far still unknown a priori to numerical computation;

(ii) a modified Green's function approach to eliminate the occurrence of irregular frequencies in mono, twin and multi-hull structures.

These investigations resulted in a prediction technique and a multiple Green's function method being proposed which are now briefly described.

2. Prediction of irregular frequencies

The predicted value at which irregular frequencies occur are usually quoted only for very simple geometries, i.e. a rectangular section, and a vertical circular cylinder. The authors have extended this information by exact analytical formulations to predict the occurrence of irregular frequencies in a triangle(12), a triangular cylinder and a sector of a circular cylinder(13). Further, by using the known expressions for a rectangular section the irregular frequencies are predicted in an arbitrary body section by introducing an equivalent rectangle assumption (12). That is,"the irregular frequencies in an arbitrarily-shaped 2D section are equal to those of an equivalent rectangle of an equal sectional area (A_s) with equivalent beam (B_e) and draft (h_e) ".

The equivalent rectangle formulation is given as

$$\omega_m = \{gk \coth (kh_e)\}^{\frac{1}{2}}, \qquad k = \frac{m\pi}{B_e}, \quad \text{for } m = 1, 2, \ldots, \qquad [1]$$

where ω_m is the mth irregular frequency and

$$B_e = (C_s)^{\alpha} B, \qquad h_e = A_s/B_e$$

where $C_s = A_s/Bh$ is the cross-section coefficient, B is the beam (on the waterline) and h is the draft measured from the midpoint of the beam. α is an empirical correction coefficient and the recommended value is $\alpha = (1+\ell nm)/8$.

A parallel technique for 3D bodies, referred to as the equivalent box approximation(13), has produced the equivalent box formulation of

$$\begin{cases} \omega_{pm} = \{gk \coth(kh_e)\}^{\frac{1}{2}} \\ k = \pi \left\{ \left(\frac{p}{L_e}\right)^2 + \left(\frac{m}{B_e}\right)^2 \right\}^{\frac{1}{2}} \quad \text{for } p=1,2,\ldots \text{ and } m=1,2,\ldots, \end{cases} \qquad [2]$$

with $L_e = \beta_1(\beta_o)^{\alpha_1} L$, $B_e = \beta_2(\beta_o)^{\alpha_2} B$, $h_e = \nabla/(L_e B_e)$

and $\alpha_1 = L^2/(L^2+B^2)$, $\alpha_2 = B^2/(L^2+B^2)$,

$$\beta_o = (C_w)^{C_o}, \quad \beta_1 = (C_c)^{C_1}, \quad \beta_2 = (C_m)^{C_2},$$

where ∇ is the displacement volume, $C_w = A_w/LB$, $C_m = A_m/Bh$, $C_c = A_c/Lh$ are the waterplane, the midsection and the central longitudinal section coefficients with A_w, A_m and A_c the relevant plane areas. The correction coefficients are assumed expressed as

$$C_o = 3/4, \quad C_1 = \{1+6\left|\frac{L-B}{L+B}\right| \ln(p)\}/8 \quad \text{and} \quad C_2 = \{1+6\left|\frac{L-B}{L+B}\right| \ln(m)\}/8 .$$

In their limiting cases, equations [1] and [2] provide the exact solutions for a rectangle or a box respectively, otherwise they represent approximate solutions of high accuracy. The irregular frequencies occurring in a multi-hulled structure may be approximately the sum of those of each individual sub-hull.

3. A multiple Green's function

By extending Ogilvie and Shin's asymmetric Green's function form (4,10,12) a procedure has been developed(14) to derive a multiple Green's function expression given by

$$G^*(p,q,\bar{p}_1,\bar{p}_2,\ldots,\bar{p}_N) = G_o(p,q) + \sum_{j=1}^{N} \tilde{G}(p,q,\bar{p}_j), \qquad [3]$$

where $p=(y,z)$, $q=(\eta,\zeta)$ are two points on the sectional contour, $\bar{p}_j = (\bar{y}_j,0)$ (for $j=1,2,\ldots,N$) is an arbitrary point on the body interior free surface. G_o is the ordinary Green function, in deep water expressed as

$$G_o = \frac{1}{2}\ln\left[\frac{(y-\eta)^2 + (z-\zeta)^2}{(y-\eta)^2 + (z+\zeta)^2}\right] + I_1 + i\, I_2, \qquad [4]$$

