AN INTERIOR INTEGRAL EQUATION METHOD FOR WATER WAVE RADIATION AND DIFFRACTION PROBLEMS

by

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1. INTRODUCTION

The Green function integral equation governing the water wave-structure interaction problem can be expressed in an exterior, surface or interior integral equation form. Conventionally, in the field of marine hydrodynamics the surface integral equation is always employed in numerical computation because of the diagonally dominant property of the resultant matrix equation and sufficient experiences gained in practical applications. When an interior integral equation is adopted, the kernal function is never singular. However, according to Mei [(1)] the interior integral equation has not been used in water wave problems perhaps due to the following reasons:

(i) the resultant matrix equation would no longer be diagonally dominant,
(ii) the choice of the interior field points could be too arbitrary.

An effort to apply the interior integral equation has been made by Martin [(2)] who introduced from acoustics a null field equation method based on the original interior integral equation. Unfortunately, divergent solutions were found for both thin and wide elliptical sections. It seems that the derived null field equation may be valid only for circular sections and slightly perturbed geometries based on a semi-circle or some other simple geometries corresponding to the chosen basis of series functions. These limitations of the null field equation approach have been well discussed in electromagnetics, optics and acoustics (for example, by Bates et al, Phil. Trans. Royal Soc., 1977; van den Berg et al, J. Opt. Soc. Am., 1979; etc.).

In the present paper theoretical basis and numerical techniques to apply the interior integral equation to general geometric forms of ships and offshore structures are described.

2. INTEGRAL EQUATION AND DIAGONAL DOMINANT PROPERTY

The Green function integral equation governing the radiation or diffraction wave potential $\phi e^{-i\omega t}$ can be expressed as

$$\left\{\begin{matrix} 4 \\ 2 \\ 0 \end{matrix}\right\} \pi\phi(P) - \int_{S_w} \phi(Q) \frac{\partial G(P,Q)}{\partial n_Q} ds = -\int_{S_w} v_n(Q)\; G(P,Q)\, ds \qquad \text{for } P(x,y,z) \in \left\{\begin{matrix} D \\ S_w \\ \bar{D} \end{matrix}\right. \qquad (1)$$

These three forms in eq. (1) may be referred to as the exterior, the surface and the interior integral equations corresponding to the locations of the field point P outside the body mean wetted surface S_w (i.e. in the exterior fluid domain D), on S_w and inside S_w (i.e. in the interior domain $\bar{D}$). The Green function has the form (Wehausen & Laitone, 1960):

$$G(P,Q) = 1/r_{PQ} + H(P,Q;k) = 1/\{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2\}^{1/2} + H(P,Q;k) \qquad (2)$$

As far as the surface integral equation is concerned the Green function possesses a 1/r singularity for the case $(x,y,z) = (\xi,\eta,\zeta)$.

Rewrite eq. (1) approximately in a discretised form (for i = 1,2,...N) as[†]

$$\alpha\phi(Q_i) - \sum_{j=1}^{N} \phi(Q_j) \int_{\Delta S_{Q_j}} \frac{\partial}{\partial n_{Q_j}} (G(P_i,Q_j) - \delta_{ij}/r_{P_iQ_j})\, ds = -\sum_{j=1}^{N} v_n(Q_j) \int_{S_{Q_j}} G(P_i,Q_j)\, d$$

with $\delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \ i \neq j \end{cases}$

and define the <u>SELF-INDUCED CONTRIBUTION FACTOR</u>

$$\alpha = \left\{\begin{matrix} 4\pi \\ 0 \end{matrix}\right\} - \int_{\Delta S_{Q_i}} \frac{\partial}{\partial n_{Q_i}} (1/r_{P_iQ_i})\, ds \qquad \text{for } P_i \in \begin{cases} D \text{ (neighbourhood of } S_w) \\ \bar{D} \text{ (any positions)} \end{cases} \qquad (4)$$

