Numerical Problems of First Order Diffraction Theory

E.R. Jefferys, Conoco (U.K.) Limited

First order three dimensional diffraction theory is embodied in many programs which can calculate hydrodynamic coefficients for bodies of arbitrary geometry. In the hands of their creators, these programs usually perform well and generate plausible coefficients which compare well with analytic predictions for very simple shapes. Resulting motions correlate well with experiment, particularly for voluminous, convex bodies such as barges and wave energy devices where viscous effects are not important.

This does not seem to be the case for semi-submersible and TLP structures. A survey for the ISSC, republished in Ref. 1 demonstrated significant and disturbing variability between the hydrodynamic coeff-icients, motions and forces predicted for an example TLP by 17 diffraction programs. Figure 1 shows typical results for surge added mass, damping and exciting force amplitude. The values, particularly of added mass, seemed to change significantly with the level of discretisation but not all partic-ipants supplied their panel model.

The ITTC Ocean Engineering committee has compared experimental results, strip theory and diffraction results for an example vessel; the three diffraction programs also seemed to overestimate added mass (ref. 2), leading to longer natural periods than Morison or experiment predict. However not all the discrepancy seems explicable by modelling errors (Priv. comm. R.G. Standing) of the type discussed below.

Further investigation within Conoco has yielded more detailed information; this note compares hydrodynamic coefficients from four different programs, each using a significantly different discretisation of another TLP. There are three panel representations of 240, 484 and 992 facets and one which uses a quadratic finite element model of the fluid near the structure.

Figure 2 shows normalised values of added mass and damping, modulus of exciting force and response in surge. The more 'accurate' representations generally yeild smaller values of added damping and exciting force. All the hydrodynamic coefficients are reasonably consistent and display similar features; note in particular the strong 'slop' mode shown by the added mass and damping curves. Added mass in all modes decreases with increasing number of facets, N and the quadratic program displays a still lower added mass. The added mass at infinite frequency is most significantly affected and the variability about $M(\infty)$, the mean value is more consistent. If the mean value of the added mass is removed, the behaviour of the frequency dependent part is similar to that of the added damping. Note that the two highest frequency points of the 240 panel curve were computed with a 680 panel representation, so the added mass drops to just below the 484 panel curve.

Table 1 summarises the added mass values in all degrees of freedom at low frequency. An 'error' has been evaluated based on the assumption that the quadratic results are correct; errors are fairly consistent from one mode to the next. Roll and pitch are pontoon dominated; this may account for their rather bigger discrepancies. Heave added mass is a very weak function of frequency as all the relevant parts of the structure are well submerged; it is also easy to check by naive 2D theory on the assumption that interactions will be weak. Assuming an appropriate added mass coefficient for the pontoon (Sarpkaya and Isaacson yields M = 0.96 V) and using the 'capping hemisphere' approximation for the column ends gives a heave added mass of 1.03 times that of the quadratic program which lends some credibility to the idea that it is right and the others are wrong! Similar calculations can be made for added mass in surge.

Figure 3 shows a log plot of heave added mass 'error' versus number of panels; one of the programs was run with 240 and 680 panels at high frequency. Obviously it would be unwise to fit a striaght line through the points and extrapolate but spot checks on free floating heave natural period for another TLP model indicate that interpolation is reliable.

In two dimensions, it is found that 16 to 18 'facets' are needed to model a submerged pontoon accurately. If the corresponding panel side of 2.5 m were chosen, the TLP would be represented by some 2800 facets. This is three times the number used in the most detailed panel model and would increase the computing cost by a factor of between 9 and 27 depending on the balance between forumulation and solution time in the program. Consultants often say that they choose their panel size to be less than one seventh of a wavelength; this is why the 240 panel study was re-run with 680 panels at high frequency. This rule of thumb seems difficult to justify since the differences between the programs are relatively independent of frequency.

Displacements in the inertia dominated modes are predicted consistently by all the programs since the larger added masses tend to be balanced by the larger exciting forces. At all frequencies above their (slow) resonant periods, semi-submersibles respond in an inertia dominated fashion so these errors may not be too important.

In the spring dominated modes whose displacements determine the tether tensions, the motions and tensions follow the exciting forces. On a design wave basis the difference between the highest and lowest tension RAO's translates into some 4500 tons of pretension hence 4500 tons less payload or roughly 13500 tons more displacement in a 60000 tonne vessel. A stochastic design tends to smooth out the differences but they are still of the order of 20% of pretension.

This is not an advertisement specifically for finite element methods. Higher order boundary element methods may do just as good a job. And the developer of the quadratic program is as yet unwilling to assert categorically that everyone else is wrong.

Some questions are prompted by these results.

1) Do panel programs always provide an upper bound for added mass? Do finite element programs follow this 'rule'?

2) The Greens function is very simple at infinite frequency. Can we calculate $M(\infty)$ accurately and adjust everything else? What about added damping? Can we adjust the 'inertial' part of the exciting force?

3) Are zeroth order panel methods guaranteed to converge monotonically to correct answers as $N \rightarrow \infty$?

