Solitary Waves Passing over Submerged Breakwaters

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This work is part of a programme of study to provide more understanding of the hydrodynamics of steep waves when they encounter coastal structures. We have developed a computer program which models the motion of unsteady waves in 2D irrotational, inviscid flows. The model is not restricted to any particular free surface motion but we choose here to examine solitary waves. This class includes the largest 2D irrotational, inviscid wave which can steadily propagate on a fixed depth. Solitary waves are everywhere elevated above the undisturbed fluid surface, so it was thought likely that these model the waves most damaging to structures. We can accurately reproduce solitary waves with heights close to the highest wave.

The numerical method employs a boundary-integral technique to solve Laplace's equation for the velocity potential. Bernoulli's equation is used as one boundary condition at the free surface. We ignore surface tension and take the atmospheric pressure above the free surface to be constant. The second boundary condition is to assume that fluid particles on the free surface stay on the surface. All rigid surfaces are assumed impermeable. Details of the method are given by Dold and Peregrine (1986).

The method is accurate, stable and efficient. If h is the undisturbed fluid depth then a solitary wave of height $H = 0.5h$ can propagate over a horizontal distance of $50h$ with less than 0.1% change in height. The method does not generally suffer from "sawtooth" instabilities.

The program can be run on an IBM-compatible AT personal computer. In 60 minutes elapsed time we can calculate the motion of a wave passing over a breakwater. The method also uses conformal mappings to transform flow domains with irregular beds into regions with a flat horizontal bed. This is done to make Laplace's equation easier to solve. For the examples used here the conformal mapping has a bottom comprising a semicircle (radius R) on an otherwise flat horizontal bed. See figure 1.

The solitary wave is started with its crest far enough away from the semicircle for the wave to be unaffected by the obstacle in the first few time steps. The wave moves left towards the cylinder. The initial data describing an incident solitary wave is calculated using the method of Tanaka (1986).

Let the undisturbed fluid depth $h = 1$. Then H, the incident wave height, and R the cylinder radius are dimensionless parameters. The $H - R$ parameter space is surprisingly rich. We expected all waves to steepen and break on the cylinder but this only happens for the largest waves $(H > 0.6)$ and the largest cylinders $(R > 0.8)$ and even these tend to break after they have passed over the breakwater.

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Small cylinders ($R < 0.3$) cause very large unstable waves ($H > 0.78$) to eventually break. The growth of the instability closely follows that discussed by Tanaka et al (1987) who also show that unstable waves do not necessarily break. Waves with height less than the energy maximum (i.e. $H < 0.77$) develop a train of weak dispersive waves in their wake, after they have passed the breakwater.

A very common but unexpected phenomenon occurs for many waves for cylinders with radii $0.5 < R < 0.9$. As the wave approaches the cylinder a second crest grows on the other side of the obstacle. The second crest soon dominates the first which meanwhile decays. The new crest propagates away. This exchange of crests across the cylinder is reminiscent of a large solitary wave catching up a smaller one.

Most surprising of all are those waves of height between 0.3 and 0.6, passing over cylinders between 0.7 and 0.9 radius. The wave passes over the breakwater and having cleared it a second stationary wave forms at the left-hand edge of the cylinder. This second wave steepens enough to break over backwards onto the cylinder. See figure 2.

For the example shown in figure 2, if we increase R from 0.7 to 0.8 the incident wave breaks forward before the second wave breaks backward. We are unable to continue the computations beyond the time at which a wave breaks, but experiments indicate that breaking does occur for both of the waves. The breaking is simultaneous when $R = 0.77$, for $H = 0.58$.

Wave tank experiments at Santander University confirm the backwards breaker. Some measurements of depth as a function of time, at fixed stations, also agree with prediction.

Figure 2 also illustrates the instantaneous stream lines of the flow. The pressure can also be found, and the forces exerted on the cylinder determined. For example when $H = 0.804$ and $R = 0.9$ the wave breaks on top of the cylinder. The horizontal and vertical components of dynamic force at each time are plotted in figure 3. The maximum horizontal force occurs during the wave's approach and well before breaking. Both of the maxima in horizontal and vertical components are about as large as the hydrostatic force. This shows that waves can exert large dynamical forces on submerged coastal structures.

References:

J.W. Dold & D.H. Peregrine, 1986b, Water-wave modulation, Proc. 20th Internat. Conf. Coastal Engng. Taipei, A.S.C.E., 1, 163-175.

M. Tanaka, 1986, The stability of solitary waves, Phys. of Fluids, 29, 650-655.

M. Tanaka, J.W. Dold, M. Lewy & D.H. Peregrine, 1987, Instability and breaking of a solitary wave, J. Fluid Mech., 185, 235-248.

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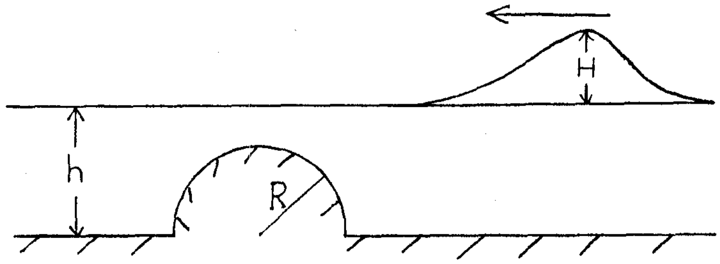

Figure 1: Solitary wave approaching submerged semicircular breakwater.