For $r \to 0^{\pm}$, that is, P_i tends to Q_i from the exterior or the interior domain, α becomes 2π which is identical with the second form of eq. (1). For simplicity, a circular flat panel ΔS_{Q_i} is chosen together with a field point P_i of local coordinats $(0,0,\bar{z})$ as illustrated in Fig. 1. It can be readily derived that

$$\alpha = \left\{\begin{matrix} 4\pi \\ 2\pi \\ 0 \end{matrix}\right\} - 2\pi\, \mathrm{sign}(\bar{z})\; \left(1 - \frac{|\bar{z}|}{\sqrt{a^2+\bar{z}^2}}\right) \qquad \text{for } P_i \in \begin{cases} D \text{ (neighbourhood of } S_w) \\ S_w \\ \bar{D} \text{ (any positions)} \end{cases} \qquad (5)$$

Values of α versus the non-dimensional distance $\bar{z}/a$ are shown in Fig. 2. It is apparent that the self-induced contribution factor α varies continously and smoothly from $0 \to 2\pi \to \gamma < 4\pi$ as the field point P_i moves from a far interior location, via the body surface S_w, to the exterior region neighbouring to the body surface. This conclusion holds for any arbitrary polygonal panels.

This discussion implies that if all the interior field points are chosen such that $|\bar{z}|$ is small the resultant matrix equation of the interior integral equation retains a similar diagonally dominant property to the surface integral equation approach.

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† For P_i located in the exterior region not close to the body surface S_w, the first term in eq. (3) should be written as $\alpha\phi(Q_i) + 4\pi\{\phi(P_i) - \phi(Q_i)\}$.

3. CHOICE OF THE INTERIOR FIELD POINTS

As long as all the interior field points are located close to the body mean wetted surface S_w the resultant matrix equation of the interior integral formulation has the numerical advantage of diagonal domination. In practical numerical computation, all chosen interior control points make up an interior surface $\bar{S}$ which is parallel to the body surface S_w and with a scale reduction factor C_s slightly less than 1.0 (i.e. $C_s = 1.0 - \varepsilon$ for ε being a small positive value). In the two-dimensional case, a scale reduction factor value $C_s = 0.95$ implies that the area enclosed by the interior contour is about 90% of the cross-sectional area.

In such a manner all these interior points can be automatically produced by a computational programme suite ("HYDROINT") in terms of the same input data file for the computer package based on the surface integral equation technique.

4. NUMERICAL EXAMPLES

Extensive numerical applications of the present interior integral equation method to various ship forms and complicated offshore structures have been conducted to verify the proposed approach. Two examples are displyed in the present paper. Fig. 3 shows the calculated sway, heave and roll added mass and damping coefficients for a ship section. The two sets of data obtained from the surface and the interior integral equation techniques coincide very well. Since the ordinary Green function is used irregular frequencies occur when these computed hydrodynamic coefficients exhibit abrupt variations due to mathematical failures (c.f. Wu and Price, First Workshop, 1986 and J. Applied Ocean Res., Oct. 1986). The irregular frequencies appearing in the interior integral equation calculation are higher than those related to the surface integral equation formulation because of a reduced area of the interior free-surface bounded by the chosen artificial interior surface. However, when a modified Green function (3) is adopted there exist no irregular frequency effects. This may be confirmed by data given in Fig. 4. In Fig. 4 hydrodynamic coefficients for a rectangular section derived from the surface and the interior integral equations by means of the modified Green function are presented. Excellent agreement between the two formulation calculations can be observed and there is no mathematical failure due to irregular frequency problem. In addition to the similar numerical solution stability and accuracy the computing time required for the surface and the interior integreal equations is nearly the same.

5. CONCLUSIONS

(i) The resultant matrix equation of the interior integral equation can retain diagonal dominant feature if all the interior field points are arranged close to the body wetted surface.

(ii) It is proposed to locate these interior control points on an artificial interior surface which is close and parallel to the body surface.

(iii) In contrast with the null field equation method which seems free of irregular frequency effect but may have divergent solutions for more complicated geometries in practical applications, the interior integral equation itself can not eliminate difficulties associated with irregular frequency problem in a higher frequency range but it may be applicable to arbitrary ship forms and offshore structure geometries.

(iv) In combination with the modified Green function the present interior integral equation approach can remove irregular frequency influence and thus may be applied to a wider range of wave frequencies as well as body geometries.

(v) Numerical example studies confirm that the present method can be performed in a totally same manner as the conventional surface integral equation with the same input data, similar numerical stability indicated by associated values of a condition number, a similar degree of numerical accuracy achieved in nearly the same computer time.

