A COMPLEX-VALUED INTEGRAL METHOD FOR FREE SURFACES WITH INTERSECTING BODIES

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We report continuing analytical and numerical studies of two-dimensional gravity waves with and without protruding surfaces. The complex boundary integral method using a least-squares solution to the overdetermined system appears to be more economical and robust than the Greens function boundary integral approach or to previous methods using the Cauchy integral theorem (Vinje and Brevig, 1980). Test cases show that previous algorithms are more susceptible to a numerical instability when nodes are not evenly spaced or surface curvature is large.

A boundary integral formulation is also developed directly for velocity to avoid the problem of numerical differentiation in the formulations for velocity potential. The velocity formulation also has the advantage of being higher order (piecewise linear velocity rather than piecewise potential and hence piecewise constant velocity) without any additional computational burden. The velocity potential is analytic and therefore all spatial derivatives of the velocity potential (including the conjugate velocity) are analytic as well. The Cauchy-Riemann condition ensure irrotationality and continuity conditions. This is similar to Vanden-Broeck (1980) method for steady flows. Advantages and disadvantages of this formulation are discussed for flows with and without free surfaces. One difficulty in the velocity formulation is replacing the Bernoulli equation for the dynamic free surface boundary condition. We replace this by a form of the Euler equation along the free surface,

$$Dv_s/Dt = -g\ dy/ds - v_n D\theta/Dt,$$

where v_s and v_n are the velocities tangential and normal to the surface and $D\theta/Dt$ is the local angular velocity of the free surface. We show comparisons on steady and wave problems to show the stability and convergence of the velocity formulation in comparison to the velocity potential formulation. We find that the methods are approximately equally accurate depending on the problem and resolution. Apparently, the higher-order representation of the velocity formulation is not that advantageous as shown by other panel method studies. We examine the use of other singularities to try to improve the method.

Flows with free vortex singularities are more easily handled by the velocity formulation as the singularities become simple poles rather than branch cuts. We show some examples with vortex elements close to a cylinder in an infinite field with and without overdetermined systems using both methods. The velocity formulation proves to be more accurate although oscillations are more severe next to the singularity. Another major drawback to the velocity formulation is that it cannot easily be extended to three dimensions. We solve for both velocity components in two dimensions with a single complex equation. The three-dimensional problem would require three scalar equations.

There has been concern about the contact line, where a free surface intersects a solid body (Dommeruth and Yue, 1986; Greenhow and Lin, 1983; Roberts, 1986). These studies indicate the existence of a weak singularity at that point. While velocity gradients can be large there, we can show for a number of cases that the velocity is completely regular. For the case of the "impulsively accelerated wavemaker" (actually a wavemaker with a step function in acceleration) Chwang (1983) showed the existence of a singularity but he stated this was outside the computational domain. Here, we show that at the contact line the flow is nonsingular. Also in sloshing flows in a container, we show by simple analytic continuation arguments that the flow at the contact line is analytic.

We also argue that the impulsive velocity wavemaker is not a well-posed problem not only because infinite forces are required to start the wavemaker but because the boundary conditions are not compatible with the initial conditions for a field operator with no time derivative. In this sense we find a contradiction in the analyses of those cited above. Specifically, the requirement that the velocity potential on the free surface be zero to leading in an expansion for small time cannot be obtained systematically.

We show how to modify this problem by introducing slight compressibility or by making it a nearly-impulsive wavemaker and show the difficulties of solving either modified problem.

It then appears that most of the computational difficulty at the contact line comes from the well-documented problem that boundary integral methods have with sharp corners. We show with some test problems with sharp corners that the least squares method we invoke shows improvement in convergence and stability in contours with and without sharp contours. This is especially true when the nodes are unevenly placed. Further studies using time marching to study periodic travelling waves show that the least squares method shows improved accuracy over the method of Vinje and Brevig. However, we have not noticed numerical instabilities (zigzag of the free surface) using either method even when crude explicit time marching is used in problems without sharp corners. This may be due to the fact that the kernel of the Cauchy operator used in the complex-valued formulation is essentially similar to the dipole method used by Baker, et al. (1982) who found similar stability properties. This does not appear to be the case when the Cauchy integral theorem is used for contours with sharp contours caused by solid body intersections of the free surface. The least squares method appears to suppress these instabilities. In any case, errors in conservation of energy are reduced by one to two orders of magnitude by the least squares method when spatial and temporal resolutions are the same.

One difficulty in extending this algorithm to larger (three dimensional) problems is that the least squares solution using the Householder transformation requires order N cubed operations, while other techniques use iteration on the Fredholm equations of the second kind "type" of equations to solve a determinate linear system in order N squared operations. We also the least squares problem iteratively, minimizing the residual using the conjugate gradient method (Fletcher and Reeves, 1964), which is similar, but more powerful than the method of steepest descent. Both the residual and the gradient of the residual can be computed in order N squared operations. We present results showing convergence in few iterations (typically two) for the overdetermined or either of the determinate systems solved previously. We find that the iterative procedure is faster for very small N, especially for the overdetermined system.

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After having been occupied with other problems than those related to numerical simulation of steep or breaking waves for some time it is a bit disappointing to listen to the presentations given here to day. I have the feeling that the brilliant young scientist working in this field have reached some form of branching point, where they take off in different directions without seeming to have any common target, all of them producing new computer programs which seem to differ slightly from another.

After Cokelet and Longuet-Higgins produced their breaking wave program in 1976 and Per Brevig wrote his program for simulation of breaking wave for his master-thesis in 1979, which (from my biased point of view) was an improvement, the development showed considerable promise. I will specially point to the work done at MIT, represented by the Lin, Newman and Yue paper of 1984 and to the in depth reseach that has been going on at the University of Bristol. All in all, one had the feeling that within reasonable time one might have tools available for analysis of practical problems. The recent development, as presented to day, does not seem to meet this.

Since 1976 a significant number of breaking wave programs have been written; some just for duplicating previous programs, others for testing out new formulations and different numerical methods and some aiming at extending the group of problems to be handled. To my knowledge there must of the order of ten to twenty different programs around, and probably just as many that I am not aware of. We must have had enough random shots at the target now! Is it not time to draw some conclusions about methods that work and methods that do not, and rather concentrate on extending the group of problems which we are able to solve?

I hope, as a bystander, to see a development in the direction drawn up by Lin and his coauthors in the near future, or at least, a thorough analysis of applicability of the different methods from a stability, accuracy and computational point of view to point out the ones which have the necessary potential.

I hope this discussion has not been regarded as a turning down of whatever is going on in this field to day; it is meant as a reminder that the goal of research is to extend our knowledge and potential to understand nature, and that this applies to breaking waves as well.