Removal of irregular frequencies using the modified integral equation

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Boundary-integral formulations of wave-body interactions for surface-piercing bodies fail to produce accurate solutions near the irregular frequencies. Irregular frequencies have no physical meaning but are associated with the selection of the specific boundary-integral equation, and in no way reflect an irregularity in the solution of the original boundary-value problem. The problems associated with the irregular frequencies reduce, to some extent, the reliability of boundary-integral formulations. Several methods have been proposed for the removal of the effects of irregular frequencies. Ohmatsu(1975) suggested the addition of a rigid lid on the body waterplane area to suppress the interior resonance. This method is effective in two and three dimensions at the cost of using additional panels on the body waterplane area. Ogilvie and Shin(1977) removed the irregular frequencies in two dimensions by placing a point source on the interior free surface. The effectiveness of this method has not yet been demonstrated for three-dimensional bodies of arbitrary geometry. In an acoustic scattering problem, Burton and Miller(1971) obtained a modified integral equation by the linear superposition of the classical Green integral equation and its normal derivative with respect to the field point. This modified integral equation has no irregular frequencies and is applied to three-dimensional surface-wave body interactions. It turns out that the proper selection of the coupling constant in this linear combination is essential for the successful numerical implementation of the method. This method utilizes the same number of panels and unknowns as in Green equation and requires in addition the evaluation of the second spatial derivatives of the wave source potential.

Using Green's identity, the velocity potential on the body boundary can be shown to satisfy the equation

$$2\pi\varphi(x) + \iint_{S_b} d\xi\varphi(\xi)\frac{\partial G(\xi;x)}{\partial n} = \iint_{S_b} d\xi\frac{\partial\varphi(\xi)}{\partial n}G(\xi;x) \qquad (1)$$

which is Fredholm of the second kind. The normal derivative of the Green equation on the body boundary is given by

$$\frac{\partial}{\partial n_x}\iint_{S_b} d\xi\varphi(\xi)\frac{\partial G(\xi;x)}{\partial n_\xi} = -2\pi\frac{\partial\varphi(x)}{\partial n} + \iint_{S_b} d\xi\,\frac{\partial\varphi(\xi)}{\partial n}\,\frac{\partial G(\xi;x)}{\partial n_x} \qquad (2)$$

which is an integral equation of the first kind with poorer conditioning than the second-kind equation.

Each equation independently solves the radiation and diffraction boundary value problems. The irregular frequencies of the Green equation coincide with the eigenfrequencies of the interior Dirichlet problem, and those of its normal derivative with the eigenfrequencies of the interior Neumann problem. Their linear combination (3) has not irregular frequencies if the imaginary part of the coupling constant α is not zero.

$$2\pi\varphi(x) + \iint_{S_b} d\xi\, \varphi(\xi) \frac{\partial G(\xi;x)}{\partial n_\xi} + \alpha \frac{\partial}{\partial n_x} \iint_{S_b} d\xi\, \varphi(\xi) \frac{\partial G(\xi;x)}{\partial n_\xi}$$

$$= \iint_{S_b} d\xi\, \frac{\partial \varphi(\xi)}{\partial n} G(\xi;x) - 2\pi\alpha \frac{\partial \varphi(x)}{\partial n} + \alpha \iint_{S_b} d\xi\, \frac{\partial \varphi(\xi)}{\partial n} \frac{\partial G(\xi;x)}{\partial n_x} \qquad (3)$$

In the numerical solution one can directly descretize equation (3) or use the regularized form obtained by a premultiplication of these equation by a frequency independent operator, as suggested by Burton and Miller (1971). With this operation the unbounded operator involving the double normal derivative of the Green function can be rendered bounded. In light of the substantial increase in the computational effort associated with such operation $[O(n^3)]$, a direct numerical solution is here attempted. The body wetted surface is fitted with plane quadrilaterals, the velocity potential is assumed piecewise constant over the surface of each panel, and the integral equation is solved by collocation at the panel centroids. Due to the addition of the first kind integral equation, the conditioning of the modified integral equation worsens with increasing magnitude of the coupling constant α. On the other hand, for a small magnitude of α the effect of the irregular frequencies of the Green equation may not be removed. The selection of the "optimal" value for the coupling constant α has been carried out by minimizing the condition number of the modified equation. The mathematical library LINPACK is used to determine the condition number.

