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# FORWARD-SPEED EFFECT IN HEAD-WAVE DIFFRACTION OVER THE FOREBODY OF A SLENDER HULL

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At the 3rd Workshop, Ando and Cumming [1] reported the model test data in regular head waves showing a peculiarly nonlinear trend of the amplitude $|P|$ of the wave-induced pressures $P$ on the slender hull of a destroyer with increasing forward speed $U$. This paper shows that the observed trend cannot be explained by slender-body theory and method of asymptotic expansions even if the second-order terms are included in the Bernoulli equation for the hydrodynamic pressure on the hull surface.

The mathematical formulation used here for the diffraction potential is similar to Faltinsen [2], Maruo and Sasaki [3], and Beck and Troesch [4]. The Cartesian coordinates $(x, y, z)$ are fixed to the ship's hull. The $xy$-plane coincides with the calm-water surface and the $xz$-plane coincides with the ship's centreplane. The origin $O$ is located at the forward perpendicular (FP). The aft perpendicular (AP) is at $x = L$. The $z$-axis points vertically upward. Long-crested waves of small amplitude $a$ and angular frequency $\omega_0$ propagate along the $x$-axis in the direction from $-\infty$ to $+\infty$. The ship is assumed to be fixed at its mean position and advances at a constant speed $U$ in the negative $x$ direction. It is assumed that water is ideal and deep, and the flow irrotational. Surface tension is neglected. The total velocity potential $\Phi$ is expressed as

$$\Phi(x,y,z,t) = Ux + \phi_s(x,y,z) + [\phi_I(x,y,z) + \phi_D(x,y,z)]e^{i(\omega t - \nu x)} \qquad (1)$$

where $\omega = \omega_0 + \nu U$ is the frequency of encounter, $\nu = {\omega_0}^2/g$ is the wave number, $g$ is the gravitational acceleration, $Ux$ is the uniform-flow potential, $\phi_s$ is the velocity potential of the perturbation of the uniform flow owing to the presence of the hull,

$$\phi_I(x,z) = \frac{ga}{\omega_0} e^{\nu z} \qquad (2)$$

is the incident-wave potential, and $\phi_D$ is the diffraction potential.

We assume the following:

$$\begin{aligned} B/L, d/L &= O(\epsilon) \\ \omega_0 &= O(\epsilon^{-1/2}) \\ U &= O(1) \\ a &= O(\delta) \end{aligned} \qquad (3)$$

where $\delta$ and $\epsilon$ are small dimensionless parameters independent of each other, and $B$ and $d$ are the beam and draft of the hull, respectively.

In the far field, we assume that the lowest-order term of $\phi_D$ can be expressed in terms of a distribution of sources on the $x$-axis. The density of the sources is assumed to be given by $\mathrm{Re}[\sigma(x)e^{i(\omega t - \nu x)}]$. Then a two-term inner expansion of the far-field solution is given by [2]

$$\phi_D(x,y,z) \sim e^{\nu z}\left[-\frac{e^{-i\pi/4}}{\sqrt{2\pi\frac{\omega+\nu U}{\nu\omega_0}}}\int_0^x \frac{d\xi\sigma(\xi)}{\sqrt{x-\xi}} + \nu|y|\sigma(x) - C\sigma(x)\right] \qquad (4)$$

where $C = -i/2$ when $U = 0$ and $C = 1/2\sqrt{2}$ for $\tau = \omega U/g > 1/4$.

In the near field, we write the two-term solution as [2]

$$\phi_D(x, y, z) \simeq -\frac{ga}{\omega_0} e^{\nu z} + A(x)\psi(y, z; x) \tag{5}$$

and match both terms simultaneously with the far-field solution in the overlap region [3]. The coefficient $A(x)$ is to be determined through matching and $\psi$ is the solution of the homogeneous boundary-value problem:

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} - \nu^2 \psi = 0 \qquad \text{in the fluid domain, } z < 0 \tag{6a}$$

$$\frac{\partial \psi}{\partial z} - \nu \psi = 0 \qquad \text{on } z = 0 \tag{6b}$$

$$\lim_{z \to -\infty} \frac{\partial \psi}{\partial z} = 0 \qquad \text{on the sea bottom} \tag{6c}$$

$$\frac{\partial \psi}{\partial N} = 0 \qquad \text{on the hull surface } y = h(x, z) \tag{6d}$$

where $\partial/\partial N$ is the derivative along the unit normal in a cross section of the ship pointing out of the fluid. In addition, $\psi$ must match the inner expansion of the far-field solution.

