THE VERTICAL WAVE DRIFT FORCE ON FLOATING BODIES

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A submerged body interacting with surface waves experiences a vertical "suction" drift force which tends to decrease its submergence. The same drift force may cause a large vertical mean displacement of a floating body with small waterplane area, a semi-submersible offshore platform or a twin-hull ship for example. Perhaps more important in practice is the second-order slowly-varying in time vertical force on small-waterplane-area floating bodies. Its modulus may be smaller than the mean vertical force, but is likely to cause large vertical excursions of the body if its frequency coincides with the heave natural frequency. A first step towards the prediction of these second-order wave effects is the evaluation of the mean vertical drift force. The present study analyses this force by applying the momentum conservation principle. A computationally efficient interpretation of existing results for submerged bodies is suggested, and new expressions are derived for surface-piercing bodies that do not require the evaluation of the flow velocity on the body boundary.

An expression for the mean vertical heave force on a body undergoing a small-amplitude time-harmonic oscillation under an otherwise calm free surface has been derived by Kochin (1940) [cf. Wehausen and Laitone (1960), eq. 19.18] by applying the momentum conservation principle and utilizing his "Kochin functions". Lee and Newman (1971) extended Kochin's results to include the force component due to the interaction of the body- and incident-wave disturbances and derived expressions for the mean drift roll and pitch moments. The analysis of Kochin and Lee and Newman cannot be applied to surface-piercing bodies because the momentum-flux integration over the free surface is "interrupted" by the body waterplane area. The prohibitive numerical effort required for its numerical evaluation led to the derivation of drift force and moment expressions by Pinkster and Oortmerssen (1977) based on the direct integration of the second-order hydrodynamic pressure force over the body. They require the evaluation of the flow velocity on its wetted surface and a careful interpretation of its position in order to include all second-order effects. A state-of-the-art survey of second-order wave effects on floating bodies was recently conducted by Ogilvie (1983).

Submerged Bodies

Denote by φ_0 and φ_B the linearized complex incident and body velocity potentials, the latter representing both the radiation and diffraction wave disturbances. The undisturbed free surface coincides with the $z = 0$ plane, with the $z-$axis pointing upwards. The vertical drift force acting on a submerged stationary body, as derived by Lee and Newman (1971), is defined by

$$F = \Re(\mathcal{W}_0 + \mathcal{W}_B), \tag{1}$$

where

$$\mathcal{W}_0 = -\frac{1}{2} i\rho g A\,\omega\, H_B(\nu, \beta, \nu, \pi + \beta), \tag{2}$$

$$\mathcal{W}_B = \frac{\rho}{16\pi^2}\int_0^{2\pi} d\vartheta \int_0^\infty dk\, k\, \frac{k+\nu}{k-\nu}\left|H_B(\nu,\beta,k,\vartheta)\right|^2, \tag{3}$$

where A is the incident-wave amplitude, and

$$H_B(\nu,\beta,k,\vartheta) = \iint_{S_B} \left\{\frac{\partial\varphi_B(\nu,\beta,\xi)}{\partial n_\xi} - \varphi_B(\nu,\beta,\xi)\frac{\partial}{\partial n_\xi}\right\} e^{k\varsigma - ik\xi\cos\vartheta - ik\eta\sin\vartheta} d\xi. \tag{4}$$

The integration in (4) is carried out over the mean position of the body wetted surface with the unit normal vector $\vec{n}$ pointing out of the fluid domain, $\nu = \omega^2/g$, β is the incident-wave angle of propagation relative to the x–axis of a body-mounted frame, and k and ϑ are a dummy wavenumber and polar angle respectively involved in the Fourier integration of expression (3). The implicit dependence of the velocity potential φ_B on ν and β requires the solution of the radiation and diffraction boundary-value problems, but this numerical task needs to be carried out once in conjunction with expression (3) where the indicated Fourier integration is carried out with respect to k and ϑ rather than ν and β. This property renders (3) quite useful for computation, and does not seem to have been appreciated in the literature. An alternative form of (3), convenient to use with panel codes, follows if the Fourier integration and body-surface integrals in the Kochin function are interchanged, and the definition of the wave source potential $G(\mathbf{x};\xi)$ at the field point $\mathbf{x}$ due to a pulsating point source of strength -4π at ξ is invoked to obtain