Numerical computation is performed for a sphere and a truncated vertical cylinder, and the results support following conclusions. A positive purely imaginary coupling constant α generates the best results. An optimal α exists, and is determined by minimizing the condition number of the modified integral equation at the first irregular frequency of the Green equation. The value of the optimal α is found to depend on the shape of the body geometry, but a value of 0.2 is found to generate satisfactory results for the bodies tested. Figures 1 and 2 show the performance of the method in evaluating hydrodynamic forces. Figure 3 shows the removal of common irregular frequencies of equations (1) and (2) when they are coalescent.

Burton, A. J. and Miller, G. F. (1971). The Application of Integral Equation Methods to the Numerical Solution of Some Exterior Boundary-value Problems. Proc. Royal Soc. London A323, 201-220.

Ogilvie, T. F. and Shin, Y. S. (1978). Integral-equation solutions for Time-dependent Free-Surface Problems. J. Soc. Nav. Arch. Japan Vol.143, 86-96.

Ohmatsu, S. (1975). On the Irregular Frequencies in the Theory of Oscillating Bodies. Papers Ship Res. Inst., No. 48.

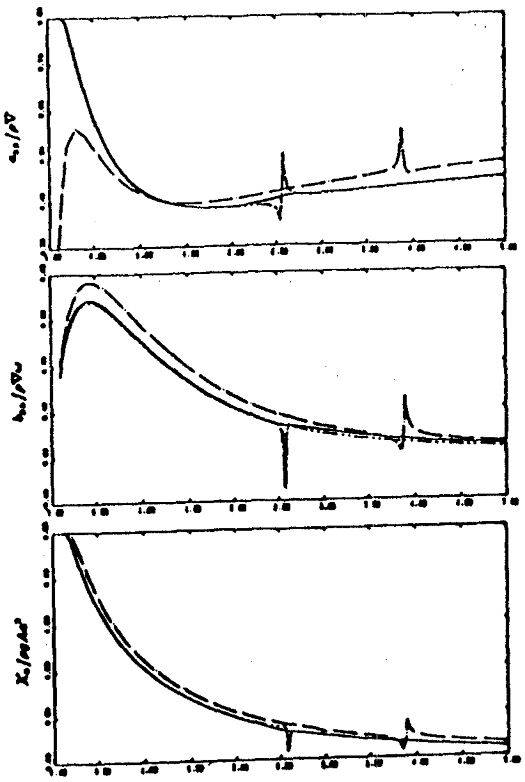

Figure 1 . Heave added mass, damping coefficients and modulus of the exciting force on the sphere, as functions of νa : (—··—), $\beta = 0$; ,(———), $\beta = 0.11$; ,(—·—), $\beta = \infty$