We write a solution of the above boundary-value problem (6a)–(6d) as (Ursell [5])

$$\psi(y, z; x) = e^{\nu z} + \int_{C_x} \nu Q(\eta, \varsigma)[G(\nu y, \nu z; \nu\eta(s), \nu\varsigma(s)) + G(\nu y, \nu z; -\nu\eta(s), \nu\varsigma(s))]ds \tag{7}$$

where $C_x$ denotes the half of the boundary curve of the submerged cross section at $x$, $Q(y, z)$ is the two-dimensional source distribution along $C_x$, and $G$ is the Green function [5]. Applying the integral-equation technique, $Q(y, z)$ is determined so as to satisfy the body boundary condition (6d). With $Q(y, z; x)$ thus determined, we obtain a Volterra integral equation of the second kind for the three-dimensional source distribution $\sigma(x)$ along the $x$-axis [4]:

$$\frac{ga}{\omega_0} + \left\{ \frac{1}{2\pi \int_{C_x} Q e^{\nu\varsigma} ds} - C \right\} \sigma(x) - \frac{e^{-i\pi/4}}{\sqrt{2\pi \frac{\omega + \nu U}{\nu \omega_0}}} \int_0^x \frac{d\xi \sigma(\xi)}{\sqrt{x - \xi}} = 0. \tag{8}$$

The kernel of this integral equation is weakly singular. An approximate solution of (8) can be obtained by the product-integration formula (Baker [6]). The potential is assumed to vanish forward of the bow. This assumption is valid for $\tau = 0$ and $\tau >> 1/4$. Having determined the values for $\sigma(x)$ and $Q(y, z; x)$, we can evaluate $A(x)$ through matching.

The perturbation potential $\phi_s$ in the near field was calculated using thin-ship theory. This approach may not be as rigorous as slender-body theory, but the qualitative features should be present. Since $\phi_s = O(\epsilon^2)$ compared with $\phi_I, \phi_D = O(\delta\epsilon^{3/2})$, possible difference between slender-body and thin-ship approximations should not be significant; moreover, thin-ship theory has been shown to be reasonably accurate for slender hulls and high speed. For a general hull form, $\phi_s$ can be determined following the approach of Havelock [7], Lunde [8], and Newman [9].

From the Bernoulli equation, the hydrodynamic pressure in the near field is obtained by

$$\begin{aligned} -\frac{P}{\rho} &= \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \Phi + \frac{1}{2} \nabla\Phi \cdot \nabla\Phi \\ &= \left( i\omega + U \frac{\partial}{\partial x} \right) \phi + \nabla\phi_s \cdot \nabla\phi \\ &\simeq e^{i(\omega t - \nu x)} \left[ i\omega_0(\phi_I + \phi_D) + 2U \frac{\partial \phi_D}{\partial x} + \frac{\partial \phi_s}{\partial y} \left( \frac{\partial \phi_I}{\partial y} + \frac{\partial \phi_D}{\partial y} \right) + \frac{\partial \phi_s}{\partial z} \left( \frac{\partial \phi_I}{\partial z} + \frac{\partial \phi_D}{\partial z} \right) \right] \qquad (9a) \\ &\simeq e^{i(\omega t - \nu x)} \left[ i\omega_0(\phi_I + \phi_D) \right] \qquad (9b) \end{aligned}$$

where $\rho$ is the water density, $\phi = (\phi_I + \phi_D)e^{i(\omega t - \nu x)}$, and the neglected terms are of order $o(\delta\epsilon^{3/2})$.

Figures 1a and 1b show typical results for the entrance and run of the hull in the range $\tau >> 0.25$, when $\lambda/L = 0.25$ and 0.50. Figure 1a shows $|P|$ versus the Froude number $F_n = U/\sqrt{gL}$ at $3/20L$ abaft the FP on the centreline, and Fig. 1b shows $|P|$ versus $F_n$ at $3/20L$ forward of the AP on the centreline. Piecewise straight dash-dot lines are drawn through the mean values of measurements at $F_n$ =0.10, 0.20, 0.29, 0.42, and 0.50, as reported by Ando and Cumming [1]. The solid lines represent the theoretical first-order approximations (9b) including only terms of order $O(\delta\epsilon)$, and the dotted lines represent the theoretical pressures (9a) including the first- and second-order terms of order up to $O(\delta\epsilon^{3/2})$. The predicted $|P|$ is almost a linear function of $U$. Generally speaking, the contribution by the second-order terms is to decrease the total pressure slightly with no significant change in the trend of $|P|$ with increasing $U$. In contrast, the experimental data show $|P|$ over the entrance of the hull to be a strongly nonlinear function of $U$: it is an increasing function of $U$ only when $U$ is below a certain limiting speed, say $U^*$. For $U > U^*$, $|P|$ becomes a decreasing function of $U$. For a given pressure transducer, the value of $U^*$ appears to increase with increasing wavelength. On the other hand, both theoretical and measured $|P|$ are increasing functions of $U$ over the afterbody, as Fig. 1b exemplifies. Further studies are needed to elucidate the nature of wave diffraction over the entrance of the advancing hull, and to properly account for the observed discrepancy between theory and experiment in the trend of the wave-induced pressure. It seems that the effect of the interference between the incident wave and the disturbance on the free surface caused by the forward motion of the ship around the entrance is not necessarily as small as the theory assumes. If the observed results can be confirmed, they will have an important effect on analytical approaches to the diffraction problem for ships.

