THE HEAVE ADDED-MASS AND DAMPING COEFFICIENTS OF A SEMI-SUBMERGED TORUS

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The situation being considered is that of a semi-submerged torus undergoing a forced periodic vertical motion. The work is an extension of the work done by A. Hulme (1985) on the submerged torus and many of his results are used here.

The method of solution was pioneered by Ursell (1949) in considering the heaving motion of a two dimensional circular cylinder. Havelock (1955) solved the problem of the heaving hemisphere by the same method and recently his solution was greatly improved by Hulme (1982).

A set of functions ϕ_n are constructed in a toroidal coordinate system appropriate for the problem. These satisfy the required differential equation, in this case $\nabla^2\phi_n = 0$, and also the free surface boundary condition which is $K\phi_n + \frac{\partial\phi_n}{\partial y} = 0$ on $y = 0$, where $K = \omega^2/g$ and ω is the radian frequency of the heave motion. The functions ϕ_n are also required to be wave-free, which in three dimensions implies that they radiate no energy to infinity. We call these functions ϕ_n wave-free potentials. Thus

$$\phi_n = (\cosh\sigma - \cos\psi)^{\frac{1}{2}}\,[(Ka\sin n\psi + n\cos n\psi)P_{n-\frac{1}{2}}(\cosh\sigma) - \tfrac{1}{2}(n-\tfrac{1}{2})\cos(n-1)\psi P_{n-\frac{3}{2}}(\cosh\sigma)$$
$$-\tfrac{1}{2}(n+\tfrac{1}{2})\cos(n+1)\psi P_{n+\frac{1}{2}}(\cosh\sigma)].$$

Here σ and ψ are toroidal coordinates, $P_{n-\frac{1}{2}}(\cosh\sigma)$ is a Legendre function and a is the radius of the ring $\sigma = \infty$.

Next the velocity potential ϕ is represented as $\phi = d\Phi + \sum_{n=1}^{\infty} d_n\phi_n$ where Φ is a ring source and d, d_n are unknown constants. An expansion in toroidal coordinates for a ring source given by Hulme (1981) is used. ϕ now satisfies all the conditions of the problem except that on the body surface and from this condition an infinite system of linear equations for the unknown coefficients d_n, can be derived.

In practice the infinite systems of equations must be truncated to, say, an $N \times N$ system and the unknowns $d, d_1, d_2, \ldots, d_{N-1}$ determined. This will then provide an approximation to ϕ, namely $\phi = d\Phi + \sum_{n=1}^{N-1} d_n\phi_n$.

The non-dimensional added-mass and damping coefficients A and B are given by

$$A + iB = -\frac{2}{Ua\pi}\,\frac{\sinh^5\sigma_o}{\cosh\sigma_o}\int_0^{\pi}\phi(\sigma_o,\psi)\,\frac{\sin\psi}{(\cosh\sigma_o - \cos\psi)^3}\,d\psi.$$

Here U is the time-independent velocity of the torus and $\sigma = \sigma_0$ is the torus surface.

Much of the work involved is concerned with writing efficient computer programs to calculate the values of special functions. The functions involved here are $Q_{n-\frac{1}{2}}(\cosh\sigma)$, $P_{n-\frac{1}{2}}(\cosh\sigma)$ together with their derivatives with respect to both σ and n.

References

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