Excitation of Upstream Nonlinear Wave Disturbances by a Ship Moving in a Shallow Channel with Sloping Side Walls

T.R. Akylas & J. Mathew
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139 U.S.A.

The generation of upstream nonlinear wave disturbances by a ship moving with transcritical speed in shallow water has received considerable attention in recent years. Previous theoretical and experimental work has focussed on the wave pattern induced by a ship propagating in a channel of finite width bounded by vertical side walls. In this case, it is now well known that a series of straight-crested solitary waves are generated periodically upstream. A review of this work and references can be found in the survey article by Akylas (1987).

In the present paper, we study the excitation of nonlinear waves by a moving pressure distribution (modelling a ship) in a shallow channel bounded by sloping side walls. There are at least two reasons which provide motivation for this work: first, the presence of sloping side walls implies that the wave pattern has to be three-dimensional everywhere; so it is of interest to know whether or not the strong upstream response found in the previous investigations is fundamentally related to the existence of straight crested solitary waves in a channel with vertical side walls. We remark that very recent numerical work by Pedersen (1988) indicates that a source moving in an unbounded sea (where no solitary waves with finite mass are possible) generates only weak three-dimensional upstream disturbances and a steady state is reached eventually. Secondly, the propagation of three-dimensional nonlinear long waves in a channel with non-rectangular cross-section is of interest in its own right. Peregrine (1968) has already pointed out that, when the channel width is much larger than the water depth (which is usually the case), strong three-dimensional effects are present; this is in accordance with experimental observations which indicate that an undular bore develops curved crests as it propagates in a channel with trapezoidal cross-section. These effects cannot be accounted for by the existing quasi-two-dimensional theories based on the KdV equation.

The geometry of the problem is shown in figure 1. Away from the sloping side wall, where the water depth is uniform, the propagation of nonlinear long waves travelling primarily along z is governed by the familiar KP equation. Using matched asymptotic expansions, it is shown that the effect of the sloping wall can be reduced to a single boundary condition at $x = 0$; the important parameter describing the presence of the beach is A/h_0^2, a measure of the area under the beach profile.

Numerical calculations have been carried out for slender pressure distributions advancing at critical speed in a channel of fixed width and for various values of the slope parameter A/h_0^2. As expected, for small A/h_0^2 the upstream response is qualitatively similar to a train of solitary waves and three-dimensional effects are weak. As A/h_0^2 is increased, upstream influence continues to be strong but the pattern loses its regularity and it is dominated by three-dimensional effects. However, for A/h_0^2 close to 1, the pattern seems to become regular again; a more or less periodic train of three-dimensional waves of elevation with curved crests are generated upstream (see figure 2.) It is important to note that waves of this sort have been observed experimentally at the head of an undular bore propagating in a channel with trapezoidal cross-section; of course, some wave breaking was observed close to the side walls where the wave steepness is large.

To explain these phenomena, we look for permanent-wave solutions of the KP in a channel with sloping walls. For small A/h_0^2, a perturbation expansion shows that no solitary waves are possible. Numerical calculations confirm this conclusion. However, for A/h_0^2 close to 1, periodic waves, very similar to those generated upstream (see figure 2) exist; as the wave period is increased, there is very little interaction between individual wave crests so that three-dimensional solitary waves (or nearly solitary waves) seem to be possible. Based on these preliminary calculations, it appears that the existence of solitary waves is not essential for the generation of upstream disturbances; however, the upstream response consists of a series of regular wave disturbances only if solitary waves are possible.

Currently, this work is under progress and further developments concerning the periodic nature of the upstream response, the effect of the sloping walls on the mass transferred upstream, comparison with experiments, and other related issues will be discused.

References

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Pedersen (1988) J. Fluid Mech., Volume 196.
Peregrine (1968) J. Fluid Mech., Volume 32.

