Numerical Solution of the Nonlinear Ship Wave Resistance Problem

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I compute the stationary potential flow around a ship at the free surface. The location of the free surface and the position of the ship (trim and sinkage) are part of the solution. The potential must meet the following boundary conditions: no flow across the body boundary, the pressure at the free surface is athmospheric, no flow across the free surface, waves appear in a sector behind the ship only (radiation condition).

Free surface conditions

At the free surface a boundary condition is used that linearizes the differences between an approximation for the potential and the location for the free surface and the exact values. This condition is applied iteratively. If the iteration converges the nonlinear boundary condition is met at the computed location of the free surface. Rankine sources are distributed in a layer above a local part of the free surface. The boundary condition is required at control points on the approximation of the free surface. The radiation condition is enforced by an extra row of control points at the upstream end of the grid and a row of point sources at the downstream end of the grid. Details of this method can be found in [2,1].

Boundary condition at the body surface

A new boundary integral method is introduced here: To avoid complicated evaluations of transcendental functions as in other panel methods I want to use numerical integration.

$$\phi(\vec{q}) = \int_S M(\vec{p})\, G(\vec{p},\vec{q})\, dS_p \tag{1}$$

is the potential of the (unknown) source distribution M on the body surface. On the body surface it generates the normal velocity

$$\phi_n(\vec{q}) = \vec{n}(\vec{q})\, \nabla_q \phi(\vec{q}) = \oint_S M(\vec{p})\, \vec{n}(\vec{q})\, \nabla_q G(\vec{p},\vec{q})\, dS_p - \frac{1}{2} M(\vec{q}) \tag{2}$$

If the normal velocity is given as the boundary condition the important part of the solution is the tangential velocity:

$$\phi_t(\vec{q}) = \vec{t}(\vec{q})\, \nabla_q \phi_q = \oint_S M(\vec{p})\, \vec{t}(\vec{q})\, \nabla_q G(\vec{p},\vec{q})\, dS_p \tag{3}$$

If G is the potential of a three dimensional point source $G = -1/4\pi r$, the singularity in the integrand of (2) for $\vec{p} \to \vec{q}$ vanishes for plane panels and therefore (2) can be integrated numerically.

The singularity in (3) remains even for plane panels with constant source strength. Therefore numerical integration can not be used.

This problem is removed by the following means:
A source distribution of constant strength on a sphere generates no tangential velocity on the sphere:

$$\int_{\text{sphere}} \vec{t}(\vec{q}) \nabla_q\, G(\vec{k},\vec{q})\, dS_k = 0 \tag{4}$$

If the sphere touches the body surface tangentially at point Q and its center lies within the body a function P exists, which gives the projection of points $\vec{p}$ of the body surface along the radius onto the sphere ($\vec{k} = P(\vec{p})$). There is also a function r, which gives the projection of a surface element S_p of the body surface onto the sphere ($S_k = rS_p$). r has the sign of the skalar product of the normalvectors on the body and the sphere. Thus (4) can be transformed into an integral along the body surface:

$$\oint_S \vec{t}(\vec{q})\,\nabla_q G(P(\vec{p}),\vec{q})\,r\,dS_p \tag{5}$$

Multiplying this expression with $M(\vec{q})$ and subtracting it from (3) gives:

$$\phi_t = \nabla_q\phi_q\,\vec{t}(\vec{q}) = \oint_S M(\vec{p})\,\vec{t}(\vec{q})\,\nabla_q G(\vec{p},\vec{q}) - M(\vec{q})\,\vec{t}(\vec{q})\,\nabla_q G(P(\vec{p}),\vec{q})\,r\,dS_p \tag{6}$$

Now the integrand vanishes for panels with constant source strength and (6) can be integrated numerically. Of course a closed body surface is required when this equation is applied.

We get a simple efficient numerical method by dividing the surface of a smooth, closed body into $i = 1\ldots N$ panels. For all panels the area f_i, the coordinates of its midpoint $\vec{x}_i$, the unit normal $\vec{n}_i$ and two approximately orthogonal tangents $\vec{s}_i$ and $\vec{t}_i$ are determined and the radius of the tangential sphere R_i is chosen.

