WAVE INTERACTION WITH A SEMICIRCULAR SHELL NEAR BOTTOM BOUNDARY

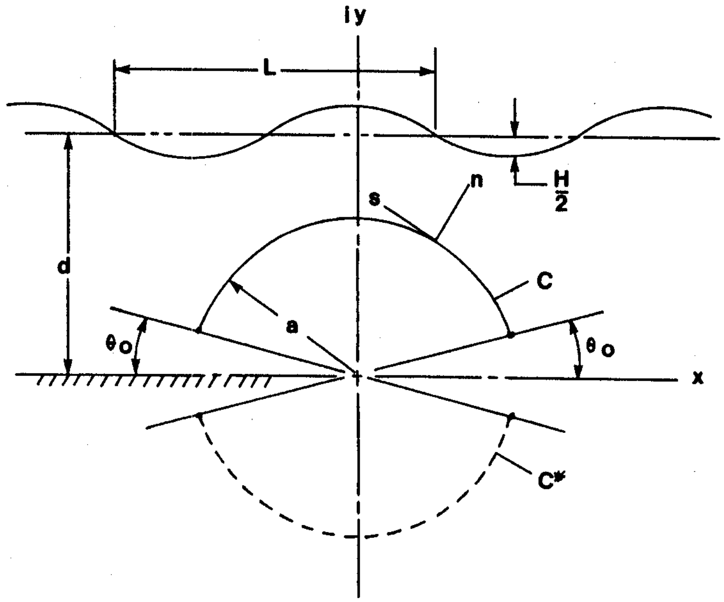
SUBRATA K. CHAKRABARTI
CBI RESEARCH CORPORATION
PLAINFIELD, ILLINOIS 60544

Offshore structures used for storing oil usually possess large volumes near the ocean floor. These structures may be open at the bottom with flow communications between inside and outside. On the other hand, they may be sealed at the edge resting firmly on the bottom. The design of such structures depend on whether their bottom experiences hydrodynamic interaction from the waves. There have been recent interest in the investigation of this hydrodynamic effect. Naftzger and Chakrabarti (1975) considered the linear diffraction effect of semispherical shell near the ocean floor and showed that the vertical forces on the shell are reduced considerably when compared to a seated hemisphere. The analytical results were correlated with experimental data from the wave tank tests. The pressure inside a slightly open shell was shown to be represented nicely by the mean pressures at the bottom of the hemisphere.

This paper presents the results of the theoretical study of wave forces on fixed two-dimensional structures. The study focuses upon submerged structures in the open ocean and, in particular, shell-like structures on or near the ocean bottom. The basic theoretical framework is based on the main assumptions that (1) the flow is irrotational throughout the greater portion of the flow field and (2) the wave height to water depth ratio is small, so that the linear wave model is applicable. We further assume that the only regions in which the flow is rotational are the boundary layers on obstacles and the ocean bottom, resulting in a circulation around the shell. Under these assumptions, the interaction between the sea and a submerged body reduces to a problem in potential theory. This problem can be solved generally by Green's method. Herein, the application of this method to shells is developed and results obtained for a semicircular shell are discussed and compared with experimental data for an open hemicylinder. The manner in which the theoretical model for a raised shell approaches that for a seated (or sealed) one is discussed with regard to the actual flow at its edges.

For moderately deep submergence of a half-cylinder on the ocean bottom, an approximate solution for the potential, ϕ, can be established using the series method. Since far from the object the reflected wave will be just a fraction of the incident wave, the free surface boundary condition on the total potential, ϕ, will be approximately satisfied by just the incident, ϕ_p, and the boundary condition on the scattered potential, ϕ_s, may be relaxed.

For plane flow, there exists a complex formulation for ϕ in terms of ϕ_p, the Green's function, G, and a function f which corresponds to the value of ϕ_s on the object surface. The function, f, is real and can be expressed in terms of a Fredholm equation of the second kind with integrals over the contour, C, and its reflection, C* (Fig. 1).



