ON THE CALCULATION OF THE SECOND-ORDER FREE-SURFACE INHOMOGENEITY FOR 3D SHIP MOTION PROBLEMS

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INTRODUCTION

In recent years there is a great interest in the prediction of second-order interactions of waves and ocean structures. Most of the published work deal with the case of vertically axisymmetric bodies. Some authors are interested in the calculation of the second-order exciting forces only while others solve for the second-order velocity potential.

The method presented in this paper addresses the calculation of the second-order potential and especially the way of calculation of the integral of the second-order free-surface inhomogeneity, for the case of finite water depth. It is based on the derivation of elementary potentials accounting for the second-order free-surface effects due to the interaction of two pulsating sources or the interaction of a source with the incident wave potential. The determination of these potential functions simplifies greatly the solution of the BVP for the second-order velocity potential, since the remaining part of the solution becomes tractable by the common methods of the first order solution. The method implies no restrictions as to the application to bodies of arbitrary shape.

STATEMENT OF THE PROBLEM

The main difference between first- and second-order Boundary Value Problems for the corresponding velocity potential is the existence of a nonhomogeneous term in the free-surface boundary equation. Because of this term, an integration over the free surface is required when applying Green's theorem for the evaluation of the second-order potential or simply the exciting forces. In the present paper we describe a method for the analytical evaluation of this integration in the part of the free surface outside a circle of certain radius surrounding the body. The integration inside this circle can be done numerically in a sufficient way.

Decomposition of the Potential

It is well established that the first-order velocity potential can be approximated by the superposition of a finite number of pulsating sources distributed over the wetted surface of the structure (Green's function method). Now, each of these elementary sources can be thought to interact with the others and with the incident first-order wave potential through the nonlinear terms in the free-surface condition. The results of these interactions contribute to the second-order velocity potential. There are other contributions to the second-order potential too, but they can be calculated by methods completely analogous to those of the first-order problem. Thus, the most important of the second-order velocity potential can be thought to be decomposed into a number of elementary potentials arising from the interaction of two pulsating sources or one pulsating source and the incident wave.

The Elementary Potentials

Let Oxyz be a right handed Cartesian coordinate system with O at the free-surface and Oz pointing vertically upwards. Let $F(x_F, y_F, z_F)$ and $E(x_E, y_E, z_E)$ be the points where two pulsating sources are located (see Fig. 1). We denote by $\varphi_{SS}^{(2)}$ the second order potential resulting from the interaction of the two sources and $\varphi_{SI}^{(2)}$ the second-order potential resulting from the interaction of the source at point F and the incident wave potential.

Let $\alpha_{SS}^{(2)}$ and $\alpha_{SI}^{(2)}$ be the nonhomogeneous terms in the free surface boundary condition for $\varphi_{SS}^{(2)}$ and $\varphi_{SI}^{(2)}$ respectively

$$g\,\frac{\partial \varphi_{SS}^{(2)}}{\partial z} - 4\omega^2 \varphi_{SS}^{(2)} = \alpha_{SS}^{(2)}\,, \qquad z = 0$$

$$g\,\frac{\partial \varphi_{SI}^{(2)}}{\partial z} - 4\omega^2 \varphi_{SI}^{(2)} = \alpha_{SI}^{(2)}\,, \qquad z = 0 \tag{1}$$

$$\alpha_{SS}^{(2)} = -\frac{j\omega}{2g}\,\varphi_F^{(1)} F \varphi_E^{(1)} + j\omega\,\vec{\nabla}\varphi_F^{(1)}\,\vec{\nabla}\varphi_E^{(1)}$$

$$\alpha_{SI}^{(2)} = -\frac{j\omega}{2g}\,[\varphi_I^{(1)} F \varphi_F^{(1)} + \varphi_F^{(1)} F \varphi_I^{(1)}] + 2j\omega\,\vec{\nabla}\varphi_I^{(1)}\,\vec{\nabla}\varphi_F^{(1)} \tag{2}$$

$$F = \frac{\partial}{\partial z}\left(g\,\frac{\partial}{\partial z} - \omega^2\right)$$

where $\varphi_F^{(1)}$, $\varphi_E^{(1)}$ mean the first order velocity potentials (Green's functions), due to the sources at F and E and $\varphi_I^{(1)}$ the first order incident wave potential.

We shall calculate the values of $\varphi_{SS}^{(2)}$ and $\varphi_{SI}^{(2)}$ at a point P located vertically under O ($x_P=0$, $y_P=0$, $z_P\leq 0$). The assumptions made here, namely $y_F=0$ and $x_P=0$, $y_P=0$ do not affect generality, but simplify greatly the presented results. More complicated formulas are developed for the general case.

