Wave-Current Interaction Effects on Large Volume Structures

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Abstract

A theoretical method to analyze motions and loads on a large volume structure in current and regular incident deep water waves is presented. The structure is free to oscillate harmonically in six degrees of freedom. The fluid motion is incompressible and the effects of flow separation are neglected. It is discussed under what current and wave conditions the flow around the body will not separate. It is pointed out that the Keulegan Carpenter number KC and the ratio between the current velocity U and a representative amplitude U_M of the oscillatory fluid motion are important parameters. It is shown that for bodies without sharp corners that there exist many practical situations involving wave-current effects on large volume structures, where flow separation does not occur. The conclusions are partly documented by experimental results.

The theoretical solution for the velocity potential is written as a series expansion in the wave amplitude ζ_a and the current velocity U. The problem is solved to first order in ζ_a and first order in U. It is assumed that ζ_a/L, the wave slopes of the different wave systems and the Froude number $U/\sqrt{Lg}$ are asymptotically small. Here g is the acceleration of gravity and L is a characteristic length of the body. In the case of a floating half sphere L may be chosen as the diameter. A consequence of the analysis is that any effects of the steady wave systems are neglected. Further, $\tau = \frac{\omega U}{g} < \frac{1}{4}$, where ω is the frequency of oscillation of the body. This implies that the body generates wave systems in all directions.

The steady motion potential ϕ_S satisfies the rigid free surface condition. Since the effect of flow separation is neglected, ϕ_S can be found by a standard numerical method. In the numerical results for a floating half sphere to be introduced later in the text, an analytic solution for ϕ_S was used.

The time dependent velocity potential is split into components $\Phi_k \eta_k (k = 1,6)$ associated with the six motion modes η_k, the incident wave potential $\Phi_0 e^{i\omega t}$ and a diffraction potential $\Phi_7 e^{i\omega t}$. Here, i means the complex unit and t the time variable. It can be shown that $\Phi_k (k = 1,6)$ satisfy correctly to $O(U)$ the following free surface condition

$$-\omega^2 \Phi_k + 2i\omega \nabla\phi_S \cdot \nabla\Phi_k + i\omega[\frac{\partial^2 \phi_s}{\partial x^2} + \frac{\partial^2 \phi_S}{\partial y^2}]\Phi_k + g\frac{\partial \Phi_k}{\partial z} = 0 \text{ on } z = 0 \qquad (1)$$

Here (x, y, z) is a right-handed coordinate system with z-axis positive upwards and the origin in the mean free surface. The sum of the diffraction potential $\Phi_7 e^{i\omega t}$ and the incident wave potential $\Phi_0 e^{i\omega t}$ satisfies also eq.(1). In the body boundary condition, the interactions with the steady motion potential are taken care of. In addition a radiation condition is specified.

We write the solution at some distance from the body as a sum of multipoles (including sources) with singularities inside the body. The miltipoles satisfy the radiation condition and the free surface condition (1) with $\nabla\phi_S = U\underline{e_r}$. Here $\underline{e_r}$ means a unit vector in the current direction. For a general body several singularity points are used. In the numerical example with a floating half sphere to be presented later in the text, only one singularity point in the center of the sphere ($i.e.(0,0,0)$) was used. The Green's function $Ge^{i\omega t}$ representing a source function can then be written as

$$G = -\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{du\ g\ \lambda_0}{(g-2\omega U\ cos\ u)}[e^{\varsigma_0}E_1(\varsigma_0)-\frac{1}{\varsigma_0}]$$

$$+2i\int_{-\pi}^{\pi}H(-cos(\theta-u))\frac{\lambda_0 e^{\varsigma_0}}{(g-2\omega U cos u)}du \tag{2}$$

Here

$\lambda_0 = \frac{\omega^2}{(g-2\omega U\,cos\,u)}$

$x = rcos\theta$

$y = rsin\theta$

$\varsigma_0 = \lambda_0(z + ircos(\theta - u))$

Further, H is the Heaviside step function and E_1 is the exponential integral. The current direction is assumed to be along the x-axis when deriving eq.(2). A similar expression for G has been derived by Grekas (1). Higher order multipoles are obtained by differentiating the source expression with respect to the singularity coordinates. The coefficients in the multipole expansion were determined by combining it with the following integral expression

$$4\pi\Phi_k(x_1,y_1,z_1) = \int_S[\Phi_k\frac{\partial}{\partial n}\frac{1}{R} - \frac{\partial\Phi_k}{\partial n}\frac{1}{R}]dS(x,y,z) \tag{3}$$