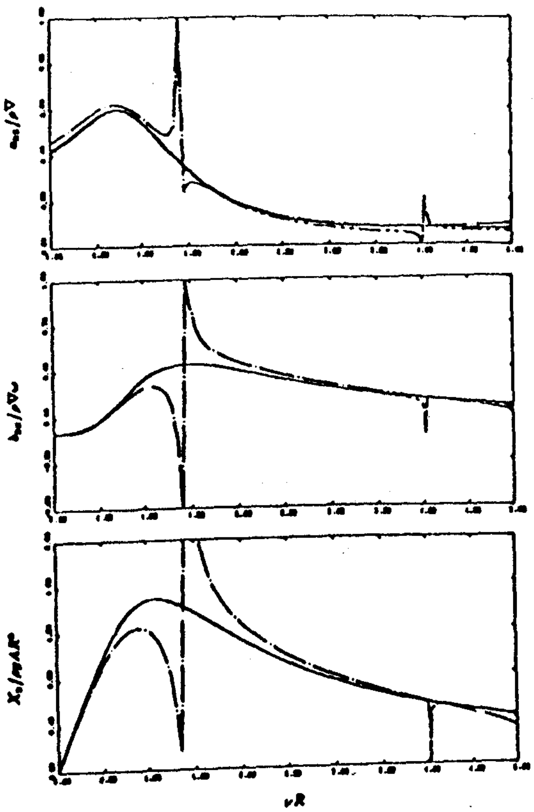

Figure 2. Sway added mass, damping coefficients and modulus of the exciting force on the cylinder, as functions of νR : (—··—), $\beta = 0$; ,(———), $\beta = 0.15$; ,(—·—), $\beta = \infty$

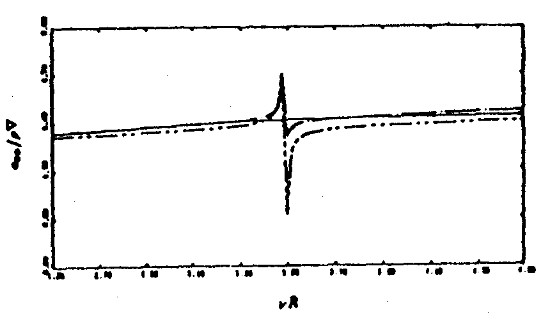

Figure 3. Heave added mass on the cylinder near the common irregular frequency of equations (1) and (2), as functions of νR : (—··—), $\beta = 0$; ,(———), $\beta = 0.15$; ,(—·—), $\beta = \infty$

Hung: Can you tell us something about the behavior of condition number in a multiple body situation?

Lee: Our experience with the TLP shows that conditioning of the discrete problem of the TLP is better than that of the single cylinder near the irregular frequencies [the irregular frequencies of the TLP are close to those of the cylinder, see Korsmeyer, *et al* (1988).] We think that the influence of the pontoon makes the condition better; but for multiple bodies without connections, it is expected that the conditioning will be similar to that of a single component body near irregular frequencies.

Papanikolaou: In your scheme you need an "optimal" value of α in order to minimize the condition number of the modified integral equation. How would you determine α for a body of arbitrary shape and what is the effort involved?

Lee: At this point, we do not have a simple method to predict the optimal α for bodies of arbitrary shape. A method would be to find the location of the first irregular frequency by solving the interior eigenvalue problem, then perform a numerical experiment to find the optimal α at that irregular frequency. However, $\alpha = 0.2$ is found to be a generally good estimation for a wide class of bodies, including cylinders with draft/radius ratios of $0.5 \rightarrow 3$ and spheroids with beam/length ratios of $1/8 \rightarrow 1$.

Kleinman: Did you observe a frequency dependence of the optimal coupling coefficient (for a fixed body) similar to that obtained by Kress, *et al* in the acoustic case?

Lee: The optimal coupling coefficient *is* dependent on the frequency, but determination of the optimal α at each frequency requires greater computational effort than solving the system. Kress derived a formula for optimal α in the acoustic Dirichlet problem for a sphere, (where there is no double normal derivative). However, it is difficult to employ his derivation in our problem, so we suggest and justify a simple optimal α.

Beck: Do you have any indication as to the extra computational effort which is required to solve your modified integral equation as compared with the normal integral equation when one is not near an irregular frequency?

Lee: The additional numerical effort in using the modified integral equation is the computation of the double normal derivative and the normal derivative with respect to the field point of the Green function. The additional computation can be broken down into two parts: the first part is frequency independent and requires a large additional effort (200%), but this needs to be performed only once; the second part is frequency dependent and requires a small additional effort (20% for finite depth and much less for infinite depth). The average additional effort is 20 to 25% when computations are performed for 20 frequencies in infinite depth.