SECOND ORDER TIME HARMONIC FORCES ON BODIES IN WAVES

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While the calculation of second order forces on bodies in waves has been a subject attracting interest for some years, many of the results published so far have been concerned with the time independent components, or time harmonic components neglecting all or part of the contribution due to the second order velocity potential. Further, some of the few published 3D solutions of forces due to the second order potential are based on dubious theoretical grounds and have been the source of much controversy.

An analytical second order solution of the time harmonic forces on a fixed surface piercing cylinder in a regular wave has recently been given by Eatock Taylor and Hung [1]. Here, particular attention will be paid to the extension of this work to arbitrary bodies free to move in a bichromatic sea, where forces of both sum and difference frequency components exist.

A suitable method of solving for the forces resulting from second order velocity potential is that due to Lighthill [3]. Essentially, we make use of Green's second identity in transforming a body integral of the unknown second order diffracted potential to integrals on the body and free surface, noting that these integrals have integrands that can be expressed as functions of the known first order solutions. The forces may therefore be obtained without solving for the second order diffracted potential at all.

Although theoretically the solution then appears relatively trouble free to obtain, in practice a major obstacle arises in ensuring the convergence of the free surface infinite integral. Other problems also exist. For instance, when the body is free to move, one of the integrals on the body surface has an integrand that contains the double spatial derivative of the first order velocity potential. In both of these cases, special attention must be paid to the development of fast and accurate numerical solutions.

The numerical implementation of our second order solution is based on a post-processor to DYHANA, a suite of diffraction programs [2] developed in UCL. Since there is a choice in choosing between two hybrid element methods (Boundary Integral Element (BIE) method and Boundary Series Element (BSE) method) for solving the first order diffraction problem within DYHANA, one of the requirements of the post-processor is that it must be compatible with data generated by either method.

To evaluate the free surface integral, for example, different treatments are required for the BSE and BIE data. In the BSE idealization, the body in question is always surrounded by a 3D finite element region with a vertical cylindrical exterior boundary. In this situation, the integral is performed by 2D quadrature within the boundary, but it is far more efficient and accurate to perform the surface integration outside the boundary by first integrating explicitly in the azimuth angle, followed by numerical quadrature along a radial line. It has also been found advantageous to use the procedure [1] of representing the semi-infinite radial integral by two parts; quadrature within a finite range plus a residual term representing the integral to infinity. This last integral to infinity is evaluated by employing only the leading order terms of the large argument asymptotic expansion of the integrand, which may then be reduced to a Fresnel integral and calculated explicitly.

In the BIE method, an additional step is required for the integration on the free surface, since the body is enclosed by 3D·box shaped finite element regions. To evaluate the free surface integral, a fictitious vertical cylindrical boundary is constructed enclosing all the finite element regions. The first order velocity potential is then evaluated on this cylindrical boundary. By use of the orthogonality of the eigenseries in the azimuth and depth, the coefficients to the eigenseries of the first order velocity potential are obtained and thus the surface integral outside the cylindrical boundary can be performed as in the BSE method. Within the cylindrical boundary, the free surface integration is obtained by numerical quadrature.

It has been found that with this application of the BIE method, the fictitious cylindrical boundary must not be too close to the finite element region. This is due to the discontinuity in the integral when the source point is moved onto the surface on which the integral equation is written. It is possible to avoid this problem by making modifications to the Green function, as shown by Noblesse [4], but this has not yet been implemented in our solution.

Another potential difficulty which has concerned us is accurate evaluation of the body integral term containing the double spatial derivative of the first order velocity potential. This contribution to the second order force arises from one of the correction terms in the Taylor series expansion of the body boundary condition. We found that despite the employment of quadratic finite elements, convergence of the results was poor because of the error in these second derivatives. By use of Stoke's theorem, however, it is possible to express the integral in an alternative form, which involves a modified body integral and an integral around the waterline and intersection with the sea bed. In this way, convergent results for the force have been obtained without the requirement of a very fine mesh on the body surface.

Results given in [1] show that in regular waves the contribution of the second order potential to the double frequency second order drift force can be large. In particular (contrary to some suggestions published elsewhere), the force may not be reasonably approximated by neglecting the term due to the free surface integral. Also illustrated by those results is the phenomenon of the double frequency force converging more slowly than the first order force with increase in water depth and submergence. It is evident that if the difference frequency is small, the same behaviour can be expected in the sum frequency force component in bichromatic waves.

The behaviour of the low frequency component, however, is less obvious. Although the second order velocity potential does not contribute to the mean horizontal forces, the influence of components at non-trivial difference frequencies is of great practical importance in the assessment of low frequency responses of compliant systems. An exact solution, however, does not appear to have been published previously for 3D bodies. From our preliminary results, we find that while approximations can give good agreement to the forces when the average frequency is high and the difference frequency is low, the contribution due to the second order velocity potential can be very significant in other circumstances.

In some situations, it has also been found that when only part of the second order potential is used in the calculation, (for example the second order incident potential has been included by Standing et. al. [5]) serious errors in the forces may arise.

Nondimensional results are given below for the surge quadratic transfer function $H(\omega,\omega+\Delta\omega)$ of a hemisphere of radius a in water of depth 3a. The BIE idealization using a mesh of 20 quadratic elements is also shown. It may be seen that the various approximations either over or underestimate the results based on the complete second order potential, depending on the frequency components ω and $\Delta\omega$.