$$I_1 = 2\int_o^\infty \frac{e^{\mu(z+\zeta)}}{\nu-\mu} \cos\mu(y-\eta)\, d\mu,$$

$$I_2 = -2\pi e^{\nu(z+\zeta)} \cos\nu(y-\eta),$$

whilst $\tilde{G}$ is the additional Green's function written as

$$\tilde{G}(p,q,\bar{p}_j) = e^{\nu\zeta} e^{-i\nu|\eta-\bar{y}_j|} \left\{ C_{j1}\, \mathrm{Sgn}(\eta-\bar{y}_j) \left(\frac{\partial G_o}{\partial \eta}\right)_{\substack{\eta=\bar{y}_j\\ \zeta=0}} + C_{j2}\left(\frac{\partial G_o}{\partial \zeta}\right)_{\substack{\eta=\bar{y}_j\\ \zeta=0}} \right\}, \qquad [5]$$

where $\nu = \frac{\omega^2}{g}$ is the wave number.

The integer N relates to the multi-hull body with N separate hulls and $\bar{y}_j$ is located on the interior free surface of the jth sub-hull (body).

For a single-hull body N=1, when $\bar{y}_1 = 0$, the multiple Green's function form reduces to that proposed by Ogilvie and Shin.

4. Numerical Results

Reasoning and detailed numerical verification of the equivalent rectangle and box formulations are to appear in references 12 and 13. Because of its simple and explicit form the interested reader can readily obtain the irregular frequencies for an arbitrary body geometry. For example for the illustrated section at station 16 of a ship form with $B/h \simeq 2.17$, $A_s/Bh \simeq 0.7$, the predicted irregular frequencies are $\omega\sqrt{B/2g} = 1.43$ and 1.86, practically coinciding with the irregular frequencies found in numerical calculations at 1.43 and 1.89 (see Figure 1).

To verify the multiple Green's function formulation, an example of a twin hull body consisting of a rectangular hull and a triangular hull is chosen. Calculations of the hydrodynamic coefficients by the authors' program (1983) based on an ordinary source-dipole method are given in Figure 2 and by a modified program (1984) by means of the present multiple Green's function, equations [3]-[5], are shown in Figure 3.

In figure 2 the singular phenomenon occur around three frequencies, whereas in Figure 3 only the resonant wave effect remains. Because of the new formulation the other two singular behaviours due to irregular frequency now disappear.

5. Conclusions

By means of the present formulations irregular frequencies for an arbitrary body geometry can be determined by the proposed relationships and the present multiple Green's function technique is capable of removing the irregular frequency influence but allows the wave resonant effects to remain. Therefore one can simply ignore numerical calculations around these predicted irregular frequencies, or alternatively, use a conventional singularity method up to the frequency region of the first predicted irregular frequency and then continue and extend the computation by introducing the multiple Green's function method up to the frequency required.

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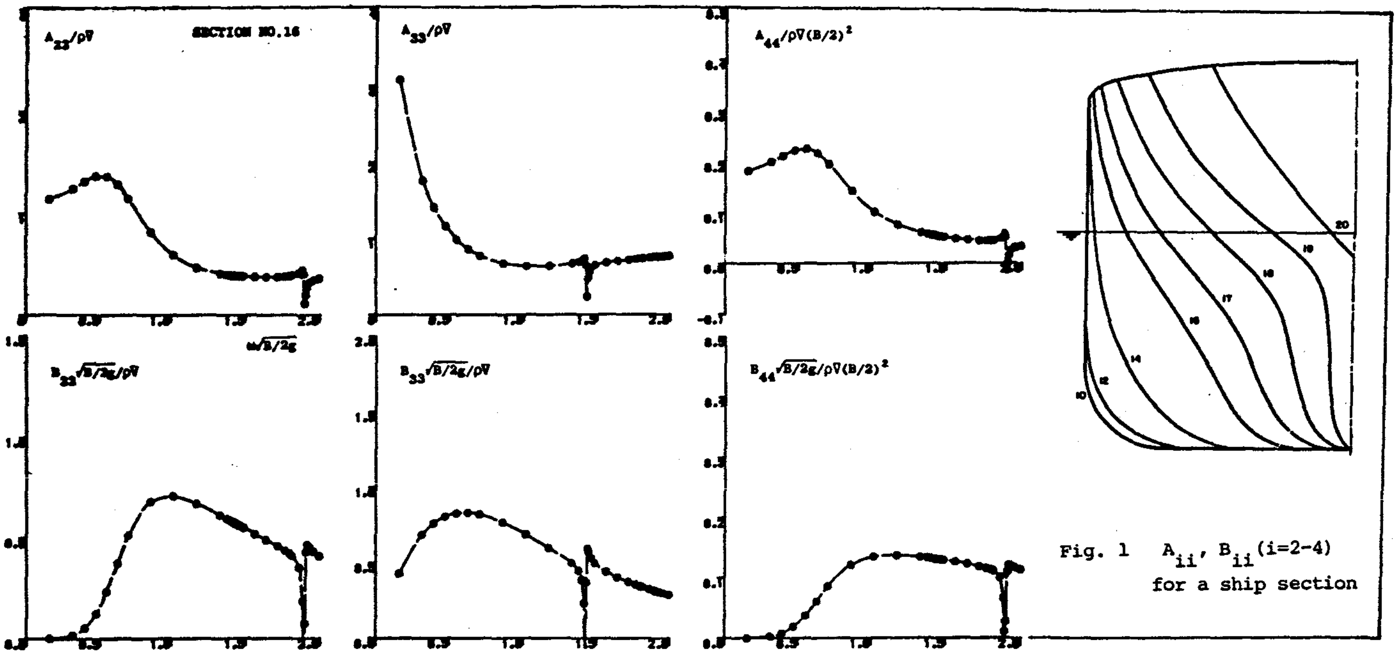