For both the half-cylinder and shell, as $d/a \to \infty$, $f(s)$, which is real, approaches a much simpler form, $f_0(s)$. Consequently, $f_0(s)$ corresponds to the solution for deep submergence. In the case of the half-cylinder, $C+C^*$ is a closed contour; wherefore, the integrals can be evaluated using Cauchy's Integral Theorem. For deep submergence, one readily obtains the same result given by the series method for large d/a. In the case of the shell, it appears that $f_0(s)$ generally cannot be evaluated without the aid of a computer.

For the semicircular shell, the picture is further complicated by separation at the edges of the shell and a discharge of vorticity. As the flow continues, the shed vorticity will alter the flow pattern in such a way as to move the stagnation point to the edge, making the velocity gradient there finite. Provided that no separation occurs on the upper surface, the flow near the edge will then be irrotational except for a small wake. Since the velocities near the edges of the shell can be large, the convective inertia, in general, cannot be neglected and the full Bernoulli equation should be used to determine the dynamic pressure on C.

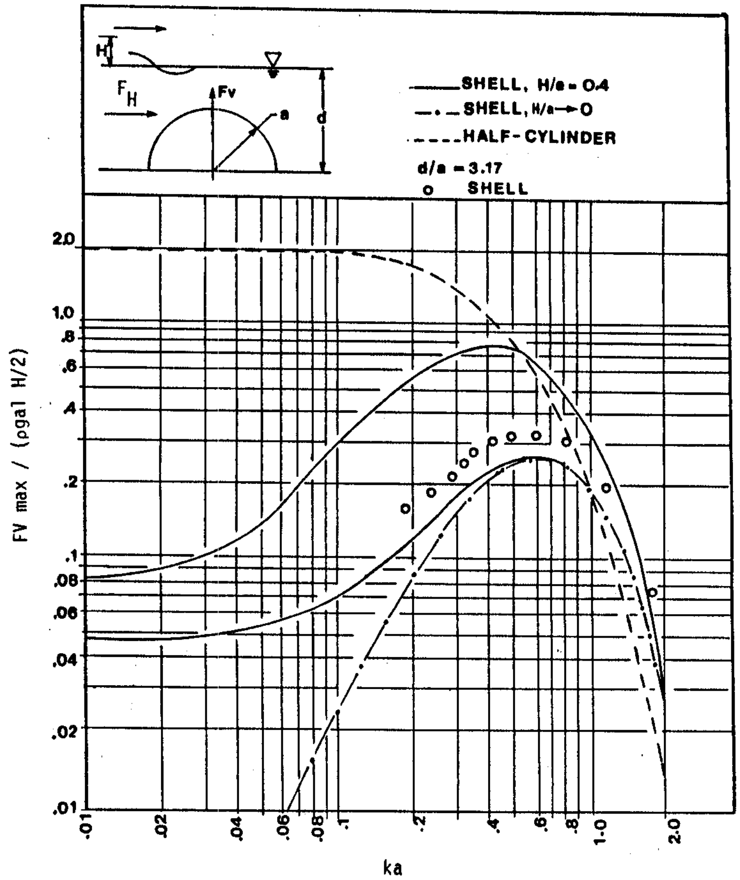
Generally, regions of separation will occur on both sides of the shell accompanied by positive and negative vorticity. For any period, the net vorticity discharge will be equivalent to some circulation around the shell. One finds that to pass continuously from the solution for a raised shell to that for a seated one, when the wave is at a crossover point, necessitates a smooth flow at the edge(s) and some circulation around the body. It can be shown, at least for deep submergence, that the circulation which is possible in the absence of vortices is sufficient to ensure a continuous velocity at the edges and the correct limit as the gap approaches zero. In actuality , one imagines vortices in the flow near the origin or far downstream, whose velocity normal to the shell is small.