## References


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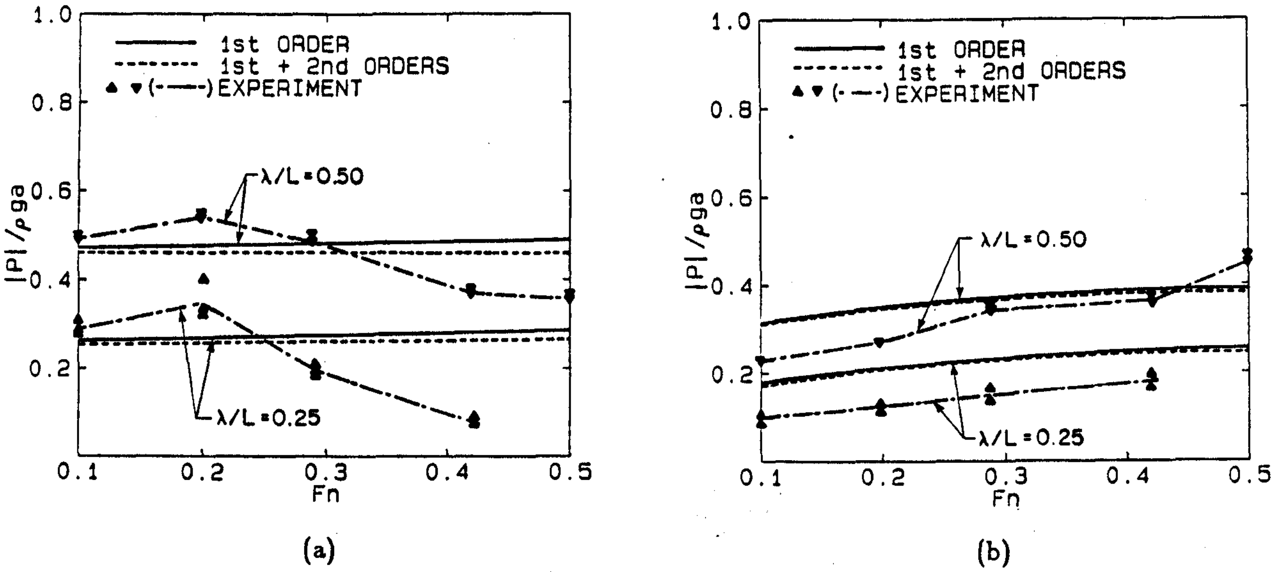


**FIG. 1. Theoretical and measured trends of wave-induced pressure with increasing forward speed (a) at $3/20L$ abaft FP on centreline and (b) at $3/20L$ forward of AP on centreline; $\lambda/L = 0.25$ and 0.50.**

## DISCUSSION

Söding: A simple method of unknown accuracy to include the effect of the steady surface waves due to high ship speed on the radiation and diffraction potential and forces is to use the ship's section shapes up to the steady-state surface and shift them vertically to obtain a straight upper limit but, instead, a curved keel line. At least this procedure takes account of the increased added mass, damping and static restoring force of flaring sections in way of the bow wave and the greater depth of submergence of the pressure transducers there.

Ando: Actually I have tried the idea of incorporating the modified water-line owing to the bow wave into calculation just as you suggest. I used the measured water-line and adjusted the local draft accordingly. But then the calculation seemed to suggest $\partial|P|/\partial U<0$ for $U>0$ under the crest of the bow wave, whereas the experiment appears to indicate that there's some critical speed $U^*>0$ at which $\partial|P|/\partial U=0$. Nevertheless, your suggestion is a valid one and worth being explored further.

Kashiwagi: I'd like to make a comment on the discrepancies between experiments and slender-ship theory predictions. The slender-ship theory you used should be considered to be based on the small forward-speed assumption. Another formulation, in which the linearized complete free-surface condition is preserved also in the inner problem, was separated with very good agreement found in the journal of naval architects in Japan. That calculation is an application of Chapman's high-speed theory applied to the radiation problem. So I think the inclusion of U- and $U^2$-terms in the free-surface condition may be the key to the improvement of the theory.

Ando: At this stage, I'm open to any suggestion and appreciate your comments. I'm not acquainted with the references you cite, so I'd like to consult them before I reply.

Hearn: In your presentation you suggest that the results you expect to obtain will be the same whether you use the indicated ship theory or 3D theory. We implemented the Faltinsen Skjørdal method in 1984/85 (See OTC 1986). The 3D results (OMAE 87) when looking at wave excitation were quite different as were the second order forces calculated. I think it is dangerous to assume that the good agreement for hydrodynamic reactive coefficients shown by Kashinagi this afternoon should be used to justify good agreement for all predictable quantities. We use very different 3D + 2D methods and this is not our experience.

Ando: You're absolutely correct. I was perhaps overjoyed to see such an instance of good agreement between slender-body theoretical and 3-D calclulations as in Prof. Kashiwagi's paper. I'll look up the references you've cited.