Using Green's third theorem over the fluid domain, we obtain:

$$-4\pi\varphi_{SS}^{(2)}(P) = \frac{1}{g}\iint_{S_F} G_2(P,Q)\,\alpha_{SS}^{(2)}(Q)\,dS_Q$$

$$-4\pi\varphi_{SI}^{(2)}(P) = \frac{1}{g}\iint_{S_F} G_2(P,Q)\,\alpha_{SI}^{(2)}(Q)\,dS_Q \tag{3}$$

where S_F is the free surface ($z=0$), Q a point of the free surface, and $G_2(P,Q)$ the Green's function of a pulsating source at Q, referred to field point P and for frequency 2ω.

We use John's series |4| to represent $\varphi_F(Q)$, $\varphi_E(Q)$, $G_2(P,Q)$. Only one term in the series has proved to be sufficient for the desired accuracy, since higher terms converge rapidly to zero for the distances of Q from P,F,E that we use in our application. Nevertheless the theory is also expanded to include more than one terms.

$$\varphi_F^{(1)}(Q) = G(Q,F) = \gamma(z_F)\,H_0(k_1 r_F)$$

$$\varphi_E^{(1)}(Q) = G(Q,E) = \gamma(z_E)\,H_0(k_1 r_E) \tag{4}$$

$$G_2(P,Q) = \beta(z_P)\,H_0(k_2 R)$$

where k_1 and k_2 the wave numbers of frequency ω and 2ω respectively and H_0 the Hankel function of first kind and of zero order.

Next, we introduce Graf's addition theorem |5| to represent $H_0(k_1 r_F)$ and $H_0(k_1 r_E)$ as :

$$H_0(k_1 r_F) = J_0(k_1\rho_F)H_0(k_1R) + 2\sum_{n=1}^{\infty} J_n(k_1\rho_F)H_n(k_1R)\cos n\vartheta$$

$$H_0(k_1 r_E) = J_0(k_1\rho_E)H_0(k_1R) + 2\sum_{n=1}^{\infty} J_n(k_1\rho_E)H_n(k_1R)\cos n(\vartheta-\varphi) \qquad (5)$$

From eq. (4c) we see that $G_2(P,Q)$ is independent of ϑ, so we can write

$$\iint_{S_{F_2}} G_2(P,Q)\,\alpha\, dS_Q = \int_{R_0}^{\infty} G_2(P,Q)\,R\,dR\int_{-\pi}^{\pi}\alpha\, d\vartheta \quad,\ \alpha=\alpha_{SS}^{(2)} \text{ or } \alpha=\alpha_{SI}^{(2)} \qquad (6)$$

Using equations (2), (4), (5), we can prove that

$$\int_{-\pi}^{\pi}\alpha_{SS}^{(2)}\, d\vartheta = \sum_{n=0}^{\infty} A_n H_n^2(k_1R)$$

$$\int_{-\pi}^{\pi}\alpha_{SS}^{(2)}\, d\vartheta = \sum_{n=0}^{\infty} B_n J_n(k_1R)\,H_n(k_1R) \qquad (7)$$

We introduce eq. (7) in eq. (3) and use asymptotic expansions for the Bessel and Hankel functions, so the above equations take the form of series expansions :

$$G_2(P,Q)\,R\int_{-\pi}^{\pi}\alpha\, d\vartheta = \sum_n C_n \frac{e^{icR}}{R^{n+\frac{1}{2}}} \quad,\quad \alpha=\alpha_{SS}^{(2)} \text{ or } \alpha=\alpha_{SI}^{(2)} \qquad (8)$$

After that, it remains to calculate a series of integrals of the form :

$$\int_{R_0}^{\infty}\frac{e^{icR}}{R^{n+\frac{1}{2}}}\, dR$$

These integrals can be claculated by partial integration and use of Fresnel Sine and Cosine functions, as suggested in Ref. |3|.

Discussion of Results

The method described here has been applied to several cases with different positions of the sources, wave lengths, wave angles and water depths. The results were chequed against numerical integration using FINGREEN for the calculation of the Green's functions. The agreement was excellent in all cases.

Figures 2 and 3 show the real part of the value of the integrals

$$\int_{-\pi}^{\pi} G_2(P,Q)\,\alpha_{SS}^{(2)}\,R\,d\vartheta \quad \text{and} \quad \int_{-\pi}^{\pi} G_2(P,Q)\,\alpha_{SI}^{(2)}\,R\,d\vartheta$$

respectively, as functions of R calculated by the present method (solid line) and numerical integration (dotted line). The results are almost identical even for small R approximately equal to max $\{\rho_E, \rho_F\}$.

The case presented here is :

$r_F = 30$ m, $r_E = 60$ m, $z_F = -10$ m, $z_E = -7$ m, $\varphi = 1.1$ rad, water depth $h = 100$ m, wave amplitude $\alpha_W = 1$ m, wave length $\lambda = 120$ m, wave angle $\beta = 2.5$ rad.

