IMPACT OF NONLINEARITY UPON WAVES TRAVELING OVER A SUBMERGED CYLINDER

by

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Incoming deepwater Stokes waves passing over a restrained submerged circular cylinder is considered experimentally. The submergence of the cylinder is small. Hence, a strong local nonlinearity is introduced at the free surface above the cylinder and free higher order harmonic waves are generated. The dominating features of the wavefield far away from the circular cylinder is:

Upstream of the cylinder: An incoming Stokes wavetrain. No reflected waves, even to higher order (Chaplin 1984).

Downstream of the cylinder: Shorter free second harmonic waves of considerable amplitude is riding on the transmitted Stokes wave.

The principal aim of this investigation is to present measurements of the amplitude of the second order wave behind the cylinder and the amplitude and phase of the transmitted Stokes wave. The measured quantities are compared with second order potential theory results (Vada 1987).

For small amplitude a the surface elevation η of the incoming Stokes wave is given by

$$\eta(x,t) = a \cos (kx-\omega t) + \frac{1}{2} a^2 k \cos 2(kx-\omega t) \quad (1)$$

where $k=\omega^2/g$ is the wavenumber, ω is the frequency, g is gravity, x is horizontal coordinate, t is time. Far downstream of the cylinder the surface elevation is then composed of a transmitted Stokes wave with amplitude a_1 and phase δ_1 and generated free higher harmonic waves, that is

$$\eta(x,t) = a_1 \cos (kx-\omega t+\delta_1) + \frac{1}{2} a_1^2 k \cos 2(kx-\omega t+\delta_1)$$
$$+ a_2 \cos (4kx-2\omega t+\delta_2) + \text{higher order modes} \quad (2)$$

where a_2 and δ_2 are respectively amplitude and phase of the generated second harmonic free wave. Let D denote the distance between the undisturbed free surface and the top of the cylinder, and R be the cylinderradius. Measurements are carried out for

$0.05 < a/D < 0.6$, $0.2 < kR < 1.7$, $0.5 < D/R < 1.0$.

The main findings are:

for $a/D < 0.2$:

$$a_1 \simeq a, \qquad \delta_1 \simeq f(kR,D/R), \qquad a_2 \simeq \frac{a^2}{R} g(kR,D/R)$$

where f and g are predicted by to second order potential theory.

For $0.2 < a/D < 0.6$ and $0.3 < kR < 1.3$ we find:

a_1/a is decreasing with increasing a/D

δ_1 has a strong decrease with increasing a/D

$a_2/R \simeq h(kR,D/R)$

Hence, a_2 is approximately constant for $a/D > 0.2$ (for fixed values of kR and D/R). For $0.2 < a/D < 0.44$ we observe that a steep wave is formed straight above the cylinder. For $a/D > 0.44$ ($ak<0.44$) this wave breaks at the cylinder.

We note that two important and competing mechanisms are present when $a/D > 0.2$, namely the nonlinearity at the free surface and the dissipation in the cylinder's boundary layer. For long incoming waves, i.e. $kR < 0.5$, we observe a very weak decay of a_1/a (with increasing value of a/D), hence a weak dissipation at the cylinder. Then the nonlinear effect is dominant, being responsible for the dramatic change in the far-field ampliude, as displayed in the figure (kR=0.4, D/R=0.5).

For shorter waves ($kR > 0.5$), we observe a pronounced decay in a_1/a, i.e. a pronounced dissipation at the cylinder. Then it is not so clear what effect is the most important for the value of the second order harmonic wave. However, a steep wave is present a the cylinder, giving rise to strong effect of nonlinearity.

Chaplin (1984) has delt with the accoustic streaming due to the oscillatory boundary-layer at the cylinder. It is observed in the present experiments that the steep wave straight above the cylinder in some cases introduces large velocities, leading to flow-separation at the cylinder. This leads to an enhancement of the circulation around the cylinder. The flow separation is observed for KC=0.6 (Keulegan-Carpenter Number) kR=0.7, D/R=0.5.

The experiments are carried out in a wave flume at Department of Mechanics, Institute of Mathematics at Oslo University. The flume is 13m long and 470mm broad. The average depth of the water is in all experiments 490mm. At one end the flume is equipped with a wave maker which is operated under program control. At the other end there is a 2m long absorbing beach. The horizontal cylinder is placed close to the middle of the flume and occupies its total width. The cylinder diameter is 196mm.