REFERENCES

1. Mei, C.C. "Numerical methods in water-wave diffraction and radiation", Ann. Rev. Fluid Mech., Vol. 10, 1978, pp. 393-416.
2. Martin, P.A. "On the null-field equations for water wave radiation problems", J. Fluid Mech., Vol. 113, 1981, pp.315-332.
3. Wu, X.J. and Price, W.G. "A multiple Green function expression for the hydrodynamic analysis of multi-hull structures", J. Applied Ocean Res., Vol. 9, No.2, 1987, pp. 58-66.

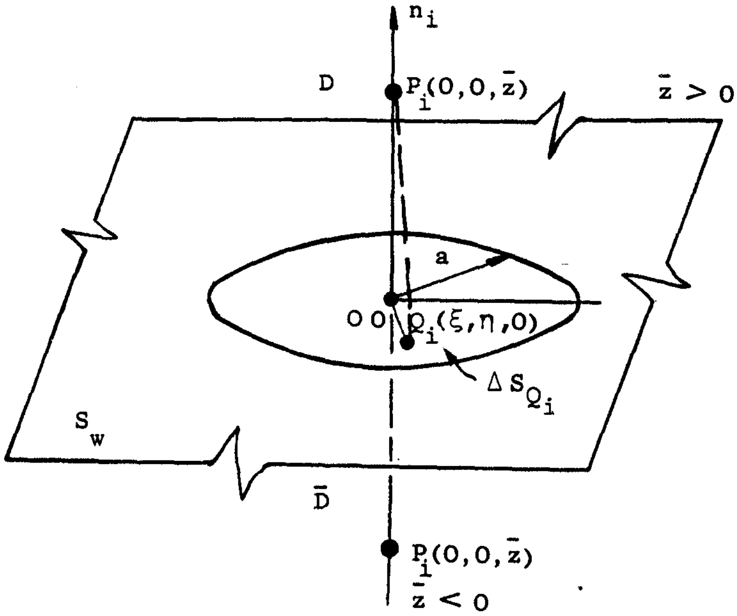

Fig. 1 A circular flat panel on the body wetted surface S_w together with a field point $P_i\,(0,0,\bar{z})$ exterior (i.e. $\bar{z} > 0$) or interior (i.e. $\bar{z} < 0$) to the body surface. a is the radius of the panel.

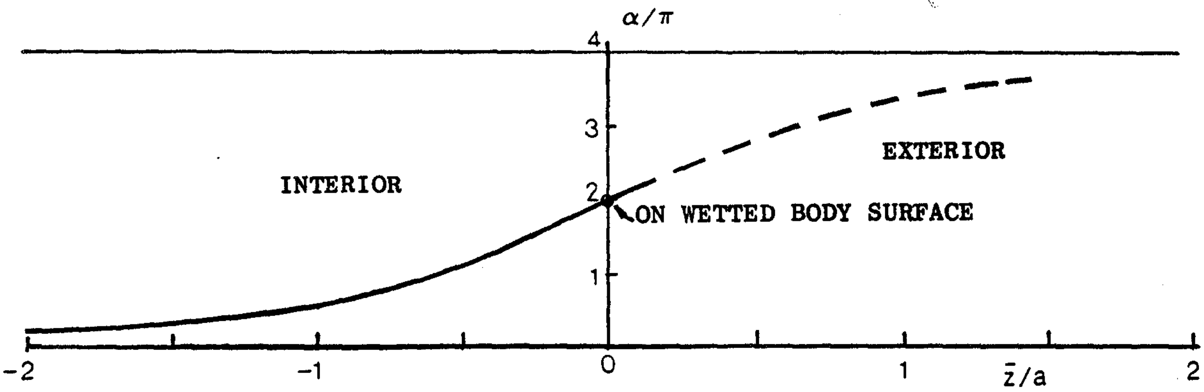

Fig. 2 Values of the self-induced contribution factor versus the distance between the field point P_i and a circular panel ΔS_{Q_i} of radius a calculated from eq. (5).