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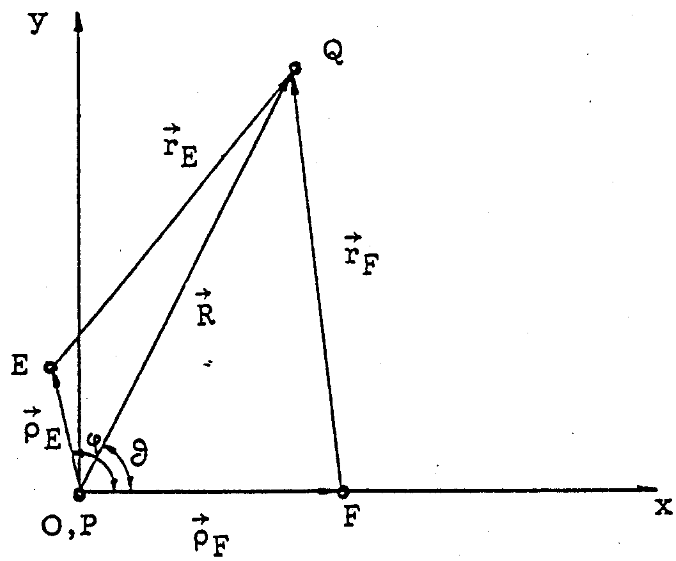

Figure 1

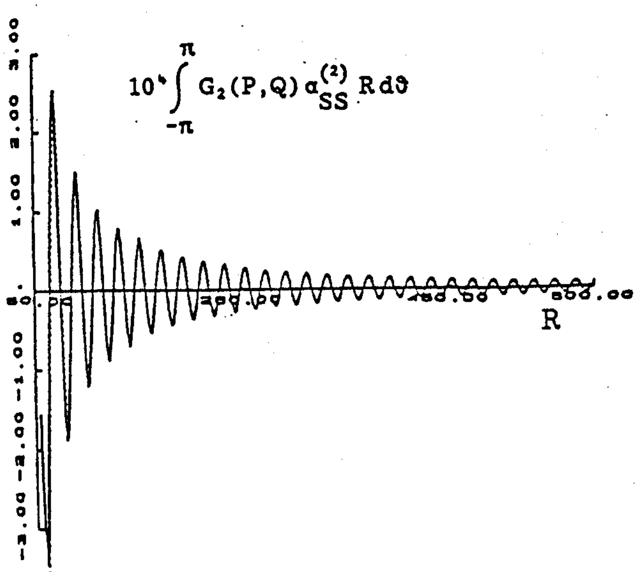

Figure 2

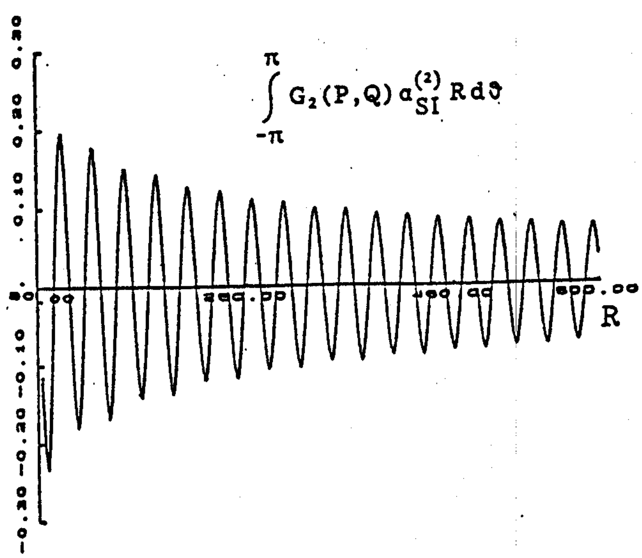

Figure 3

Newman: There seems to be a close connection between this work and Sclavounos' analysis of "Second-order Green Functions." The latter work also suggests how to make the transition from finite to infinite depth.

Zaraphonitis & Papanikolaou: There is a connection to Sclavounos' work (Ref. [2]), as far as the concept of the "second-order Green functions" is concerned. In the present work an algorithm is presented to evaluate efficiently the free-surface inhomogeneity by analytical means. The method is valid for arbitrarily-shaped bodies, but only for finite water depth. We do not know of any related work extending this procedure to the infinite depth case.

Martin: You decompose your potential into two parts, one of these involves the inhomogeneous free-surface condition (with $a_S^{(2)}$ on the right-hand side) but no immersed body. How do you define $a_S^{(2)}$ over the portion of the free surface which was previously occupied by the body?

Zaraphonitis & Papanikolaou: It is assumed that the free-surface boundary condition is valid also within the body, *i.e.* along the extension of the free surface, $y = 0$, inside the body. Thus, a "variable pressure distribution" problem is solved leading to the first part of the velocity potential. On the basis of this, a modified kinematic body boundary condition for the second part of the velocity potential can be formulated and solved in complete analogy to the first-order problem.

Moreover, instead of solving for two separate second-order potential sub-problems, a direct solution to the problem can be provided on the basis of Green's third theorem by assuming pulsating dipoles of unknown strength on the body surface and pulsating sources of known strength both on the body and the free surface. Then, the free-surface inhomogeneity is part of the integrand for an integral going from the body to infinity, as given in this paper.