Finally, it is important to emphasize that viscous forces make an important contribution to the TLP response in large waves; inclusion of Morison drag has a significant effect on the tether tension RAO's and it seems likely that the relatively unimpressive correlation between theory and model test is caused by other viscous effects which are not yet fully understood (ref. 3). These differences are certainly significant from the 'scientific' point of view and the margins they imply add to the cost of platforms by increasing pretension margins for 'prediction error'. Clearly, first order wave loading is by no means a solved problem, even in the diffraction regime!

References

1) Eatock Taylor, R. and Jefferys, E.R., 'Variability of Hydrodynamic Load Predictions for a Tension Leg Platform', Ocean Engineering, Vol. 13, No. 5, pp 449-490, 1986.

2) Report of the 16th ITTC Ocean Engineering Committee.

3) Chaplin, J.R., 'Nonlinear Forces on a Horizontal Cylinder Beneath Waves', J. Fluid Mech. 1984, Vol. 147, 449.

Program	Surge	Sway	Heave	Roll	Pitch	Yaw
	DM/M_o %	DM/M_o %	DM/M_o %	DM/M_o %	DM/M_o %	DM/M_o %
240	51.9	50.26	54.6	61.5	60.5	48.1
484	30.9	34.80	37.0	46.1	39.2	32.6
992	25.4	25.9	24.4	35.2	32.5	23.6
Q	-	-	-	-	-	-