$$\mathcal{W}_B = \frac{\rho}{16\pi^2}\iint_{S_B} d\mathbf{x}\left(\frac{\partial\varphi_B^*}{\partial n_x} - \varphi_B^*\frac{\partial}{\partial n_x}\right)\frac{\partial}{\partial z}\iint_{S_B} d\xi\left(\frac{\partial\varphi_B}{\partial n_\xi} - \varphi_B\frac{\partial}{\partial n_\xi}\right)(G - 1/r)d\xi, \tag{5}$$

where $r = |\mathbf{x} - \xi|$. The function $G - 1/r$ is analytic in the entire fluid domain, thus over the surface of any submerged body, and all integrals indicated in (5) can be carried out by quadrature. Moreover, its values and derivatives can be evaluated concurrently with the kernels G and $\partial G/\partial n$ in boundary-element integral equations and strored for subsequent use in (5).

Expression (3), on the other hand, is quite convenient to use for the derivation of the mean vertical force on submerged elementary singularities often utilized to approximate the wave flow due to deeply submerged bodies in long waves or due to slender bodies. Using (4) to evaluate the Kochin function for a submerged pulsating unit source and vertical dipole, we obtain the vertical drift forces

$$F_S = -\frac{\rho}{8\pi}|\sigma|^2\nu^2\frac{d}{dx}\left[\tfrac{1}{x} + 2e^{-x}Ei(x)\right]_{x=2\nu d}, \tag{6}$$

$$F_D = -\frac{\rho}{8\pi}|\mu|^2\nu^4\frac{d^3}{d^3x}\left[\tfrac{1}{x} + 2e^{-x}Ei(x)\right]_{x=2\nu d}, \tag{7}$$

where $Ei(x)$ is the exponential integral. The vertical drift force on a horizontal dipole can be shown to be half the force on the vertical dipole. The force on the source turns out to be suction-like for all values of νd, where d is its submergence. For the dipoles the force becomes repulsive over a small frequency range near the long wavelength limit. Similar results can be derived for higher-order singularities as well as distributions of them.

Surface-Piercing Bodies

The application of the momentum conservation principle for the vertical drift force leads to infinite integrals over the mean position of the free surface exterior to the body waterplane, involving quadratic products of the linearized velocity potentials and their derivatives. They can be reduced to integrals over the mean position of the body wetted surface S_B, the waterline C_W and the mean position of the free surface by an appropriate application of Green's theorem. They take the form

$$F = \Re(\mathcal{W}_W + \mathcal{W}_0 + \mathcal{W}_B), \tag{8}$$

where

$$\mathcal{W}_W = \frac{\rho}{4} \oint_{C_W} (\varphi_0^* + \varphi_B^*) \frac{\partial(\varphi_0 + \varphi_B)}{\partial n} dl, \tag{9}$$

$$\mathcal{W}_0 = -\frac{1}{2} i\rho g A\omega \, H_B(\nu, \beta, \nu, \pi + \beta) - \frac{\rho}{4} \oint_{C_W} \left(\frac{\partial \varphi_B}{\partial n} \varphi_0^* - \varphi_B \frac{\partial \varphi_0^*}{\partial n} \right) dl, \tag{10}$$

$$\mathcal{W}_B = -\frac{\rho}{4} \iint_{S_F} \left(\varphi_{Bz} \frac{\partial \varphi_B^*}{\partial n} - \frac{\partial \varphi_{Bz}}{\partial n} \varphi_B^* \right) ds. \tag{11}$$

Expressions (8)-(11) do not include the quadratic correction to the hydrostatic force which can be shown to be of cubic order for a wall-sided body. With the exception of (11), expressions (9) and (10) involve as unknowns the values and normal derivatives of the velocity potential φ_B on the body boundary which can be determined very accurately by standard panel codes. For submerged bodies, expression (9) vanishes and (10) reduces to (2).

The integral in (11) extends over the entire free surface exterior to the body waterplane. Its integrand is oscillatory, decays slowly at infinity and is inefficient to evaluate by quadrature. The "conservation" form of (11) suggests that the integration over the free surface can be replaced by an integration over the body boundary and over a control surface at infinity. The latter leads to a purely imaginary contribution that does not contribute to the force, and the former requires the evaluation of φ_{Bz} and φ_{Bnz} on the body boundary which may be inaccurate in conjuction with panel methods. By virtue of Green's identity and the form of (11), the integral over the body surface can be displaced into the fluid a distance a few times larger than the typical panel dimension to ensure the accurate evaluation of derivatives of φ_B.

