THE FORMULATION OF MEAN DRIFT FORCES AND MOMENTS FOR FLOATING BODIES

S.M. Hung* and R. Eatock Taylor

London Centre for Marine Technology, University College London

In this paper, an attempt is to be made to clarify the confusing situation of the existence of different near and far field formulations for mean drift forces. The keys to the systematic development of consistent formulations by these methods are shown, and the results obtained by the present formulations are presented and discussed.

1 Introduction

It is well known that two major methods exist for the calculation of mean (time independent) drift forces on floating bodies in waves. Based on the principle of conservation of momentum, the far field method was originated by Maruo [5] for the calculation of horizontal forces. It was subsequently extended by Newman [7] to yaw moment, and by Lee and Kim [4], Molin and Hairault [6] and Sclavounos [10] to the calculation of the vertical force and overturning moment. On the other hand, the so called near field method suggested by Pinkster and van Oortmerssen [9] relies simply on the direct integration of fluid pressure on the body surface.

Not surprisingly, relative advantages and disadvantages can be found between the two methods. It has been pointed out by Sclavounos [10] that the far field method may be more efficient and less demanding on numerical discretisation. On the other hand, the near field method is potentially more useful if one wishes to extend the solution to the calculation of time harmonic second order forces. In any case, the existence of the two methods should, in principle, be useful for cross checking theoretical derivation and computational implementation.

Since both the far and near field methods in their most general form are based on the same usual assumptions of irrotational flow and boundary conditions, they should yield identical solution of all forces and moments for any sufficiently defined hydrodynamic problem. However, experience [3][11] has shown that agreement is frequently far from perfect. A lone exception to this appears to be the exact agreement obtained by Drake, Eatock Taylor and Matsui [1], who considered a vertical cylinder free to pitch at the sea bed.

Of course, many reasons may exist for the disagreement of results. The obvious amongst them is the different numerical error incurred in the evaluation of the formulations in the two methods. However, a close study of some well known near field formulations published reveals that none of them agree in entirety with each other. While the limitations of results given by Pinkster have been pointed out by Standing et al [11], more recent results by Molin [6] and Ogilvie [8] are again different. A similar situation can also be observed for the far field formulation of vertical drift force and overturning moment. While some variations in these formulations may be attributed to the use of different stated assumptions, they do not always provide the complete explanation.

From this state of affairs, it is clear that the solution of mean drift

* Now at: Brown & Root Ltd., 125 High St., London SW19 2JR

forces on bodies in waves is still by no means well established, and the clarification of the differences between the various near and far field formulations is of some importance. It is also an opportune moment to investigate the possibility of obtaining good agreement between the near field and the far field results. The derivation of both near field and far field formulation in the present work suggests that previous results in vertical force and overturning moment are often incomplete.

2 The near field formulation

Although the near field formulation is conceptually the simpler of the two methods for the calculation of mean forces, as pointed out by many authors [10][11][8] careful consideration must be given to the integration of the fluid pressure on the moving body surface. Denoting the first order rotational amplitude of the body with respect to the fixed reference axis Oxyz in roll, pitch and yaw as Ω_4, Ω_5 and Ω_6, and the resulting transformation matrices for describing a vector X moving (to X') with the body fixed axis Ox'y'z' as T_4, T_5 and T_6 respectively, we may write $X' - X_c = T(X-X_c)+\chi^{(1)}$, where $X_c=(x_c,y_c,z_c)$ is the centre of rotation, $\chi^{(1)}=(\chi_1,\chi_2,\chi_3)$ is the first order translational motion and T is a function of T_4, T_5 and T_6.

Due to the lack of commutativity in rotation, for example

$$T_4T_5 \neq T_5T_4 \tag{1}$$

and in accordance with Stoke's expansion, a decision must be made towards the choice of the sequence of Euler's angles in the rotational degrees of freedom, such that the instantaneous location of a moving vector on the body surface may be consistently described. Following [11], the sequence of roll, pitch and yaw will be adopted here, so that we assume $T=T_6T_5T_4$. It is noteworthy that if one employs a different sequence of the Euler's angles, or if the rotations are assumed to be about the moving axis [8], T will be different to second order and so will be the general formulation of the second order mean vertical force and overturning moment.

If we make a further assumption that the body in question is not penetrating the sea bed or other bodies during its motion, and all spatially independent pressures are assumed to be zero, the formulations for vertical axisymmetric bodies can be written as

$$F^{(2)}= \rho\frac{g}{2}\oint_{WL} \Xi_R^{(1)^2} N dl - \rho\int_{S_m}\{\tfrac{1}{2}(\nabla\Phi^{(1)})^2+(\chi^{(1)}+T^{(1)}(X-X_c))\cdot\frac{\partial}{\partial t}(\nabla\Phi^{(1)})\}nds + T^{(1)}(F_D^{(1)}+F_S^{(1)}) - \rho g z_c A_W(0,0,\frac{\Omega_4^2+\Omega_5^2}{2}) \tag{2}$$