The difficult problem in passing to the seated solution obtains when there is a crest or trough over the body, or, more precisely, when the pressure on the body is symmetric fore and aft. First, in this phase, the circulation will be small and generally, vortices must be introduced adjacent to the semicircular shell, to ensure a smooth flow at the edges. The introduction of such vortices is not an altogether simple matter. Secondly, as their gaps become small, the flow beneath the shells is reduced until the viscous forces at the bottom arrest all flow underneath. The inside and outside of the shell are then separate flow regions, but no flow is assumed inside. This assumption is based on the notion that the vorticity is confined to the viscous region beneath the edge so that essentially any flow inside the shell will be acyclic and irrotational. However, such a flow cannot exist in a fluid bounded by a surface on which the normal velocity is zero. Therefore, to the extent that flow around the edge(s) is absent, the normal velocity there will be zero and no flow will exist inside.

In the absence of flow inside, the only dynamic pressure possible under the shells is a uniform fluctuation. This situation will be referred to as an "almost" sealed condition, as opposed to a completely sealed or seated condition for which there is no fluid beneath the edge and no dynamic pressure inside. In general, the "almost" sealed condition must be approached through solutions with vorticity, not only to ensure a smooth flow at the edge, but also to have the appropriate pressure differences there. When the pressure is symmetric fore and aft, the flow will tend to stagnate at the bottom fore and aft such that the difference in pressure between inside and outside is minimal. Hence, as the gap size approaches zero, the uniform pressure fluctuation inside should equal the symmetric part of the pressure outside at the stagnation points. Wherefore, the limit for the differential pressure will be the outside pressure for the completely sealed case less the value of its symmetric part at the stagnation points at the bottom. For the semicircular shell, this limit and that for the flow in the absence of vorticity are the same due to the fact that the lengths of both edges tend toward stagnation when the pressure is symmetric.

The analytical results for raised shells are based on the semicircular surfaces whose centers of curvature lie at the bottom. Thus, it is only for small openings between the structure and the bottom that the results will be a good approximation to those for a raised hemicylinder. However, it is precisely this situation which is of most interest.

The depth to radius ratio for the test setup was 3.17. Although this value is not large enough for the free surface boundary to be neglected, the results given by the deep submergence solution for $d/a = 3.17$ will provide a meaningful comparison with the experimental data for the vertical force. It is estimated that the average opening on the sides of the shell for the test setup was 5 degrees and that the mean wave height to radius ratio was approximately 0.4. Based upon these estimates, a numerical evaluation of the shell solution was carried out, and the amplitudes of the resulting horizontal and vertical forces were computed. The normalized amplitudes for two values of the circulation are shown in Fig. 2 plotted against ka. The forces are normalized with respect to $\rho gal\ H/2$, where l = length of the half-cylinder. The convective inertia term which becomes important when ka is small and the wave height large, is included in the computation of the forces on the shell. In the first case, which corresponds to the lower (solid) curve, the circulation around the shell is zero. In the second case, corresponding to the upper curve, the circulation is such that a stagnation point is located at an edge. The center-line curve denotes the limit for no circulation as $H/a \to 0$; this limiting curve for all practical purposes coincides with the curve for $H/a = 0.4$ over the range of ka plotted. The amplitudes of the horizontal and vertical forces (per unit length) on the corresponding seated hemicylinder is also shown in the figure as broken lines.



The curve for which the circulation is such that the stagnation point lies at an edge is thought to represent an upper bound at least for large ka. The fact that one of the experimental points falls above this curve at ka ≃ 1.8 suggests that the free surface, which has been neglected, may serve to shift the theoretical values up, providing a close agreement between the data and the curve for no circulation. It is assumed that this will be the case, implying that the circulation, whose value generally cannot be determined by potential theory, is approximately zero for moderate and large ka. Clearly, more experimental data, particularly for large d/a ratios, is needed to check the theoretical results obtained thus far. The extent to which the vorticity actually remains near the edge, probably can be determined only by experiments involving some sort of flow visualization.

Naftzger, R.A. and Chakrabarti, S.K., "Wave forces on a Submerged Hemispherical Shell", Proceedings on Conference on Ocean Engineering III, ASCE, Newark, Deleware, June 1975.