For bodies with complicated geometries an alternative method for the evaluation of expression (11) is more appropriate. The integrand of (11) would vanish if $\partial^2 \varphi_B / \partial z^2$ was equal to $\nu \partial \varphi_B / \partial z$ on $z = 0$. This equality does not hold for the total body-wave velocity potential φ_B, but does for its oscillatory wavelike component. Represent φ_B on the free surface by a distribution of sources of strength $\sigma(\mathbf{x})$ on the body boundary, and utilize the definition of the wave source potential [cf. Wehausen and Laitone (1960), eq. (13.17"')]

$$G(\mathbf{x}; \xi) = \frac{1}{r} + \frac{1}{r'} - \frac{4\nu}{\pi} \int_0^\infty dk \, \frac{\nu \cos k(z + \varsigma) - k \sin k(z + \varsigma)}{\nu^2 + k^2} K_0(kR)$$
$$- 2\pi i \nu e^{\nu(z+\varsigma)} [J_0(\nu R) - iY_0(\nu R)], \tag{12}$$

where $K_0(x), J_0(x)$ and $Y_0(x)$ are the Bessel functions of zero order. The last term in (12) is the wavelike oscillatory component of the wave source potential which decays exponentially in the z-direction. The corresponding wavelike component of φ_B behaves similarly, and when substituted in (12) does not contribute to the vertical force. Combining the identities,

$$\frac{1}{r} + \frac{1}{r'} = \frac{2}{\pi} \int_0^\infty dk \, \cos kz \, \cos k\varsigma \, K_0(kR), \tag{13}$$

$$K_0(kR) = \int_0^\infty du \, \frac{uJ_0(uR)}{u^2 + k^2}, \tag{14}$$

where $R^2 = (x - \xi)^2 + (y - \eta)^2$ with the remaining terms in (12), (11) can be reduced to the form,

$$\mathcal{W}_B = \mathcal{W}_B^1 + \mathcal{W}_B^2, \tag{15}$$

$$\mathcal{W}_B^1 = -\frac{\rho}{8\pi^2} \int_0^{2\pi} d\vartheta \int_0^\infty k \, dk \int_0^\infty du \, \frac{u}{u^2 + k^2} \, \mathcal{S}_C(u, k, \vartheta) \mathcal{H}_S^*(u, k, \vartheta), \tag{16}$$

$$\mathcal{W}_B^2 = \frac{\rho\nu}{4\pi^2} \int_0^{2\pi} d\vartheta \int_0^\infty k^2 \, dk \int_0^\infty du \, \frac{u}{u^2 + k^2} \left[\nu \left(\mathcal{S}_C \mathcal{H}_S^* + \mathcal{S}_S \mathcal{H}_C^* \right) + k \left(\mathcal{S}_C \mathcal{H}_C^* - \mathcal{S}_S \mathcal{H}_S^* \right) \right]. \tag{17}$$

The "modified Kochin functions" $\mathcal{S}_{C,S}$ and $\mathcal{H}_{C,S}$ are defined as follows

$$\mathcal{S}_{C,S}(u, k, \vartheta) = \iint_{S_B} d\mathbf{x} \, \sigma(\mathbf{x}) \begin{pmatrix} \cos kz \\ \sin kz \end{pmatrix} e^{-iux\cos\vartheta - iuy\sin\vartheta}, \tag{18}$$

$$\mathcal{H}_{C,S}(u, k, \vartheta) = \iint_{S_B} d\mathbf{x} \left(\varphi_{Bn} - \varphi_B \frac{\partial}{\partial n} \right) \begin{pmatrix} \cos kz \\ \sin kz \end{pmatrix} e^{-iux\cos\vartheta - iuy\sin\vartheta}. \tag{19}$$

Expressions (16)-(17) involve the evaluation of infinite regular integrals with integrands that depend on the values and normal derivatives of the velocity potential φ_B on the body boundary, and can be regarded as a generalization to expression (3) for a surface-piercing body. Computations are presented of the vertical force on submerged and surface-piercing bodies based on expressions (1)-(5) and (8)-(19) respectively, used in conjuction with the MIT radiation-diffraction code.

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