# INTERACTION BETWEEN REGULAR WAVES, CURRENT AND A TWO-DIMENSIONAL FREE SURFACE-PIERCING BODY.

by

Rong Zhao and Odd M. Faltinsen

Division of Marine Hydrodynamics
Norwegian Institute of Technology
N-7034 TRONDHEIM - NTH

## SUMMARY

An infinitely long horizontal cylinder of arbitrary cross-section is studied theoretically and numerically in regular beam sea waves and uniform current. Infinite water depth is assumed. The effect of shed vorticity is neglected. It is argued that this is correct to O(U) where U is the current velocity far away from the body. The results are applied to calculate linear wave excitation loads, added mass and damping coefficients, first order motions, mean drift forces and roll moments as well as wave drift force damping.

The flow field is solved as a potential flow problem correct to first order in wave amplitude. The velocity potential is divided into a steady part $\phi_S$ and a time dependent part $\phi_T$. The $\phi_S$-problem satisfies the rigid free surface condition correct to O(U). The free surface condition correct to $O(U^2)$ for a "sufficient" submerged body can be written as

$$U^2 \frac{\partial^2 \phi}{\partial y^2} + g \frac{\partial \phi}{\partial z} = 0 \qquad \text{on } z = 0 \tag{1}$$

Here $\phi$ is defined by $\phi_S = Uy + \phi$. Further g is acceleration of gravity and (y,z ) is an orthogonal coordinate system where the z-axis is vertical and positive upwards. The mean free surface corresponds to z=0.

For a free surface-piercing body equation (1) does not apply. The reason is that equation (1) is based on U to be the first order approximation to the steady flow field. This cannot be true for a free surface-piercing body where there must be zero horizontal steady velocity at the intersection points between the free surface and the body surface. In our numerical study for a free surface-piercing body we have only used the free surface condition for $\phi_S$ correct to O(U).

The linear free surface condition for the time dependent problem $\phi_T$ can be written correct to $O(U^2)$ as

$$- \omega^2 \phi_T + 2\, i\omega \frac{\partial \phi_s}{\partial y} \frac{\partial \phi_T}{\partial y} + \left(\frac{\partial \phi_s}{\partial y}\right)^2 \frac{\partial^2 \phi_T}{\partial y^2}$$

$$+ 3 \frac{\partial \phi_s}{\partial y} \cdot \frac{\partial^2 \phi_s}{\partial y^2} \frac{\partial \phi_T}{\partial y} + \frac{\partial^2 \phi_s}{\partial y^2} i\omega \phi_T \tag{2}$$

$$- \frac{1}{2} \left(U^2 - \left(\frac{\partial \phi_s}{\partial y}\right)^2\right) \left(\frac{\partial^2 \phi_T}{\partial y^2} + \frac{\omega^2}{g} \frac{\partial \phi_T}{\partial z}\right) + g \frac{\partial \phi_T}{\partial z} = 0 \quad \text{on } z = 0$$

This is based on that the time dependence of $\phi_T$ is $e^{i\omega t}$, where i is the complex unit and t is the time variable. Equation (2) applies both for a free surface-piercing and a submerged body. Far away from the body as well as for a "sufficient" submerged body the free surface condition becomes

$$- \omega^2 \phi_T + 2\, i\omega U \frac{\partial \phi_T}{\partial y} + U^2 \frac{\partial^2 \phi_T}{\partial y^2} + g \frac{\partial \phi_T}{\partial z} = 0 \quad \text{on } z = 0 \tag{3}$$

Grue and Palm [1] have used this free surface condition in their numerical study for submerged cylinders.

In addition to using the free surface conditions (2) and (3) we have also used the free surface conditions that follows by neglecting terms of $O(U^2)$ in (2) and (3).

The body boundary condition for $\phi_T$ is written in terms of components of $\vec{n}$ and $\vec{m}$ (see Newman [2]). The $\vec{n}$ vector is the normal vector to the body surface. The $\vec{m}$ vector represents effects of interactions between the steady flow and the oscillatory flow. Care is needed in the calculation of $\vec{m}$.