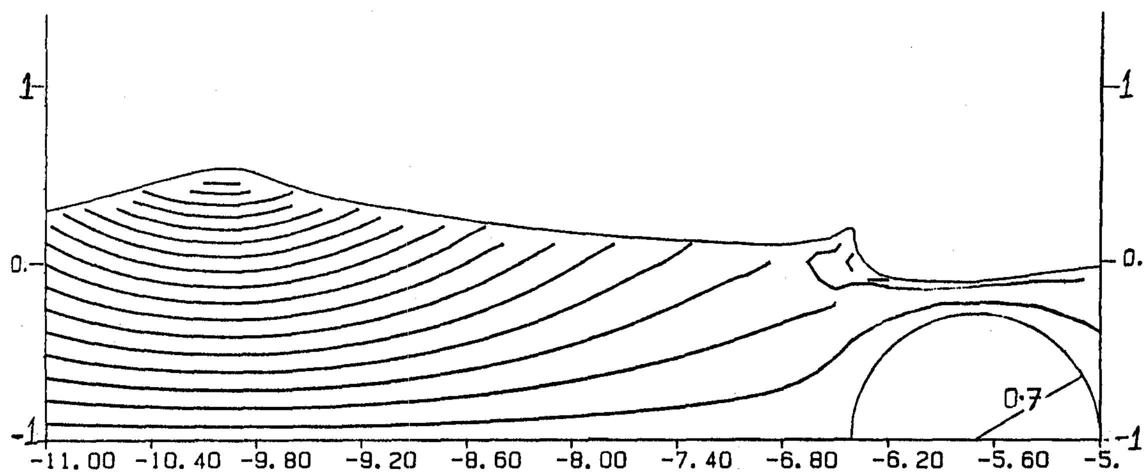

Figure 2: The backwards breaker for H = 0.58 and R = 0.7. The instantaneous stream lines are also shown. Natural scaling.

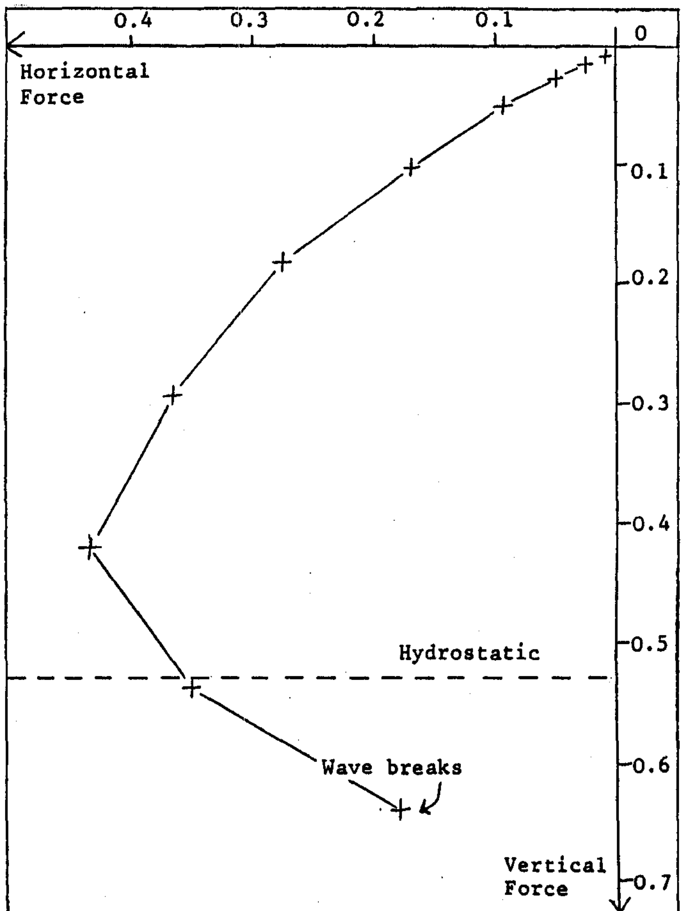

Figure 3: The dynamic force on a cylinder
H = 0.8, R = 0.9. Times 0,0.5,1,...,4.5.

Pawlowski: I would like to know if at the intersection of the free surface and the beach the ***computed*** components of the fluid velocity recover a zero normal component with respect to the solid boundary.

Cooker: The physical problem is conformally mapped into a domain with a flat, horizontal bed. The contact angle of the free surface and the beach is conserved by the mapping. The free surface is now reflected in the flat bed forming a symmetric domain. The geometric symmetry ensures that the collocation point on the beach remains on the beach. Hence, our integral equation solved in the transformed plane, automatically contains the kinematic boundary condition, and Φ_N and Φ_ξ are consistent not only with the equations of motion ($\nabla^2\Phi = 0$), but with the boundary conditions as well.