THE COMPLETE SUM AND DIFFERENCE FREQUENCY WAVE FORCE QUADRATIC TRANSFER FUNCTIONS FOR AN AXISYMMETRIC BODY

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Introduction

The second-order wave effects on compliant offshore platforms and moored ocean vessels have been a topic of increasing interest and extensive studies in the past decade. Because of the complexity of the second-order boundary-value problem involving inhomogeneous free-surface conditions, a number of approaches which either neglect, approximate or otherwise avoid the direct calculation of the second-order potential have been proposed. Recently, we developed a complete second-order diffraction-radiation method for the calculation of sum and difference frequency wave forces on axisymmetric bodies and their resultant motions in the presence of bichromatic waves. The nonlinear second-order potential is obtained explicitly so that, in addition to forces and moments, local quantities such as second-order pressures, velocities, and surface elevations are also available. We present results for the second-order wave force quadratic transfer functions in bichromatic seas for a floating hemisphere and a conical island. The validity of several existing approximations are discussed.

Problem Formulation

Assuming potential flow and weak nonlinearities, we write the following perturbation expansion and decomposition for the total velocity potential Φ:

$$\Phi = \epsilon\ (\ \Phi_I^{(1)} + \Phi_D^{(1)} + \Phi_R^{(1)}\ ) + \epsilon^2\ (\ \Phi_I^{(2)} + \Phi_D^{(2)} + \Phi_R^{(2)}\ ) + \cdots \qquad (1)$$

At first order, the diffraction potential represents the scattered waves due to the presence of the fixed body, and the radiation potential the radiated waves due to first-order body motions. At second order, $\Phi_D^{(2)}$ represents the second-order diffraction potential for the body undergoing first-order motions, while $\Phi_R^{(2)}$ is the second-order radiation potential due to second-order motions in the absence of ambient waves. In the presence of two incident plane waves of different frequencies, the linear and second-order potentials can be expressed as follows:

$$\Phi^{(1)} = \mathrm{Re} \sum_{j=1}^{2} \phi_j^{(1)}\ e^{-i\omega_j t}$$

$$\Phi^{(2)} = \mathrm{Re} \sum_{j=1}^{2} \sum_{l=1}^{2} \{\ \phi^-\ e^{-i\omega^- t} + \phi^+\ e^{-i\omega^+ t}\ \} \qquad (2)$$

where $\omega^{\pm}=\omega_j \pm \omega_l$ and the sum and difference frequency problems can be treated seperately. The boundary-value problem for the first-order body disturbance potential, ϕ_B, which combines the diffraction and radiation potentials is given by:

$$\nabla^2 \phi_B^{(1)} = 0 \qquad \text{in the fluid}$$

$$\left(-\omega^2 + g \frac{\partial}{\partial z} \right) \phi_B^{(1)} = 0 \qquad \text{at } z=0 \ (S_F)$$

$$\partial \phi_B^{(1)} / \partial z = 0 \qquad \text{at } z=-h \ (S_b)$$

$$\partial \phi_B^{(1)} / \partial n = - \partial \phi_I^{(1)} / \partial n - i\omega \, \mathbf{n} \cdot (\boldsymbol{\xi}^{(1)} + \boldsymbol{\alpha}^{(1)} \times \mathbf{x}) \qquad \text{on the body } (S_B)$$

$$\lim_{\rho \to \infty} \sqrt{\rho} \, (\partial/\partial\rho - ik) \, \phi_B^{(1)} \to 0 \qquad \text{at infinity } (S_\infty) \tag{3}$$

where $\xi^{(1)}$ and $\alpha^{(1)}$ are the complex amplitudes of the first-order translational and rotational motions. The boundary-value problem for the sum and difference frequency second-order diffraction potential is given by:

$$\nabla^2 \phi_D^{\pm} = 0 \qquad \text{in the fluid}$$

$$\left(-\omega^{\pm 2} + g \frac{\partial}{\partial z} \right) \phi_D^{\pm} = Q^{\pm} \qquad \text{at } S_F$$

$$\partial \phi_D^{\pm} / \partial z = 0 \qquad \text{at } S_b$$

$$\partial \phi_D^{\pm} / \partial n = - \partial \phi_I^{\pm} / \partial n + B^{\pm} \qquad \text{at } S_B$$

$$\phi_D^{\pm} \sim \{ F_H^{\pm}(\theta,z) \, e^{ik^{\pm}\rho} + F_P^{\pm}(\theta,z) \, e^{i\rho(k_j \pm k_1 \cos\theta)} \} / \sqrt{\rho} \qquad \text{at } S_\infty \tag{4}$$

where $k^{\pm}$ is the wavenumber corresponding to the frequency $\omega^{\pm}$. The inhomogeneous free-surface and body-boundary forcing terms, Q and B, are composed of quadratic products of the first-order potentials or motions, and can be obtained by using Taylor's expansion of the exact free-surface and body boundary conditions with respect to their mean positions. For example, the sum-frequency free-surface forcing term Q^+ can be calculated from:

$$Q^+ = (q_{j1}^+ + q_{1j}^+) / 2$$

$$q_{j1}^+ = \frac{-i\omega_1}{2g} \phi_1^{(1)} \left(-\omega_j^2 \frac{\partial \phi_j^{(1)}}{\partial z} + g \frac{\partial^2 \phi_j^{(2)}}{\partial z^2} \right) + i\omega_1 \nabla \phi_j^{(1)} \cdot \nabla \phi_1^{(1)} - q_{IIj1}^+ \Big|_{z=0} \tag{5}$$