The wave field far away from the body follows by using the free surface condition (3) and the radiation condition.

The velocity potential $\phi_T$ at a point $(y_1, z_1)$ in the fluid domain is written as

$$2\pi\ \phi_T(y_1,\ z_1) = \int_{S_B+S_F+S_\infty} \left\{ \frac{\partial\phi_T}{\partial n} \log r - \phi_T \frac{\partial}{\partial n} \log r \right\} ds \qquad (4)$$

This means that a distribution of basic sources and dipoles are distributed over the body surface $S_B$, the free surface $S_F$ and vertical control surfaces $S_\infty$ at infinity. On the body and free surface $\partial\phi/\partial n$ can be replaced by the body boundary and free surface condition, respectively. The free surface is divided into a near-field where equation (2) applies and an outer domain where equation (3) applies. In the outer domain the velocity potential due to the body is written as a combination of wave sources and dipoles as well as wave-free singularities. The singular point of the singularities is inside the body. The coefficients of the different terms in the outer domain representation of $\phi_T$ are found as part of the equation system that follows by letting points $(y_1,\ z_1)$ in (4) approach points on $S_B$ and $S_F$. It is shown that this hybrid technique is an efficient and accurate way of solving the boundary value problem for $\phi_T$.

In order to control the numerical results for the wave excitation loads a generalisation of the Haskind relation has been derived. The prediction of the damping coefficents has been controlled by energy considerations. The horizontal mean wave drift force has been calculated both by a direct pressure integration method and an expression that follows from conservation of momentum and energy in the fluid.

The numerical results have been compared with Grue and Palm's results for a submerged circular cylinder. The agreement is very satisfactory when the same free surface condition has been used. If we use the free surface condition (2), the results agrees well with Grue and Palm's results when $h/R \geqslant 2$. Here h means the distance from the mean free surface to the cylinder axis and R is the cylinder radius. When $h/R = 1.25$ there are significant differences by using condition (2) and (3). As expected the solution breaks down when we use condition (3) for a free surface-piercing body. The reason is the same as we described for the $\phi_S$-problem.

We have also investigated when we can neglect terms of $O(U^2)$ in the free surface condition. It is a good approximation to keep only terms of $O(U)$ when $\tau = \omega U/g < \sim 0.1$. Using the free surface condition correct to $O(U)$ represents a simplification of the solution procedure. In that case there is for instance only one wave system upstream and downstream. When we use the free surface condition correct to $O(U^2)$ more wave systems occurs. This is well known from the literature. The wave lengths of two of these wave systems become small when $U \rightarrow 0$. This causes numerical problems. Similar problems do not occur when we use the free surface condition correct to $O(U)$. Solving the problem correct to $O(U)$ has also the advantage that this is consistent with neglecting the effect of shed vorticity. Correct to $O(U^2)$ we should take into effect the shed vorticity.

The studies presented in this article are part of present investigations that deals with second order nonlinear interaction between irregular waves, current and a two-dimensional body. It is meant to represent the first step towards a practical numerical solution procedure that evaluate the combined effect of waves and current on marine structures.

REFERENCES:

1. Grue, J. and Palm, E. (1985):
"Wave radiation and wave diffraction from a submerged body in a uniform current". J. Fluid Mech. Vol. 151, pp. 257 - 278.

2. Newman, J.N. (1978):
"The theory of ship motions", Advances in applied mechanics, Volume 18, Academic Press, Inc.

## Discussion

Falnes: The results presented show that the current influences the excitation force much more than it influences the radiation resistance. Does this mean that the reciprocity relation between the radiation resistance and the excitation force (Newman 1962) needs to be modified in the case when a current exists?

Zhao & Faltinsen: Yes. We need to generalize the Haskind relation. This has been done in the presented work.

Falnes: In a given situation (given current, wave direction, frequency) if the first order (excitation) force is increased/decreased is then also the second order drift force increased/decreased?

Zhao & Faltinsen: In general we can't say that. The second order drift force depends on the body motion as well as the excitation force. Due to, for instance, coupling between motion modes an increase in heave excitation force may not necessarily mean an increase in heave motion. The further consequence for the drift force is not obvious.