where $S = S_B + S_{F1} + S_C$ and $R = \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}$. Further S_B is the mean wetted body surface and S_C is a vertical cylindrical control surface extending from the mean free surface to the sea bottom with cylinder axis coinciding with the z-axis. S_{F1} is the mean free surface inside S_C. At the intersection between S_C and S_{F1}, the simplified free surface condition with $\nabla\phi_S = U\underline{e_r}$ is assumed valid. Further $dS(x,y,z)$ is a surface element and $\underline{n}$ is the normal vector to dS. The positive direction of $\underline{n}$ is into the fluid. Equation (3) is rewritten by replacing $\partial\Phi_k/\partial n$ with the free surface condition (1) on S_{F1} and the body boundary condition on S_B. At S_C, the multipole expansion of Φ_k is used. By letting points (x_1,y_1,z_1) in (3) approach points on the bounding surface S, we obtain a Fredholm integral equation of the second kind. In the numerical solution S_B and S_{F1} are divided into plane quadrilateral elements. The velocity potential is assumed constant over each element. First order derivatives of Φ_k along the free surface is numerically approximated in terms of Φ_k on adjacent elements. The integral equation is satisfied at the midpoint of each element. This gives N number of equations. Number of unknowns are $N + N_C$, where N_C is the number of terms used in the multipole expansion. Sufficient number of equations for the unknows are obtained by matching inner and outer solutions, $i.e.$, equation (3) and the multipole expansion, at the control surface S_C. This is done by the least square method. The consequence of using a simplified free surface conditon (1) with $\nabla\phi_S = U\underline{e_r}$ is discussed.

Having obtained Φ_k, we can find added mass and damping, wave excitation loads, motions and wave drift forces. Numerical results for a floating halfsphere are given. The results show that drift forces in particular, have a strong dependency on the current velocity. The trend is similarly as pointed out by Zhao and Faltinsen (2) in the two-dimensional case.

References

(1) Grekas, A. (1981): "Contribution a l'etude Theorique et Experimentale des Efforts du Second Ordre et du Comportement Dynamique d'une Structure Marine Sollicitee par une Houle Reguliere et un Courant" Thèse de Docteur Ingenieur (Ecole Nationale Superieure de Mecanique).

(2) Zhao, R. and Faltinsen, O.M. "Interaction between Waves and Current on a Two-Dimensional Body in the Free Surface." Accepted for publication in Applied Ocean Research.

Reed: Please clarify the definition of the steady, second-order force. Does F_1 include "drag" due to current? How large is the drag due to steady flow relative to the second-order force; and are separation effects included?

Zhao & Faltinsen: Separated flow effects are not included in the calculation of the mean second-order force. Our experiments indicate that the flow will not separate around bodies without sharp corners if the Keulegan-Carpenter number is low and the current velocity is smaller than the amplitude of the horizontal wave velocity component at the free surface. If the flow is not separating in combined wave and current, it would be incorrect to add current forces in still water to predict mean second-order forces.

The relative order of magnitude between the drag due to steady flow and the second-order forces will depend on the Froude number, based on steady flow, and the Keulegan-Carpenter number. The ratio between the drag force due to current and the second-order force can be written as $\frac{C_D}{2C_W}\pi^3\frac{F_n^2}{KC^2}$ (F_n = Froude number based on current velocity and draft, KC = Keulegan-Carpenter number based on incidental wave oscillatory motion at the free surface, C_D = drag coefficient in current only, C_W = non-dimensionalized second-order force). If $C_D = 0.2$, $C_W = 0.5$, $F_n = 0.06$, $KC = 0.5$ we see that this ratio is 0.09.

Kashiwagi:

1. Will your method give reasonable results if the contribution from the steady perturbation potential is neglected in the free surface condition?

2. Did you confirm that your results satisfy the Haskind-Newman relations with forward speed?

Zhao & Faltinsen:

1. We have tried to neglect the effect of the local steady flow around the body, *i.e.* to approximate the steady flow by the far-field steady flow over all of the free surface, but the numerical results were not satisfactory. Energy relations between the damping coefficients and the radiated waves were not satisfied.

2. In the calculation of wave excitation loads we have generalized the Haskind-Newman relation. We found that the generalized Haskind-Newman relation agreed satisfactorily with results following from direct pressure integration.

Wu: I would like to congratulate the authors for showing us interesting solutions to a difficult problem of great importance. In view that little is known even for a beam current transversely incident on a ship's hull, without any surface waves, I wonder if it would be desirable to first deal with the surface current force alone before such further complications are included, as those effects due to the orbital velocity of fluid particles on the unsteady movement of the flow separation points (or lines) and in turn on the variations in the resultant hydrodynamic forces and moments.

Zhao & Faltinsen: It is not necessarily easier to consider the effect of current only. When $U/U_M < 1$, (U = current velocity, U_M = amplitude of horizontal wave velocity component at the free surface) the flow around bodies without sharp corners is not likely to separate for small Keulegan-Carpenter numbers. Our experiments with a hemisphere show this. If $U/U_M \geq 1$ and if the flow separates, it may be more desirable to first analyze the effect of current only. In our analysis we have assumed that the flow does not separate.

Sclavounos: In wave-current-body interaction problems it is often assumed that the principal effect of the current can be accounted for merely by the Doppler shift. The linear forces in your computations seem to indicate otherwise. Is it possible to identify the principal mechanism responsible for the differences between $U = 0$ and $U > 0$ in the linear and drift forces?

Zhao & Faltinsen: We think an important mechanism is the effect of the local, steady flow around the cylinder. The use of a Doppler shift would, therefore, not be sufficient to explain the results.