Fig. 1 A_{ii}, B_{ii} (i=2-4) for a ship section

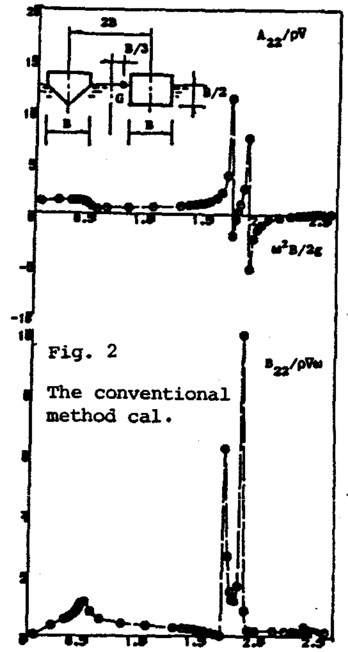

Fig. 2

The conventional method cal.

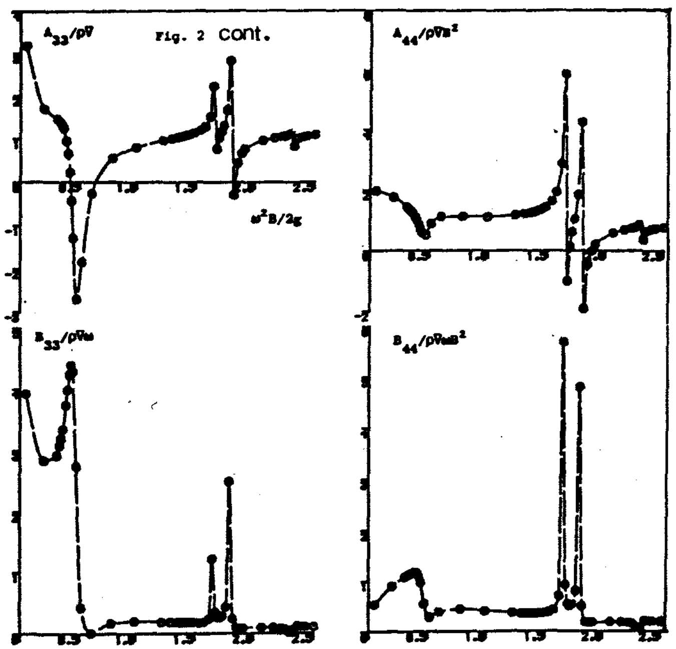

Fig. 2 cont.

