A STRIP THEORY FOR WAVE RESISTANCE

E.O.Tuck

Applied Mathematics Department
University of Adelaide
AUSTRALIA

Suppose that we have to solve the linearised thin-ship boundary-value problem, namely to find a velocity potential $\phi(x, y, z)$ satisfying the three-dimensional Laplace equation

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

in $z < 0, y > 0$, subject to the free-surface condition

$$\kappa\phi_z + \phi_{xx} = 0$$

on the plane $z = 0$, and the linearised body boundary condition

$$\phi_y = U f_x(x, z)$$

on the ship's centreplane $y = 0$, together with suitable radiation conditions at infinity..

Let us forget for the moment that Michell solved this problem exactly and completely in 1898, and seek an apparently *ad hoc* approximate solution. Namely, in the spirit of the strip theory of ship motions, we simply replace the three-dimensional Laplace equation by the two-dimensional Laplace equation, dropping the term ϕ_{xx}, and see what then happens. Physically, this corresponds to assuming that any disturbance created by the ship travels only laterally; it concentrates attention on diverging waves, and neglects transverse waves.

Anyway, the solution can be written down immediately in the form

$$\phi = \phi_0 + \phi_1$$

where ϕ_0 is the solution with $\phi = 0$ on the free surface, namely

$$\phi_0(x, y, z) = \frac{U}{\pi} \int_{-\infty}^{0} f_x(x, \zeta) \log \sqrt{\frac{y^2 + (z - \zeta)^2}{y^2 + (z + \zeta)^2}}\, d\zeta$$

and ϕ_1 is the free-surface correction, namely

$$\phi_1(x, y, z) = -\frac{2\kappa U}{\pi} \int_0^\infty dk \int_{-\infty}^{0} d\zeta e^{k(z+\zeta} \cos ky \int_0^x f_\xi(\xi, \zeta) \frac{\sin\left[\sqrt{\kappa k}(x - \xi)\right]}{\sqrt{\kappa k}}\, d\xi$$

It is easy to check that all required conditions are satisfied. Incidentally, x is a time-like variable, and one way to derive the above is to identify the two-dimensional problem in the (y, z) plane as identical to a transient wavemaker problem, where the wavemaking plane $y = 0$ is executing a motion given by normal velocity Uf_x as a function of "time" x/U.

Now the linearised pressure is $p = -\rho U \phi_x$, and the wave resistance (from both sides of the symmetrical ship) is

$$R_W = 2 \int\int p(x,0,z) f_x(x,z)\, dx\, dz$$

The contribution from ϕ_0 is assumed to be zero; this follows by symmetry, by physical arguments, and formally by doing the integral provided that f_x vanishes at bow and stern, or at least that f_x takes equal and opposite values at bow and stern.

Hence

$$R_W = -2\rho U \int\int \phi_{1x}(x,0,z) f_x(x,z)\, dx\, dz$$

After an interesting piece of manipulation, this can be reduced to

$$R_W = \frac{2\rho U^2 \kappa}{\pi} \int_0^\infty \left[P^2(k) + Q^2(k)\right]\, dk$$

where

$$P(k) + iQ(k) = \int\int f_x(x,z) e^{kz + i\sqrt{\kappa k}x}\, dx\, dz$$

The above looks rather familiar; it looks a lot like Michell's integral. Indeed, one form of Michell's integral is

$$R_W = \frac{2\rho U^2 \kappa}{\pi} \int_\kappa^\infty \sqrt{\frac{k}{k-\kappa}} \left[P^2(k) + Q^2(k)\right]\, dk$$

with P, Q given by the same formula. This version of Michell's integral can be obtained by setting $k = \kappa \sec^2 \theta$ in the more common form which has θ as the wave direction.

Now it is rather obvious that the strip theory formula follows just by letting κ become small in Michell's integral. That is, the strip theory is just a high-speed limit of the thin-ship theory. The point of the derivation via a stripwise solution of the boundary-value problem, rather than just by letting $\kappa \to 0$ in Michell's integral, is that it points out how this limit is truly two-dimensional.

