SOME COROLLARIES FOR THE STUDY OF TWO–DIMENSIONAL BODIES IN WAVES

by Antonio Carlos Fernandes, PETROBRÁS/CENPES, Brazil

Newman 76 has put together for the first time the so–called reciprocity relations for linear hydrodynamic problems involving bodies in the presence of a free surface. Together with new improvements, they have been presented again by Mei 83 and later, Fernandes 85 has shown some simple new relations for the diffraction properties of three dimensional bodies. Under this light, the present work investigates the two dimensional problems a little further.

Let f_{ij} be the complex force in the j–th (generalized) direction that results when a two–dimensional body oscillates harmonically (circular frequency ω) in the i–th direction with a unit amplitude. The following decomposition is widely used:

$$f_{ij} = \omega^2 a_{ij} - i\omega b_{ij} \tag{1}$$

where the coefficient a_{ij} is called added mass and b_{ij} damping. It is possible to show for symmetric bodies that

$$b_{24}^2 = b_{22} b_{44} \tag{2}$$

where the subscript 2 corresponds to sway motion, 4 to roll and 3 is reserved for heave. This expression, which seems to be new, is a corollary of a more general result given by

$$b_{ij} = \frac{\omega}{2\rho g^2}[X_i(\pi)X_j^*(\pi) + X_i(0)\, X_j^*(0)] \quad i,j = 2,3,4 \tag{3}$$

which has been proved by Newman 76. Here, ρ is the fluid density, g is the gravity acceleration and $X_i(\theta)$ is the wave exciting force on a fixed body under the action of a unit amplitude wave coming from the θ–direction. Now introducing δ_i as the argument of $X_i(0)$ such that

$$X_i(0) = |X_i(0)| e^{i\delta_i} \tag{4}$$

and using (3), it is possible to show for symmetric bodies that

$$b_{24} = \frac{\omega}{2\rho g^2} |X_2(0)|\,|X_4(0)| [\cos(\delta_2 - \delta_4) + i\,\mathrm{sen}(\delta_2 - \delta_4)] \tag{5}$$

Since the left hand side is real, either

$$\delta_2 = \delta_4 \tag{6}$$

$$\text{or} \quad \delta_2 = \delta_4 \pm \pi \tag{7}$$

which is a very strong result and resembles those proved by Fernandes 85 for three dimension. This result has been considered by Mei 83 but there, the possibility (6) has not been displayed.

In a recent paper, Athanassoulis et alli 88 have shown several relations for th low–frequency case. These, with the use of the reciprocity relations can be improved and th results for symmetric bodies are:

$$X_2 = i\omega^2[\, a_{22}(0) + \rho A \,] + o(\omega^2) \tag{8}$$

$$X_3 = \rho g B + \frac{\rho g}{\pi} B^2 \frac{\omega^2}{g} \ln\frac{B\omega^2}{g} + o(\omega^2 \ln\omega) \tag{9}$$

$$X_4 = i\omega^2[\, a_{24}(0) - \rho I \,] + o(\omega^2) \tag{10}$$

$$a_{22} = a_{22}(0) + o(1) \tag{11}$$

$$a_{33} = -\frac{\rho B^2}{\pi} \ln\frac{B\omega^2}{g} + o(\ln\omega) \tag{12}$$

$$a_{44} = a_{44}(0) + o(1) \tag{13}$$

$$a_{42} = a_{42}(0) + o(1) \tag{14}$$

$$b_{22} = \frac{\omega^5}{\rho g^2}[\, a_{22}(0) + \rho A \,]^2 + o(\omega^5) \tag{15}$$

$$b_{33} = \rho B^2 \omega + \frac{2}{\pi} \rho B^3 \frac{\omega^3}{g} \ln\frac{B\omega^2}{g} + o(\omega^3 \ln\omega) \tag{16}$$

$$b_{44} = \frac{\omega^5}{\rho g^2}[\, a_{24}(0) - \rho I \,]^2 + o(\omega^5) \tag{17}$$

$$b_{24} = \frac{\omega^5}{\rho g^2}[\, a_{22}(0) + \rho A \,][\, a_{24}(0) - \rho I \,] + o(\omega^5) \tag{18}$$

where A is the area of the body trace, B is the body beam at the free surface, $a_{22}(0)$ and $a_{24}(0)$ are finite real constants expressing the values at the zero–frequency limit and I is a geometric property given by

$$I = Az_B + \frac{B^3}{12}. \tag{19}$$

In this last expression, $z_B = -|z_B|$ is the vertical ordinate (z pointing upwards from the free surface) of the center of buoyancy of the body trace. Note that I may be positive, negative or null and indicates how shallow (algebraically small I) the trace goes into the water.