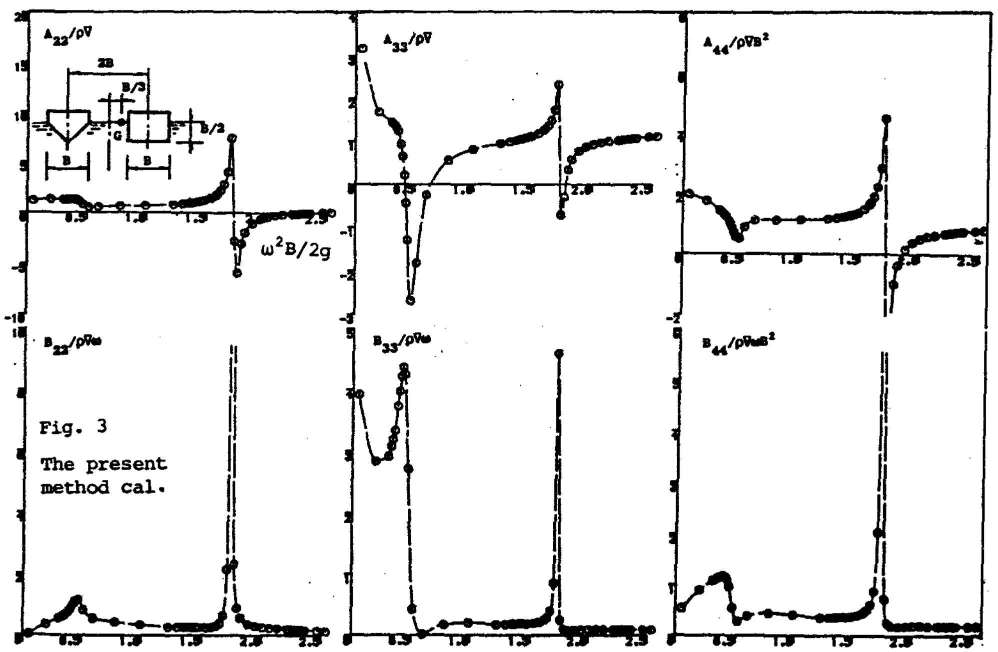

Fig. 3

The present method cal.

Discussion

Troesch: Have you done calculations for 3-D bodies with a length/beam ratio about 3-4?

Have you tried to find a method similar to Ogilvie-Shin in 3D?

X.J.Wu: Yes, I tried. In the 3D case I found that Shin's modified formulation in his PhD thesis might not work. I have extended our 2D multiple Green's function expression to the 3D case and derived several possible versions. However, I will not adopt and publish a formula before extensive studies to verify its applicability. In my experience, I found that sometimes some elegantly derived formulations may be accurate mathematically but not necessarily accurate numerically, limited in applications to some particular cases or well defined idealized shapes rather than general cases, or theoretically incorrect. In the present stage, my tentative 3D results were not stable. This may be caused by either incorrect derivation or most possibly by errors in modifying a large computer program due to the inclusion of a modified 3D Green's function.

Yeung: Would you clarify the difference between what you proposed and the approach of Ogilvie-Shin?

X.J. Wu: Ursell derived a modified Green's function for high-frequency problems. This was extended by Ogilvie and Shin for solving irregular frequencies in a single hull case. Our contribution is to extend and generalize Ogilvie and Shin's expression to derive a multiple Green's function expression capable of removing irregular frequencies in a mono-, twin- or multi-hull body (system). In our formulation, each surface-piercing subhull has an additional singularity located somewhere on the interior free surface.

Evans: If it is important to identify the irregular frequencies accurately presumably one could use the powerful variational techniques for sloshing frequencies in arbitrary containers. Although the condition is $\phi = 0$ here rather than $\phi_n = 0$ the methods should go over.

X.J. Wu: Of course, many methods can be used to yield irregular frequency values. But any of them may take considerable computing time and may not produce more information. This may be why in practical computation, no one has so far adopted a separate program to determine the irregular frequencies and then performed the hydrodynamic analysis using a singularity-method program. Our formulas, however, give accurate approximations of the irregular frequency values as well as explicit rules for the distribution and appearance of these irregular frequency influences (see Ref. 12 and 13) with very little computing effort. In fact, you can finish the calculation by calculator in a few minutes. These formulas have been implemented in my programs and proved to be accurate and efficient in practical ship and offshore applications. Generally speaking, alternative methods can be applied without the trouble of irregular frequency, such as the finite element method, Bai and Yeung's hybrid method, Ursell's multipole expansion method, etc. For deep water marine applicatons involving complicated body geometry, especially for 3D cases, singularity distribution techniques may be more suitable. Furthermore, I believe that the tendency of the further development of the 2D simplification is towards a 3D-2D combination concept (Ref. X.J. Wu, 7th Intl. Conf. Boundary Element Method, Sept. 1985). Since the most popular method in 3D analysis is the singularity approach for compatibility, it may be appropriate to use a singularity technique in the relevant 2D case as well.