It is also clear that the present theory is a kind of slender body theory. Only when the draft is small is it consistent to retain both terms in the exponent of the integral for $P(k) + iQ(k)$. Indeed, retention of the full Kelvin boundary condition on $z = 0$, while replacing the 3D Laplacian by the 2D Laplacian, is justifiable on slender-body grounds at high speed, irrespective of whether the Neumann boundary condition on the body is approximated by the thin-ship boundary condition on $y = 0$.

From the high-speed slender-body point of view, the present wave-resistance integral is just the explicit small-beam limit of results that could have been obtained for general cross-section shapes (not necessarily of small beam/draft ratio) by solving a sequence of two-dimensional boundary-value problems with the full Kelvin free-surface condition. This is just what is done in the strip theory of ship motions, and has been advocated as an attack on steady ship-hydrodynamic problems by several authors. However, it does not seem to have been considered seriously as a tool for wave-resistance computation.

The attached Figure (wave-resistance computations for a parabolic strut with draft/length= 0.05) gives some indication why. Clearly the present theory (chain-dotted) is not of very much use in the more practical Froude-number range below the main peak – which does not even exist according to this theory! The fact that this theory is not valid for low Froude number is of course not surprising, in view of the fact that it assumes small κ, and also because it specifically neglects transverse waves. It does approach the solid curve (full Michell integral) as $F \to \infty$, but disappointingly slowly. At such high speeds, the applications are to special purpose vehicles, and planing tendencies have to be considered. It is interesting that the present ideas have much in common with some theories of planing surfaces, although the simplification to the body boundary condition is then usually for small draft rather than small beam.

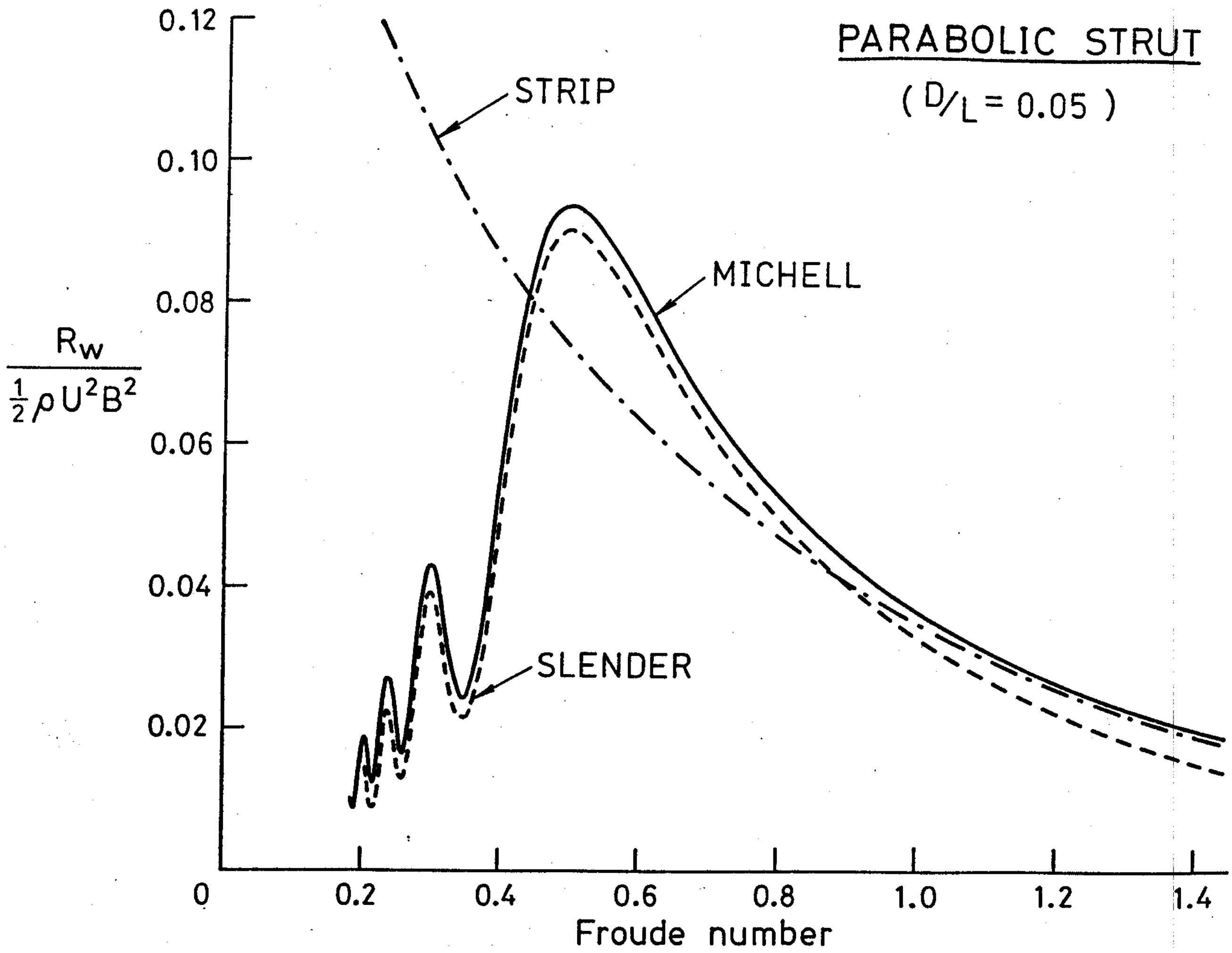
What we need is to put the transverse waves back into the picture. The connection with slender body theory suggests a pathway toward that end. Conventional slender-ship theory does not assume small κ, and hence the Kelvin boundary condition is replaced by the rigid-wall condition, at least in the near field. Waves are generated in the far field by representing the ship as a line of sources, of strength proportional to the slope of the section area curve.