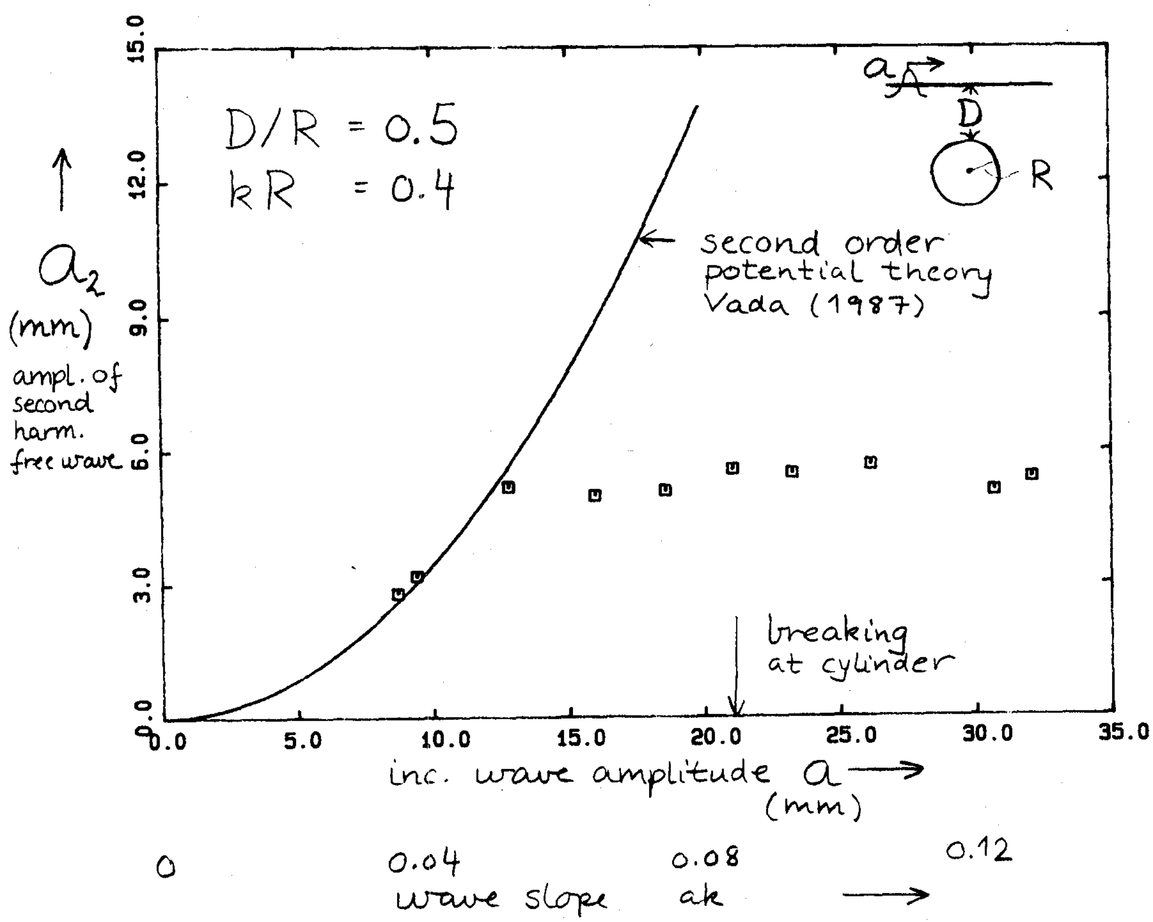
The free surface elevation is measured by three surface piercing wave gauges. One gauge is placed between the the cylinder and the wave maker at a distance approximately 0.8m from the centre of the cylinder, and two gauges are placed behind the cylinder with distances from the cylinder centre approximately 0.8m and 1.5m, respectively. The wave elevation is in some cases also measured at a larger distance than 0.8m from the cylinder centre, for comparison, to ensure that we record the farfield properties of the flow.

Data of the surface elevation at the different wave gauges are recorded over three periods of the incoming waves, before any reflected wave from the beach has reached the gauges. Then Fast Fourier Transform is applied to the time series to obtain the amplitudes of the waves behind the cylinder.

REFERENCES

Chaplin, J. R. (1984). Nonlinear forces on a horizontal cylinder beneath waves. J. Fluid Mech., **147**: 449-464.

Vada, T. (1987). A numerical solution of the second-order wave diffraction problem for a submerged cylinder of arbitrary shape. J. Fluid Mech., **174**: 23-37.



Papanikolaou:

1. Two references which cover both experimental and theoretical work on a quite similar subject are: Y. Kyonaka, Kyushu Univ., *ONR Symp.* 1982; and Papanikolaou, *1st Workshop on Water Waves and Floating Bodies,* MIT, 1986.

2. The authors show a strong dependence of the phase angle on the incident wave amplitude. Can this effect be explained? Is it possible that it is a third-order effect, the influence of those terms oscillating with frequency ω, but being of third-order with respect to the incident wave amplitude?

Ref. Papanikolaou *Proc IUTAM Symp,*Tokyo, 1987.

Grue & Granlund:

1. The references mentioned treat second-order theory and experiments for bodies moving on the free surface, and hence are related to the present research. The present research, in addition to quantifying the far-field properties of the waves, addresses the effect of the local nonlinearity which is observed at the body. The results show a dramatic deviation from second-order theory predictions when the ratio of the incoming wave-amplitude to the local depth exceeds a certain value.

2. It is expected that third-order effects are important in the vicinity of the body. However, since we observe a very steep (non-breaking) wave directly above the body, we expect that also very-high-order effects are important. In other words, a fully nonlinear theory may be needed to explain the strong decrease of δ_1.