Hydrodynamic Considerations for the Design of Pipe-Bridges

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Recently there has been renewed interest in bridging some Norwegian fjords whose depth, width and shipping access requirements preclude conventional or floating bridges. A practical method may be a pipe-bridge (rørbru) of radius 5m carrying a double-lane road inside. This flexible pipe may leave the fjord side above water, gradually penetrate the surface and be submerged by several diameters in the centre to allow ship passage overhead. Methods of pipe restraint include floats, moorings and tension leg arrangements, but the designer must answer two questions: i) can a vehicle be driven through under normal wave and current loadings? ii) can the pipe survive unusual or accidental extreme loading caused by e.g. earthquakes and ship impact? Understanding the hydrodynamics of the pipe is vital at an early design stage and it is possible to provide useful information to the structural engineer.

When calculating the normal modes of the pipe-bridge it is important to consider the strong variation of the added mass with both frequency and depth of submergence. For the fundamental, the low-frequency added mass depth variation shown in fig.1 is important; for higher harmonics of period ~ 10s the dimensionless wavenumber KR ~ ·2, which is close to the added mass peak in figs. 3 and 7.

Because fjord waves have a small directional spread, wave loading amplitude can be calculated from the two-dimensional cylinder damping curves (figs. 4,6 and 8) using the Haskind relations. Typically KR ~ O(1) which is much higher than most ocean engineering situations. The curves for added mass and damping when H/R = 0 (half submerged) and H/R > 1 (totally submerged) are of course well known, but the transition between them and for a less than half-submerged cylinder does seem to require some comment. Low-frequency asymptotic forms are quite easy to obtain for the damping using the relative motion hypothesis, Haskind relations and the Kramers-Kronig relations. Results are

$$b(KR) = \frac{B}{\rho \forall w} = (KR)^2 \pi \left(a(0) + \frac{V}{\forall}\right)^2 \quad \text{as } KR \to 0$$

for sway, and for heave with H/R > 1, and

$$b(KR) = R^2 \left(\frac{S}{R}\right)^2 \left[1 + \frac{2}{\pi}\left(\frac{S}{R}\right) KR \, \ell n \, KR\right] / \forall$$

for heave with -1 < H/R < 1. Here V is the displaced volume, $\forall$ is the total body volume and a(0) is the added mass in the appropriate mode at zero frequency. These results are valid for any two-dimensional symmetric body. a(0), which must be known can be taken from figure 1.

The dotted lines in figure 6 arise from equivalent wavemaker theory, the flexible wavemaker motion chosen to match the fluid flow in the high-frequency problem (known analytically). The resulting damping is known to be correct at low frequency also, but the good prediction of the damping for $-1 < H/R < 0$ and the wave-free intermediate frequencies for $1 > H/R > 0$ is interesting and somewhat unexpected.

The loading due to earthquake-induced vibration of the bridge ends has been studied by the exact numerical method of Vinje and Brevig. With typical periods of 1~2s the usual added mass model is adequate provided the high-frequency depth-dependent added mass is taken from fig.2.

Ship impact may result in impulsive velocity locally and impulsive acceleration away from the impact point, and the fluid loading is much harder to model in a simple way. Exact numerical calculations indicate very strong free-surface nonlinearities which may even result in breaking wave generation. Both added mass and Havelock's submerged dipole theories are significantly in error and direct numerical calculation is necessary.

Reference:

Greenhow, M. and Ahn, S-L. "*Added mass and damping of horizontal circular cylinder sections*". Ocean Engng Vol.15 no.5 pp 495-504.

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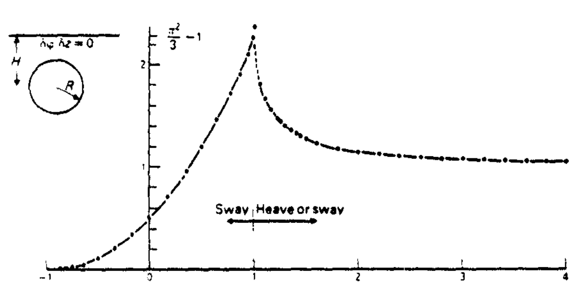

FIG. 1. Variation in added mass of a cylinder near a boundary with $\partial\phi/\partial z = 0$. —— denotes analytical results from Greenhow and Li (1987) and • denotes present numerical calculations.

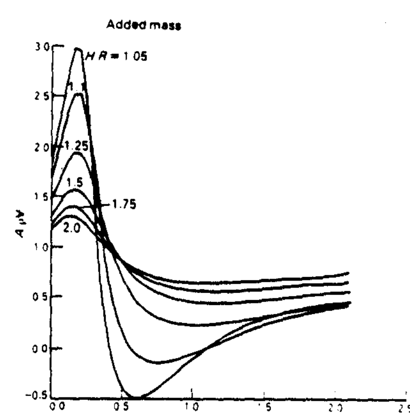

FIG. 3. Added mass variation with frequency for the fully-submerged circular cylinder

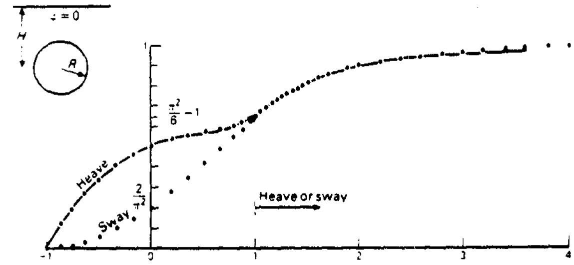

FIG. 2. Variation in added mass of a cylinder near a boundary with $\phi = 0$. Line and points as in Fig. 1.