$$M^{(2)}= \rho\frac{g}{2}\oint_{WL} \Xi_R^{(1)^2}(X-X_c)xN\, dl - \rho\int_{S_m}\{\tfrac{1}{2}(\nabla\Phi^{(1)})^2+(\chi^{(1)}+T^{(1)}(X-X\ ))\cdot\frac{\partial}{\partial t}(\nabla\Phi^{(1)})\}((X-X_c)xn)ds$$

$$+ T^{(1)}(M_D^{(1)}+M_S^{(1)}) \quad + \quad \chi^{(1)}x(F_D^{(1)}+F_S^{(1)}) \tag{3}$$

such that $\Xi_R^{(1)}$ is the relative wave amplitude, $N=n/\sqrt{(1-n_z^2)}$ where n_z is the vertical component of the inward normal on the water line, A_W is the water plane area, $T^{(1)}$ is the first order component of T, $F_D^{(1)}$, $F_S^{(1)}$ are the first order hydrodynamic forces and hydrostatic forces respectively while $M_D^{(1)}$, $M_S^{(1)}$ are the moments. The integrals in WL and S_m are performed on the mean water line and mean body surface respectively. It appears that part or all of the last term in equation 3 is usually neglected in the literature. Although this term is zero for freely floating bodies, it may contribute significantly for structures such as tension leg platforms. A complete formulation for bodies of arbitrary geometry and further discussion may be found in [2].

3 The far field formulation

As stated above, the formulation of the mean vertical force and overturning moment obtained by the near field method are generally dependent on the assumption of a sequence of Euler's angle for the definition of the moving body surface. Obviously, in order to obtain a corresponding formulation by the far field method, this assumption must also be specified. However, it appears that this assumption has not been a requirement in most of the existing far field formulations for vertical force and overturning moment for floating bodies. It therefore follows that these formulations are incomplete.

Considering the rate of change of translational (L) and rotational (R) momentum respectively, we get [7]

$$\frac{d}{d\,t} L = -\rho \int_V \{\nabla(\frac{p}{\rho} + gz)+(\nabla\Phi.\nabla)\nabla\Phi\}\ dv + \rho \int_S \nabla\Phi.U_n ds = 0 \tag{4}$$

$$\frac{d}{d\,t} R = -\rho \int_V (X-X_c)x\{\nabla(\frac{p}{\rho} + gz)+(\nabla\Phi.\nabla)\nabla\Phi\}\ dv + \rho \int_S (X-X_c)x\nabla\Phi.U_n ds = 0 \tag{5}$$

where p is the pressure and V is a control volume bounded by S, which consists of the instantaneous wetted moving body surface S' and any suitably chosen fictitious surface outside the body. Making use of Gauss's divergence theorem, we can write

$$F^{(2)} = -\rho \int_{S-S'} \{\frac{p}{\rho}n + (\nabla\Phi_n-U_n)\nabla\Phi\ \}\ ds \quad -\rho \int_S gz(0,0,n_z)\ ds \tag{6}$$

$$M^{(2)} = -\rho \int_{S-S'} \{\frac{p}{\rho}(X-X_c)xn + (\nabla\Phi_n-U_n)(X-X_c)x\nabla\Phi\ \}\ ds \\ -\rho \int_S gz\ (X-X_c)x(0,0,n_z)\ ds \tag{7}$$

Notably, the last term in both equations can contribute. If the sequence of Euler's angles is the same as that of the previous section, and again for the sake of simplicity if one considers axisymmetric bodies only, the last term in equation 6 gives rise to the last term of equation 2, and the last term of equation 7 becomes

$$-\rho g \{ \chi^{(1)} + T^{(1)}(X-X_c) \} \times (0,0,\chi_3)\, A_W \tag{8}$$

4 Results

Finally, on the basis of the formulations presented above, mean vertical forces and moments in a unit incident wave amplitude are given in Table 1 for a floating hemisphere. The first order solution is provided by a Boundary Series Element implementation similar to that of Yue, Chen and Mei [12], but specifically developed for vertical bodies of revolution. The hemisphere is of 1m radius with a mass of 2146.7kg tethered horizontally by a linear spring having a stiffness of 100N/m. The centre of rotation and CG are assumed to coincide at 0.5m below mean water level. The radius of gyration is 0.5m and the water depth is 3m.

5 References

[1] Drake, K.R., Eatock Taylor, R. and Matsui, T., Proc. of Royal Soc., 1984.

[2] Hung, S.M., Ph.D. thesis, Univ. of London, 1987.

[3] Kokkinowrachos, K. Bardis, L. and Mavrakos, S., BOSS, 1982.

[4] Lee, C.M. and Kim, Y.H., Int. Symp. in Hydrodynamics in Ocean Eng., 1981.

[5] Maruo, H., J. Ship Res., 1960.

[6] Molin, B. and Hairault, J., Proc. Int. Workshop on Ship and Platform Motions, 1983.

[7] Newman, J.N., J. Ship Res., 1967.

[8] Ogilvie, T.F., Proc. Int. Workshop on Ship and Platform Motions, 1983.