where q_{II} represents the self-quadratic terms of the linear incident wave potential. The sum and difference frequency second-order radiation potentials, which is proportional to the second-order motions, satisfy the boundary value problem:

$$\nabla^2 \phi_R^{\pm} = 0 \qquad \text{in the fluid}$$

$$(-\omega^{\pm 2} + g \frac{\partial}{\partial z}) \phi_R^{\pm} = 0 \qquad \text{at } S_F$$

$$\partial\phi_R^{\pm} / \partial z = 0 \qquad \text{at } S_b$$

$$\partial\phi_R^{\pm} / \partial n = -i\omega^{\pm} \mathbf{n}\cdot(\boldsymbol{\xi}^{(2)\pm} + \boldsymbol{\alpha}^{(2)\pm} \times \mathbf{x}) \qquad \text{at } S_B$$

$$\lim_{\rho\to\infty} \sqrt{\rho} (\partial/\partial\rho - ik^{\pm}) \phi_R^{\pm} \to 0 \qquad \text{at } S_\infty \tag{6}$$

For unit amplitude second-order motions, (6) gives the added-mass and hydrodynamic damping for the sum and difference frequency second-order equation of motion. In this sense, the analyses for the first and second-order radiation potentials are identical except for the shift of frequencies, and most of the interesting nonlinear aspects and complexities are involved in the second-order diffraction problem.

We solve the second-order diffraction problem directly by applying Green's theorem to the sum and difference frequency diffraction potential and the (linear) wave-source Green function to obtain a Fredholm integral equation of the second kind:

$$2\pi \phi_D^{\pm} + \iint_{S_B} \phi_D^{\pm} \frac{\partial G^{\pm}}{\partial n} dS = \iint_{S_B} G^{\pm}(B^{\pm} - \frac{\partial\phi_I^{\pm}}{\partial n}) dS + \frac{1}{g} \iint_{S_F} Q^{\pm} G^{\pm} dS \tag{7}$$

where the source strengths on the body and free surface are given by the body and free-surface forcing terms respectively. The most important and difficult step in the solution of (7) is the evaluation of the body and free surface integrals on the right hand side of (7). For sum-frequency problems, the free-surface integrand is highly oscillating and slowly-decaying and a direct truncation in the near field may result in significant errors. The detail asymptotic analysis and evaluation of this free-surface integral is described in Kim & Yue (1988). For the difference-frequency problem, the oscillations and decay of the free-surface integrand is much slower than that of the sum-frequency problem especially when two incident wave frequences are close. In this case, the contribution of the free-surface integral is in general small compared to other second-order contributions. For vertically axisymmetric bodies, we expand the linear and second-order potentials in terms of Fourier cosine series and integrate (7) in θ direction first to obtain successive one dimensional integral equations for each Fourier mode:

$$2\pi \phi_{Dn}^{\pm} + \int_{S_B} \rho \, dl \, \phi_{Dn}^{\pm} \frac{\partial G_n^{\pm}}{\partial n} = \int_{S_B} \rho \, dl \, (B_n^{\pm} - \frac{\partial\phi_{In}^{\pm}}{\partial n}) G_n^{\pm} + \frac{1}{g} \int_{S_F} \rho \, d\rho \, Q_n^{\pm} G_n^{\pm} \tag{8}$$

For the evaluation of the free-surface integral in (8), we seperate the contribution of the free wave from the original integrand and integrate analytically outside a truncation boundary where the integrand is local wave free. As a result, we can shrink the domain of numerical integration and save the computing time drastically. The total second-order force on a freely floating body can be obtained by integrating the pressures on the instantaneous body surface, as described in Ogilvie(1983).

The total second-order wave exciting force for second-order motions can be divided into four parts: (i) contributions from the quadratic products of the linear quantities, (ii) contributions from the second-order incident wave potential, (iii) contributions from the body-boundary forcing term of the second-order diffraction potential in (4), and (iv) contributions from the free-surface forcing term of (4).

Results and Discussion

The sum and difference frequency second-order forces (the complete quadratic transfer function) on a floating hemisphere and a bottom-seated conical island in the presence of bichromatic incident waves are calculated by our boundary-integral equation method using ring sources. For the sum-frequency springing forces, the contribution of the free-surface integral is usually dominant over other second-order contributions. As a result, many approximation methods such as that of Herfjord & Nielsen (1986) or Petrauskas & Liu (1987), who neglect this contribution may be quite restrictive. For the difference-frequency or slowly-varying drift forces, however, the contribution of the free-surface integral is not very important compared to the other second-order contributions except for the cases where the difference of two frequences is large or when large nonlinearities are expected for special geometries such as mild slope conical gravity platforms. Existing approximation methods for slowly-varying difference-frequency wave forces are also examined for their validity for different combinations of frequencies and body geometries.

References

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Papanikolaou: Accept my compliments for your fine work. I would like further clarification of the decomposition of the second-order problem.

Kim & Yue: The way in which the second-order problem is decomposed is not unique. Any decomposition is acceptable as long as the total potential satisfies the original boundary value problem. In our point of view, the present decomposition is consistent with the linear problem in the sense that $\phi_I^{(2)}$ and $\phi_D^{(2)}$ yield wave-exciting forces for 2nd-order motions. The form of the far-field asymptotic behavior of $\phi_D^{(2)}$ does not change whether we include first-order motion or not.