In the original versions of that theory (including that of the present author), this line of sources was allowed to lie in the free surface. However, any submergence of the source line, up to the draft, is equally acceptable. Letting the source line lie in the free surface is undesirable on a number of grounds. Practically speaking, it exaggerates the generation of diverging waves at all speeds, and the resulting wave resistance prediction is unacceptable. Other choices of source line submergence (e.g. the section centroid, as in the dashed curve of the present Figure) do not suffer from this defect, and there seems some evidence that the centroid is a good choice, but the lack of uniqueness in choice of this parameter is disturbing.

However, what is interesting in the present context is that we also have another theory, the strip theory, which (presumably) does do a good job of predicting the diverging waves, precisely that aspect of the conventional slender-ship theory that is worst. There is therefore a possibility that a combination (dare one say 'unification') of these two slender-ship theories could provide a practical wave-resistance tool.

Note that it is not being advocated that the thin-ship limit of the strip theory, as represented by the integral derived here, be used for this purpose. After all, that is only a sub-set of Michell's integral, and there is no reason why the full Michell result could not be used, if the ship really is sufficiently thin. The merit of the strip approach for slender ships with non-small beam/draft ratios is that, at least for the diverging part of the spectrum, it improves upon Michell's integral in generating the diverging waves more realistically, the effective wavemaker being displaced away from the centreplane.

It has long been the present author's lament that Michell's wonderful integral has been under-used in the ship hydrodynamic community. Certainly it is disappointing to see vastly more complicated computational procedures being used in situations where there is no improvement in quantitative predictive capability. If all one wants is qualitative information ('ship A has less resistance than ship B') why not use Michell? Michell is not too bad quantitatively, also. If quantitative improvements are to be sought, they should be sought in a manner firmly rooted in a physical understanding of the ship-hydrodynamic problem, not in blind number-crunching. One of the physical features that should be borne in mind is the transverse-diverging distinction in the wave pattern, and the present discussion throws light upon the role of the diverging waves in the wave resistance problem.


PARABOLIC STRUT
(D/L = 0.05)
STRIP
MICHELL
SLENDER
0.12
0.10
0.08
0.06
0.04
0.02
$\frac{R_W}{\frac{1}{2}\rho U^2 B^2}$
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Froude number

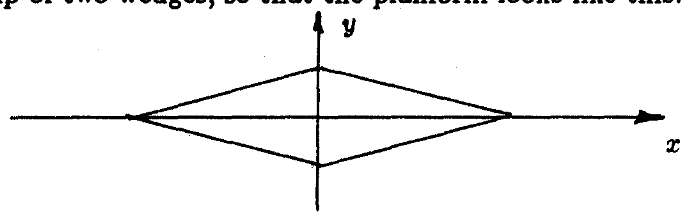
Maruo: The formulation by Tuck is equivalent to the solution in the time domain. However it is not a complete expression for a slender ship. The complete expression can be obtained by the perturbation expansion of the Fourier transformed Green function, which takes different forms at small and large wave numbers. By combining these expressions by means of matching in the far-field, the asymptotic expression for a slender ship is obtained. It consists of a part for the diverging wave and a part for the transverse wave. Numerical computation with respect to the Wigley hull based on this formulation shows a good agreement with measured results.