Table 1

Added Masses Variations

at high frequency

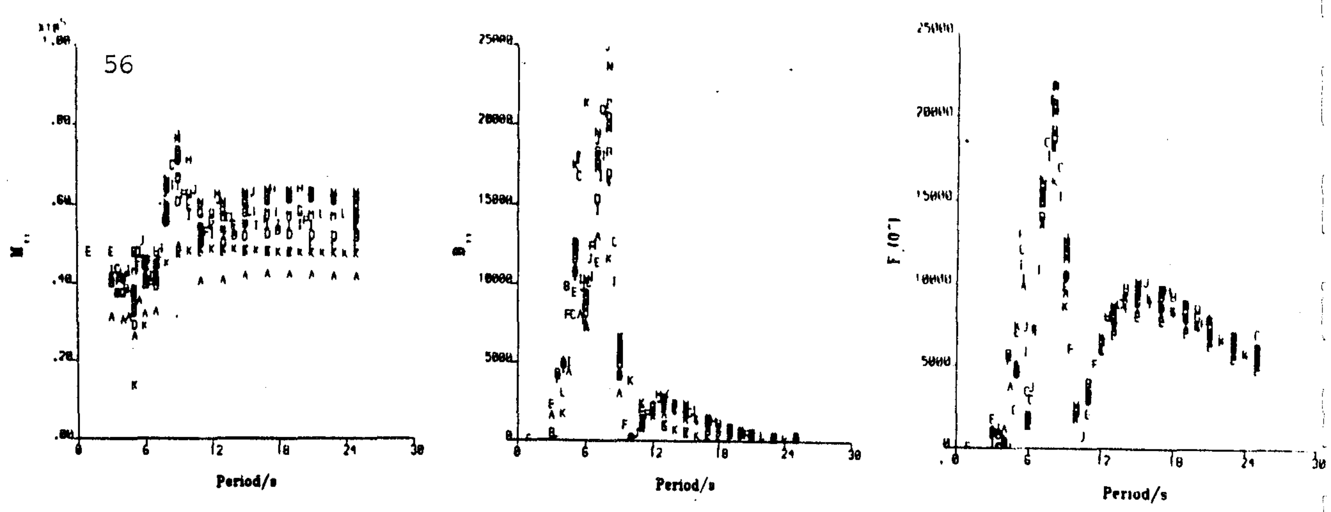

Fig. 1 ISSC TLP Results

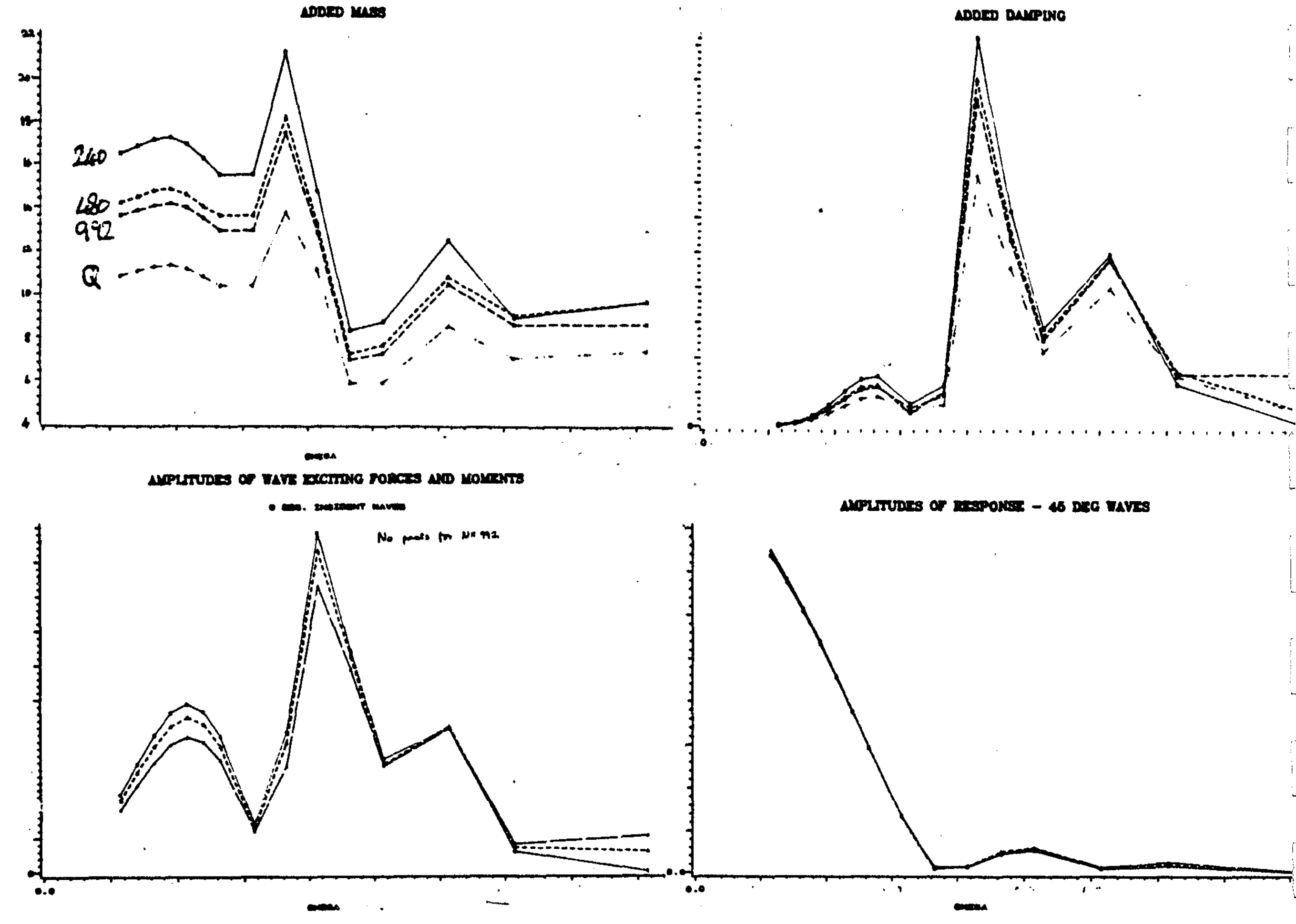

Fig 2. Normalised Added Mass and Damping, Exciting Force and Response

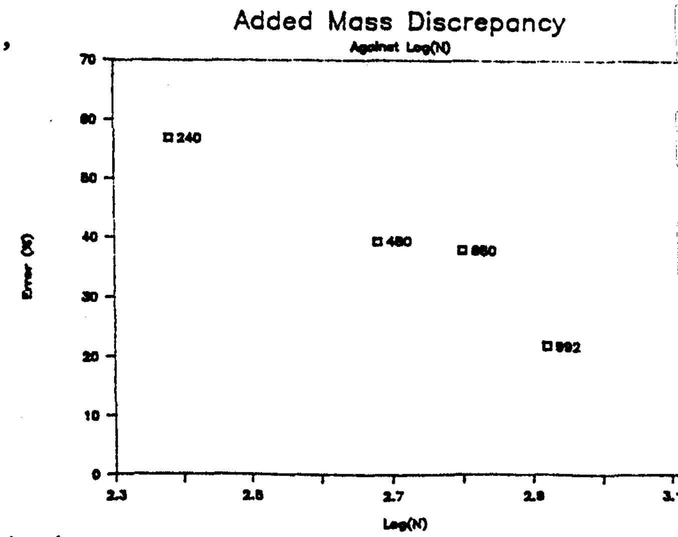

Fig 3. Heave Added Mass Variation

Discussion

Sclavounos: Dr Jefferys posed a question in his abstract about whether plane/piecewise-constant panel methods for radiation-diffraction problems converge in the limit as the number of panels N tends to infinity. I believe it is possible to show, and experience suggests it, that this discretization does indeed converge (in some cases quadratically) if the singular components of the Green function are handled consistently and carefully. The Green function is singular like 1/r near the location of the source and its image above the free surface. An additional singularity of logarithmic nature is also present at the source image. All three singularities need to be integrated analytically over the panels, should they lie close to the source or its image, to ensure the convergence of the velocity potential pointwise on the body surface and on the water line. The remaining regular wave part can be integrated by quadrature. We have found that centroid integration is sufficient.

Calisal: The dependence of the solution on the number of panels could be reduced by using, "patches" as was recently done by Dr B. Okan at BMT. This procedure shows that more efficient computations and higher accuracy can be obtained by insisting on the continuous distribution of "sources". It will be very interesting to see how this procedure works and compares for your geometry.