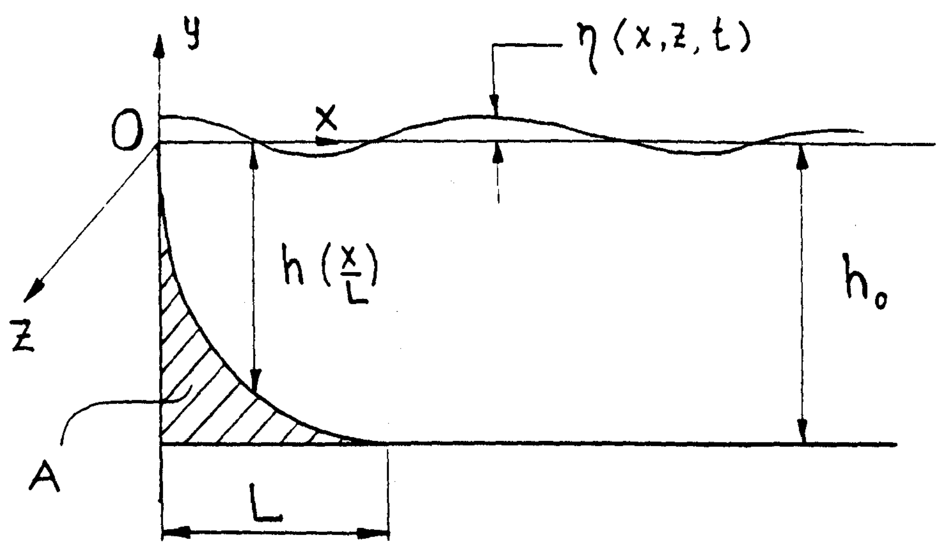

Figure 1

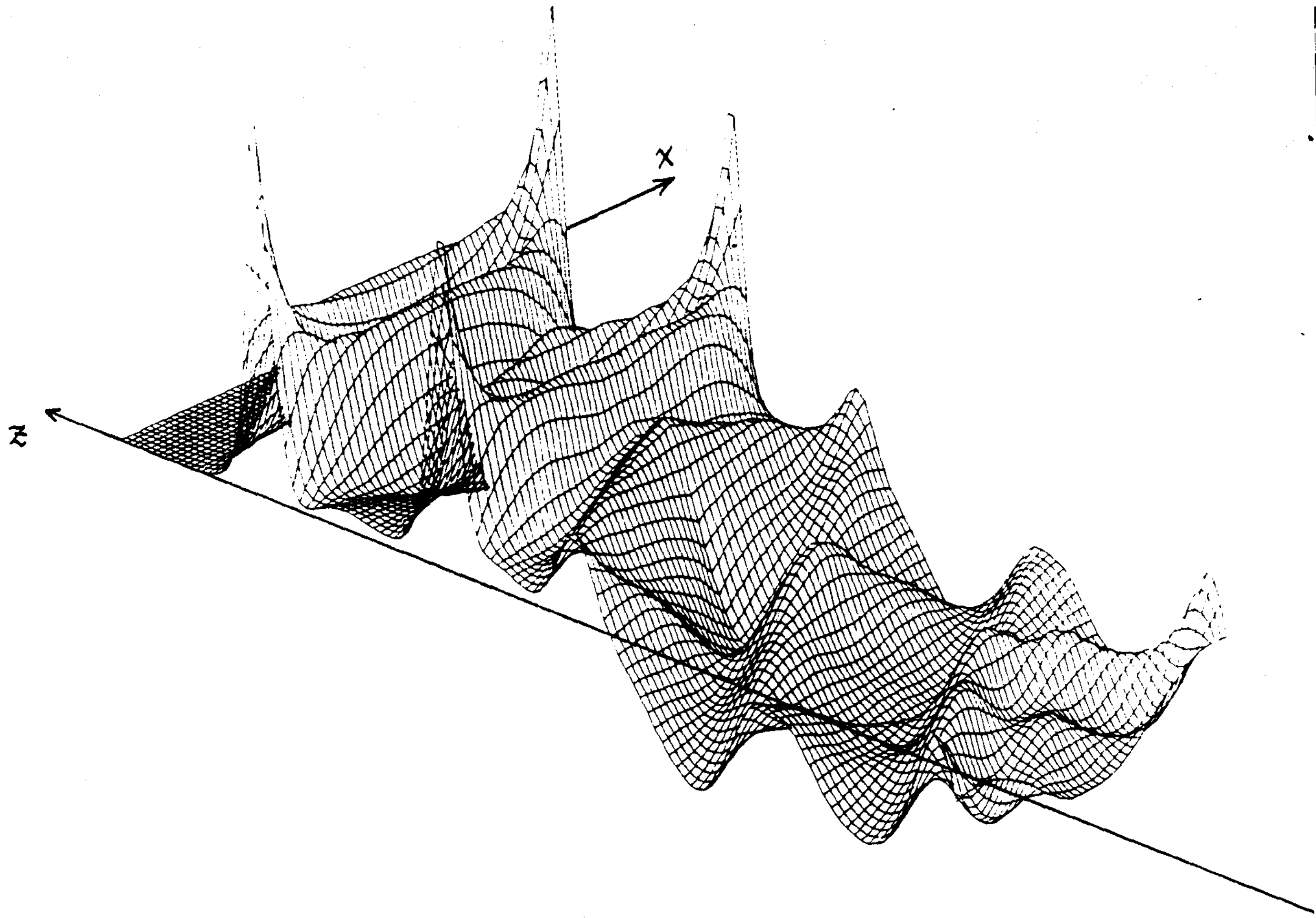

Figure 2

DISCUSSION

Pedersen: Upstream influence does not only depend on the existence of waves travelling with supercritical speed. These waves should also be able to travel without significant loss of energy. In your case diffraction at the sloping side-walls may cause a considerable loss of energy and disenable periodic generation of upstream crests and you may get only a finite number as in unbounded sea. Such a diffraction will probably be accompanied by a curvature in the crest at the middle part of the channel. Has this been observed?

Akylas & Mathew: We thank Dr. Pedersen for his insightful comments. We have carried out our calculations to quite long times (enough to observe at least ten upstream wave disturbances). We have no indication that upstream influence weakens with increasing time. On the other hand, we do observe wave crest curvature.

Tuck: Why move at "critical" speed? It has always seemed to me that the only interest in this type of study is to determine the lightest *subcritical* speed at which upstream waves are *not* generated, as a function of ship and channel geometry. In particular, surely subcritical speeds would be more interesting in the present case, since then the speed would go critical at some point on the slope, leading to a caustic phenomenon.

Akylas & Mathew: The results that we presented were computed assuming critical speed based on the uniform depth h_0. We plan to vary the source speed in order to study its effect on the upstream response. We agree with Professor Tuck that for a non-rectangular channel cross-section, critical conditions are expected to occur at a source speed $\sqrt{gh}$ with $h<h_0$.