[9] Pinkster, J.A. and Oortmerssen, van G., Proc. 2nd Conf. on Numerical Ship Hydrodynamics, 1976.

[10] Sclavounos, P.D., 2nd Int Workshop on Water Waves and Floating Bodies, 1987.

[11] Standing, R.G., Dacunha, N.M.C. and Matten, R.B., NMI report R124, 1981.

[12] Yue, D.K.P., Chen, H.S. and Mei, C.C., Int. J. for Num. Meth. in Eng., 1978.

Table 1

ω	$\lvert\chi_1\rvert$	$\lvert\chi_3\rvert$	$\lvert\chi_5\rvert$	Near field $F_1^{(2)}$	Near field $F_3^{(2)}$	Near field $M_5^{(2)}$	Far field $F_1^{(2)}$	Far field $F_3^{(2)}$	Far field $M_5^{(2)}$
2.0	1.003	1.078	.5078	1045.	5247.	-5.788	1045.	5247.	-5.797
3.0	.8238	1.752	1.390	1184.	9602.	-3093.	1182.	9593.	-3095.

Pawlowski: With respect to rotations it should be understood that angles are not properly defined as small parameters in the problem discussed by the authors. In order to visualize this, it is enough to consider a semi-circular cross-section and consider its rotation, first with respect to the intersection of the centerplane with the waterplane, and next with respect to an arbitrary point on the free surface. It is clear that in the first case the geometry of the cross-section remains unchanged; but in the second, its displacement can be made arbitrarily large. It is clear that to treat the angle as a small parameter is meaningless without specifying the axis. A consistent perturbation theory must impose the condition of small displacements of the body with respect to its characteristic dimension.

Ref. (1) J.S. Pawlowski "Basic Relations of Ship Theory Part I," Delft University of Technology 1982, Report of Shipbuilding Laboratory.

Hung & Eatock Taylor: While we agree with the suggestion that a consistent perturbation theory must, for practical use, impose the condition of a small perturbation parameter, this is totally unrelated to the observation regarding the center of rotation. Instead of being a result of the necessity of a small perturbation parameter, the problem we describe has a more fundamental nature. It is a characteristic of any rigid body moving in a multi-phase medium (in this case air and water).

Although the employment of an arbitrary center of rotation (located for solving the rigid body equations of motion) does not provide different local response and wave excitation in first-order analysis, the same cannot be said of the second-order mean vertical force and overturning moments. For example, a different value of z_c in Equation 2 of the abstract would leave the sum of all the terms except the last unchanged, while the last term is proportional to z_c. This observation leaves where z_c should be, unanswered. Physically, the change of the force is evident by considering the second-order buoyancy force due to the change of submerged volume when the body is in motion if z_c is not zero.

Korsmeyer:

1. Can a perturbation approach be justified for a problem which contains a body with a sharp corner? Note, for example, that second-spatial derivatives of the first-order potential, which appear in the right-hand side of the body boundary condition at second-order, are not integrable at a body corner.

2. Also, you mentioned a mapping scheme to take care of the singularity in the first-order potential at the corner, but you covered it only briefly. It appeared to be the mapping which corresponds to a square-root singularity, however it should be the mapping for a cube-root singularity.

Hung & Eatock Taylor: We should like to thank Dr. Korsmeyer for his questions on a subject that has been much neglected in the literature of hydrodynamics.

1. We beleive the academic's reply is no, both from consideration of body-free-surface intersection points and submerged sharp body corners. This is because in the former, the perturbation parameter employed here (ϵ, the wave slope) can approach infinity, and even as ϵ tends to zero, the exact solution is still physically undefined; and in the latter, it is indeed the case that second-order forces are indeterminate if, say, $\phi_{rr} \propto r^{-4/3}$) at the corner. The existence of sharp corners on the body surface can introduce substantial error to the numerical calculation of second-order drift forces, and to a lesser extent, first-order hydrodynamic coefficients.

Despite this seemingly sorry state of affairs, we believe, from a practical point of view, that it is still desirable to pursue an accurate numerical solution to second order. In our experience, if the corner is well submerged, first-order solutions and second-order mean forces can still be evaluated, although the rate of convergence becomes greatly reduced. [A feature of the corner singularity is that it 'pollutes' the solution.] A way of tackling this problem which appears to hold great promise has previously been reported by Liggett and Liu for steady flow problems (see below).

2. A few years ago we conducted a simple numerical experiment using the quadratic hair line crack tip elements commonly employed in fracture mechanics. Although we were fully aware of the fact that this type of element ($\phi_r \; \alpha \; r^{-1/2}$) does not provide a suitable singularity characteristic for an L-shaped corner ($\phi_r \; \alpha \; r^{-1/3}$), we felt that the exercise was worthwhile. Unfortunately, their employment seemed to provide no real benefit towards convergence.

Ref. Liggett, J.A. and Liu, P.L.F., *Boundary integral equation methods for porous media flow.* George, Allen & Unurns publ., 1983.