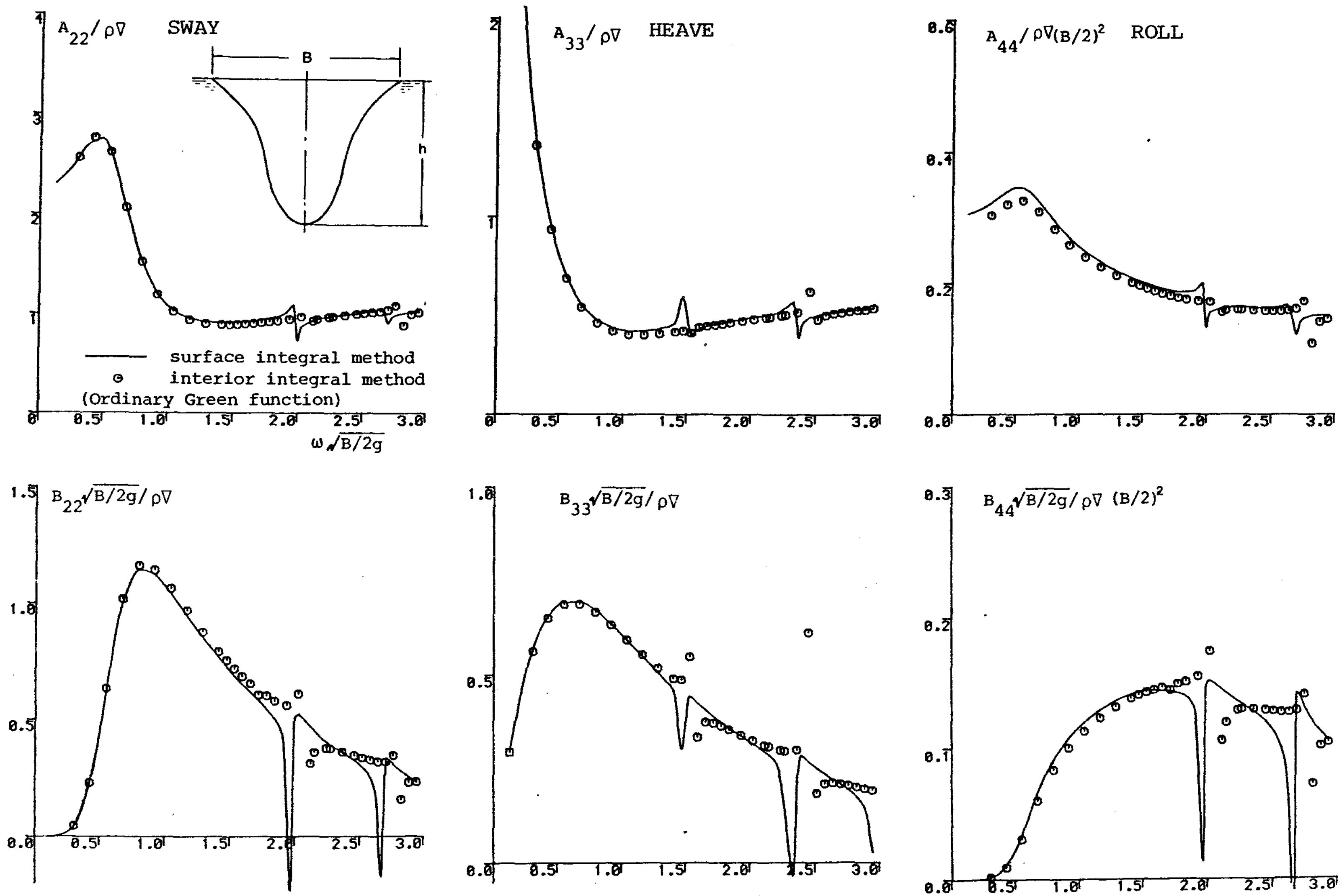

Fig. 3 Comparison between results from the surface and the interior integral equation methods (ordinary Green function).