The midpoints x_i are the source points as well as the control points for the boundary condition. The integration is replaced by simple summation.

$$v_n(\vec{x}_k) = \sum_{i=1}^{N} M_i\,\vec{n}_k\,\nabla_k G(\vec{x}_i,\vec{x}_k)\,f_i\,(1-\delta_{ik}) \;-\frac{1}{2}M_k \tag{7}$$

$k = 1,...,N$, $\delta_{ik} = 0$ for $i \neq k$ and $= 1$ for $i = k$. (7) is a linear algebraic system of equations for the unknown $M_i = M$ at $\vec{x}_i$.

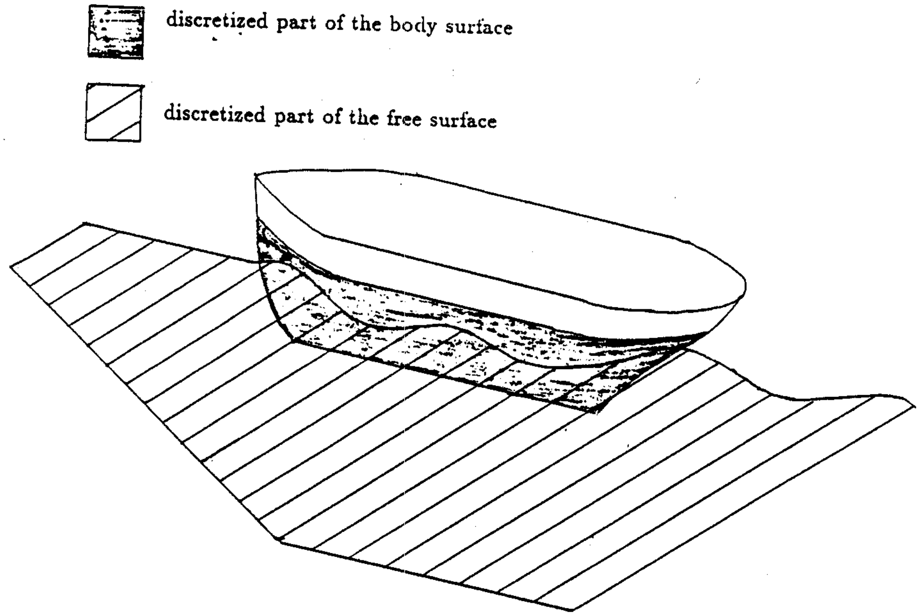
To compute velocities at the control points (6) is discretized into:

$$v_t(\vec{x}_k) = \vec{t}_k\,\nabla_k\phi = \sum_{i=1}^{N}\vec{t}_k\,[M_i\,\nabla_k G(\vec{x}_i,\vec{x}_k) - M_k\,\nabla_k G(\vec{p}_k(\vec{x}_i),\vec{x}_k)\,r_k(\vec{x}_i)]\,f_i \tag{8}$$

Thus only the simple Greenfunction $1/4\pi r$ has to be evaluated twice for each source/control point combination. The method is proved to be at least as accurate as Websters method [4], a comparison to the Hess and Smith [5] method is under way.

Combination of free surface and body

These methods are now merged to compute the free surface flow around a ship. The free surface grid control points with sources in a layer above are placed around the ship with the innermost line of control points a small distance from the waterline. The body surface is discretized some way above the undisturbed waterline.



The body boundary condition (on the discretized part) and the free surface condition are fulfilled simultaniously. Then the location of the free surface is computed from Bernoullis equation. The pressure is integrated over the actually wetted part of the hull to get the forces and moments. With these values trim and sinkage are corrected. Then the next step of iteration is started. If the iteration converges, the nonlinear free surface condition is fulfilled at the computed location and the Neumann condition is fullfilled on the actually wetted part of the hull. The forces and moments on the hull are in equilibrium.

There are only very preliminary computations for the Wigley Parabolic Hull completed at this moment.