It is important to say that several of these results have in fact been shown before in the literature, perhaps with the exception of (9), (12) and (16). The idea here is to illustrate the relations (2), (6) and (7). Hence, from (8), (9) and (10) it follows that for low frequencies,

$$\arg(X_2) \approx \pi/2 \tag{20}$$

$$\arg(X_3) \approx 0 \tag{21}$$

$$\arg(X_4) \approx \pm \pi/2 \tag{22}$$

which is coherent with (6) and (7). The sign in (19) depends on the sign of b_{24} since for low frequencies and from (18)

$$\text{sign}[b_{24}] \approx \text{sign}[a_{24}(0) - \rho I]. \tag{23}$$

One may apply the reciprocity relations in several ways. A very fruitfull is to take advantage of the fact that they are necessary conditions and hence may indicate the precision of the results of numerical methods. For instance, the next table indicates the phase difference that were obtained via a computer program (based on Green's function integral equation method) applied to several rectangles with different beam (B) to draft (T) ratios.

	kB/2:		
B/T	.1	.5	1.5
2	0.00	0.02	0.01
8/3	0.00	0.06	0.00
10/3	180.00	179.72	–0.10
4	180.00	180.01	180.07
8	180.00	180.02	179.82

Phase difference $\arg(X_2) - \arg(X_4)$ in degrees
for rectangles for several B/T ratios.

In this table, the results with two decimal figures, are for a 30–points discretization. Note that the phase difference $\delta_2 - \delta_4$ depends on both the geometry and the frequency and are discrete in nature. As a matter of fact, at this point it is interesting to note that in Vugts 68 (a very well known paper which shows experimental results for several rectangles and also other shapes) there is a small inconsistency. There in page 261, it is assumed that the phase of X_2 and X_4 in the low frequency limit are the same (no matter the geometry) since "both are proportional to the wave slope, so that in the long wave approximation the phases are indeed identical". By the present work, it is clear that the proportionality is correct but the long wave approximation may lead to opposite phase which depend on I given by (19) and used in (10).

REFERENCES

ATHANASSOULIS, G.A. ET ALLI 88, "The limiting values of added mass of a partially submerged cylinder of arbritary shape" J. Ship Research, Vol.32,No1,March,pp 1–18.

FERNANDES,A.C. 85, "Reciprocity relations for the analysis of floating pneumatic bodies with applications to wave power absorption", Proc. Offshore Mech. and Artic Engineering (OMAE).

MEI,C.C. 83 , "The applied dynamics of ocean surface waves", John Willey and Sons.

NEWMAN,J.N. 76,"The interaction of stationary vessels with regular waves", in 11–th Symposium on Naval Hydrodynamics, Un. College London, pp 427–451.

VUGTS,J.H., 68 "The hydrodynamic coefficients for swaying, heaving and rolling cylinder in a free surface", International Shipbuilding Progress 15:(167) July pp 251–276.

DISCUSSION

Greenhow: According to van Dyke (in his book) we should be careful in taking account of log terms in expansions without the corresponding polynomial terms. Some comparisons between the low frequency results and numerical results would be interesting to see how low the frequency has to be in order for these simple results to be useful.

Fernandes: I have always felt the lower order long wave approximation results somewhat disappointing because they are only useful for very small frequency range. However, with the higher order terms of the abstract the useful frequency range improved. This is so even not considering the van Dyke polynomial terms.

Kashiwagi: Is eq.(2) one of what you want to stress in this paper? If yes, my comment is that relation (2) was proved by Prof. Bessho possibly 25 or 30 years ago through the argument of integral equation. And in addition, eqs(6) and (1) where also proved by Prof. Bessho in the same paper.

Fernandes: I am not sure what is meant by "argument of integral equation". In this paper equation (2) is presented as a simple corollary of what is called the Bessho-Newman relation in Mei (1983), that is, equation (3) here. As for equations (6) and (7) I certainly agree that they are not new as I tried to stress in the presentation. But what ultimately I am trying to emphasize is that this strong result might be more used at least for checking purposes as I also tried to show in the presentation.

Falnes: As a comment to the reciprocity relation (2) $b^2_{24}=b_{22}b_{44}$ I wish to mention that this relation also holds for an axisymmetrical body, in the 3-D case. A similar relationship $b^2_{15}=b_{11}b_{55}$ applies, of course, for surge and pitch. The 6x6 matrix of radiation resistance **b** is singular, and its rank is not more than 3 (since, furthermore, there is no interation between the wave and the yaw motion of an axisymmetric body).