Reference: 10th Weinblum Memorial Lecture. "Evolution of the Theory of Slender Ships" Schiffstechnik to be published.

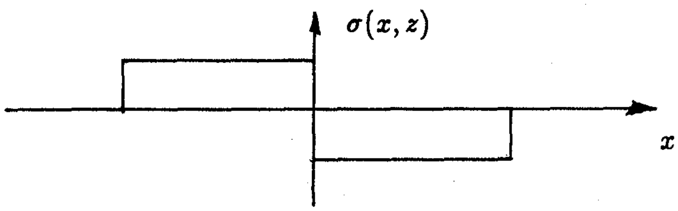
Ogilvie: Havelock produced the first slender-body theory for wave resistance by assuming a line distribution of sources. He set the source strength at each section proportional to the rate of change of cross-section area, and he placed the source at the centroid of the section. Havelock, of course, used the full Kelvin-Havelock source potential, and so he included transverse, as well as diverging waves. Thus he appears to have anticipated Tuck's main results – by about 40 years.

Several times in the last two decades, I have argued with the author that using a line of sources on the free surface was the worst possible choice in a slender-body theory for steady ship motion, and he always argued back that any other choice was somehow "inconsistent." Following one of these friendly discussions, I made the following analysis.

Consider a "ship" made up of two wedges, so that the planform looks like this:

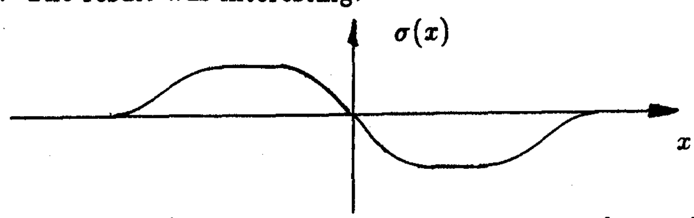


The thin-ship source distribution then looks like this



for $-h < z < 0$. Tuck's earlier slender-ship theory leads to a line distribution of sources of the same form, located on z=0, but it then yields infinite wave resistance. Such a distribution of sources is apparently usable (although perhaps not accurate) when they are distributed over the center plane but not when they are concentrated at the free surface.

So I asked, what line distribution of sources would give the same wave resistance as the centerplane distribution of sources? The result was interesting:



The smooth curve was the result. [The amount of smoothing depends on the draft of the corresponding centerplane distribution.]

A sufficiently smooth line distribution of sources at the free surface at least does not produce the pathological result of infinite wave resistance. What is more interesting is that, for the line distribution to reproduce the results of the centerplane distribution, it must extend out ahead of the ship. I do not know how far one should pursue the implications of this.

I certainly agree with Tuck that the degenerate form of Michell's integral should not be used for computing wave resistance. In fact, my attempts to develop a slender-body theory were motivated primarily by the hope of accounting better for the production of diverging waves. The formulation of the thin-ship body condition requires that the potential be expanded in a Taylor series with respect to the centerplane, *e.g.*,

$$\phi_x(x, f(x,z), z) = \phi_x(x, o, z) + f(x,z)\phi_{xy}(x, o, z) + \ldots$$

But ϕ_{xy} is very large in magnitude near the centerplane where diverging waves are significant, and so it is not valid to use the truncated series there. Thus, thin-ship theory violates one of the assumptions made in its derivation. It is for this reason that I believe attempts to solve the Neumann-Kelvin problem should be continued.

Finally, I note that Cummings, in about 1956, proposed a physical basis for incorporating the transverse waves into a slender-body theory of wave resistance. Cummings made some mistakes, but I think his argument in this part was valid, if not formally developed.