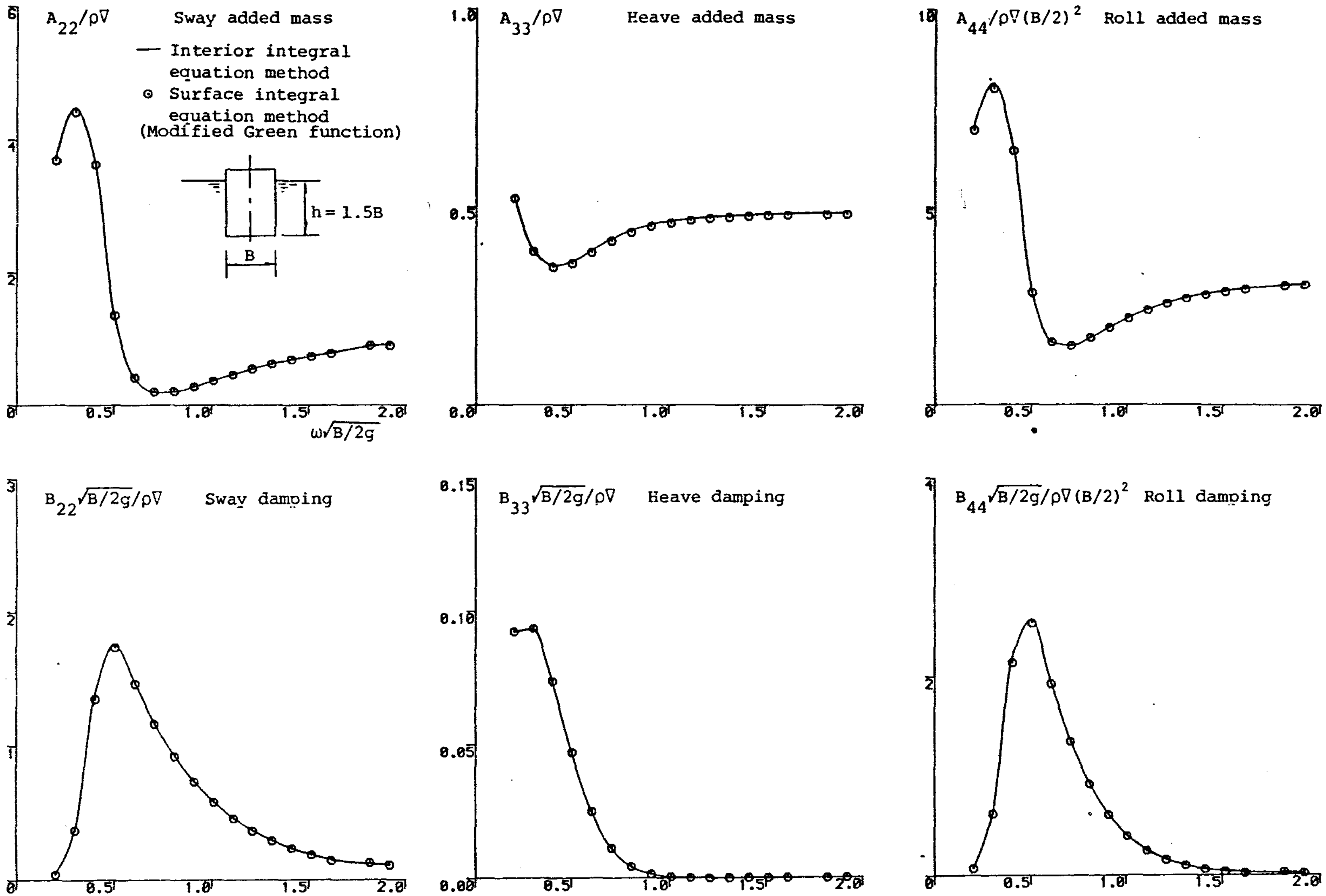

Fig. 4 Sway, heave and roll added mass and damping coefficients for a rectangular section of B/h = 2/3.

Discussion

Schultz: In your Green's function integral equation, you advocate smoothly varying the constant on the right hand side from 0 to 4π as you move from outside to inside the fluid domain. In the classical approach one goes in discrete steps 0, 2π, 4π for outside, on the boundary, and inside respectively. However, the integral for the 2π case is principal-valued while the others are not. In your approach, how do you evaluate the integral in a continuous manner as the singularity moves into the fluid domain?

X.J. Wu This is the key problem I want to make clear in this study. From the present discussion one may find that the classical statement about the jump behaviour of the self-induced contribution may be somewhat inadequate. The total self-induced contribution factor as defined by eq. (4) for general panel geometries or eq. (5) for circular panels varies smoothly and continuously though as a part of this total value the contribution from the simple source singularity exhibits a 4π jump via the body surface as shown by the second term on the right side of eq. (5).