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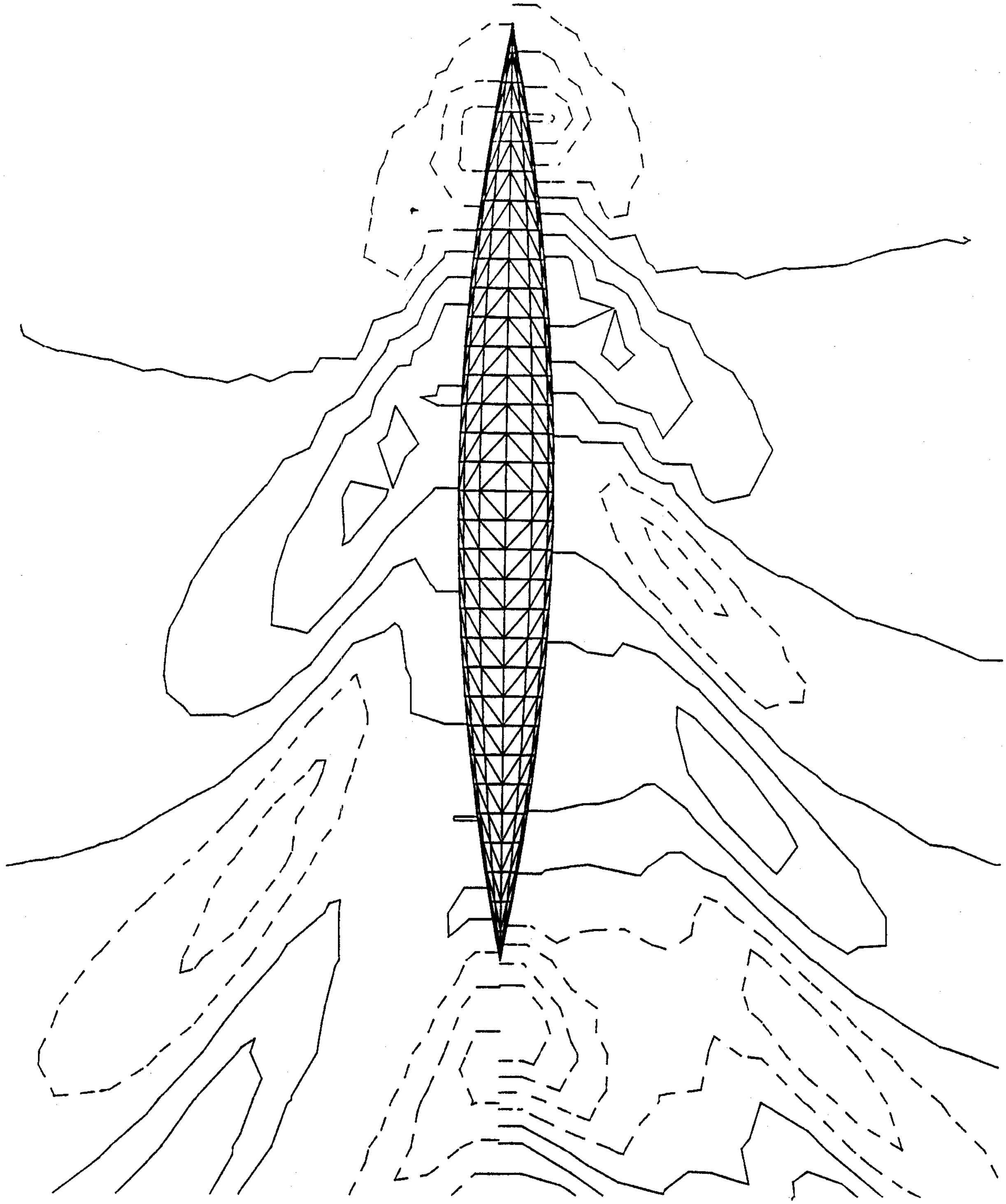

Computed wave pattern for Wigley parabolic hull. Broken lines are crests, full lines are troughs. The lines are $0.05 \cdot U^2/2g$ apart.
The left half shows the case of $F_n = 0.35$, the right half is for $F_n = 0.28$.
For $F_n = 0.35$ the wave resistance coefficient was computed to $1.3 \cdot 10^{-3}$, sinkage is .049× the draught and the trim is 0.079° aft.
For $F_n = 0.28$ the wave resistance coefficient was computed to $1.06 \cdot 10^{-3}$, sinkage is .028× the draught and the trim is 0.006° aft. For $F_n = 0.4$ the wave resistance coefficient was computed to $2.22 \cdot 10^{-3}$, sinkage is .075× the draught and the trim is 0.6° aft.