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NONDIMENSIONAL LOW FREQUENCY QUADRATIC TRANSFER FUNCTION OF A HEMISPHERE IN SURGE

$\frac{\omega\sqrt{a}}{\sqrt{g}}$ \ $\frac{(\omega+\Delta\omega)\sqrt{a}}{\sqrt{g}}$	0.5	0.7	0.9	1.1
		FIXED		
0.3	.023 .023 .023 .024 -.169-.014 .064 .050 .171 .027 .068 .055	.056 .056 .052 .062 -.359-.066 .076 .034 .363 .086 .092 .071	.124 .124 .102 .151 -.528 .034 .274 .171 .542 .129 .292 .231	.140 .140 .055 .231 -.580 .471 .780 .592 .596 .491 .782 .636
0.5	.049 .000 .049	.097 .097 .097 .097 -.182-.093-.049-.062 .207 .134 .109 .115	.179 .179 .176 .181 -.343-.112-.010-.055 .387 .211 .176 .189	.212 .212 .196 .233 -.414 .110 .284 .150 .465 .239 .345 .277
0.7		.164 .000 .164	.262 .262 .262 .258 -.157-.074-.035-.045 .305 .272 .264 .262	.313 .313 .310 .301 -.248 .015 .114 .054 .399 .313 .330 .305
0.9			.375 .000 .375	.437 .437 .437 .425 -.112-.014 .030 .020 .451 .437 .438 .426
		FREELY FLOATING		
0.3	.000 .000 .000 .000 -.083 .072 .130 .125 .083 .072 .130 .125	.002 .002-.001-.001 -.277 .016 .126 .100 .277 .016 .126 .100	.083 .083 .009-.011 -.781-.219 .141 .039 .785 .235 .141 .041	.817 .817-.219 .055 -.759 .292 .415 .105 1.115 .867 .469 .118
0.5	.000 .000 .000	.001 .001 .001 .001 -.140-.051-.023-.037 .140 .051 .023 .037	.055 .055 .036 .034 -.492-.260-.128-.190 .495 .266 .133 .192	.573 .573 .157 .264 -.486 .039 .171-.043 .752 .575 .233 .267
0.7		.002 .000 .002	.047 .047 .042 .042 -.307-.223-.182-.213 .310 .228 .187 .218	.513 .513 .343 .385 -.312-.049 .031-.085 .600 .515 .344 .395
0.9			.078 .000 .078	.513 .513 .460 .478 -.057 .042 .082 .030 .516 .514 .467 .479

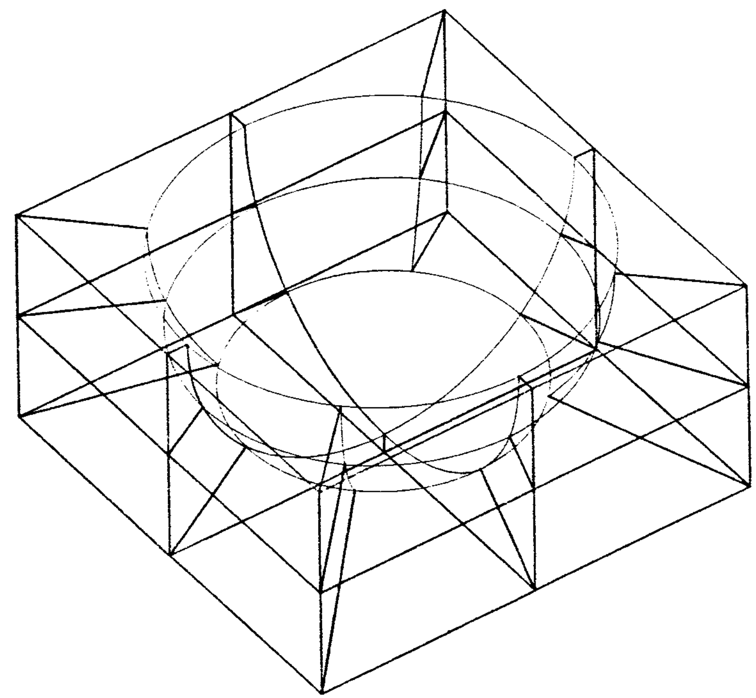
Results are non-dimensionalised by $\rho g a A^2$ where A is the incident wave amplitude

Within each box, the results are:

Column 1: excludes contributions due to second order potential
2: includes contribution due to second order incident potential
3: excludes free surface integral
4: complete

Line 1: in phase component
2: out of phase component
3: amplitude

BIE MESH OF A HEMISPHERE



Authors' further comments

Hung: It is our view that it should also be possible to employ our present method to calculate the second order velocity potential without approximations. The key lies in the definition of the assist potential (the radiation potential $\psi^{(1)}$ as denoted in [1]). For example, if we require the second order velocity potential on a particular facet on the body, we shall define $\psi^{(1)}$ by specifying $\frac{\partial\psi^{(1)}}{\partial n} = \bar{1}$ on that facet, and $\frac{\partial\psi^{(1)}}{\partial n} = 0$ on the others. We then carry on as usual to evaluate the second order force, which, of course is the force on the facet alone. Subtracting the second order forces due to the first order potential and motions from this force, and dividing it by $-i\rho\omega A$ where ω is the second-order wave frequency and A is the facet area), the second-order potential is obtained. Compared with direct methods for the calculation of the second-order velocity potential, this new route is likely to be far more cost effective when the potential is needed only at a few locations.