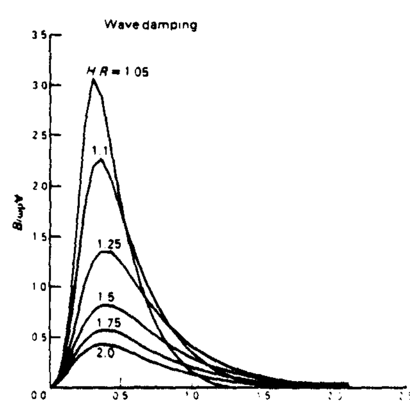

FIG. 4. Wave-damping variation with frequency for the fully-submerged circular cylinder

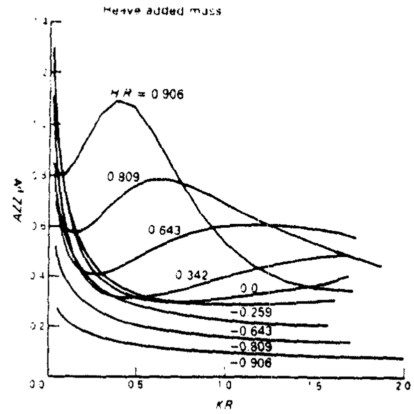

Fig. 5. Added-mass variation with frequency for a surface-piercing heaving cylinder section.

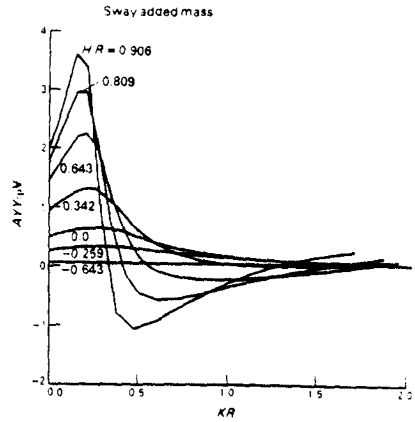

Fig. 7. Added-mass variation with frequency for a surface-piercing swaying cylinder

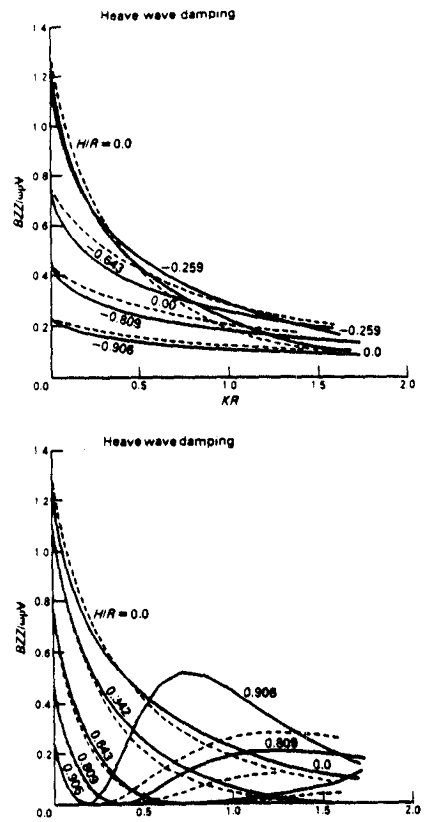

Fig. 6. Wave-damping variation with frequency for a surface-piercing heaving cylinder section. (a) less than half-submerged, (b) more than half-submerged. The results of the equivalent wavemaker method are shown dotted.

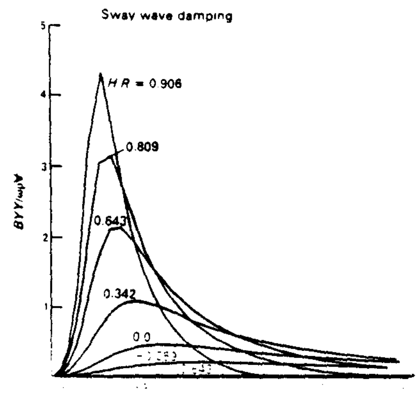

Fig. 8. Wave-damping variation with frequency for a surface-piercing swaying cylinder

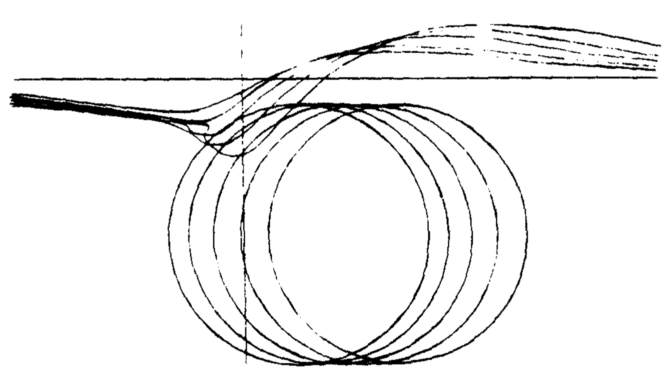

Fig. 9 Impulsively-started constant acceleration motion of a cylinder and free surface. H = 6m, a = 2ms^{-2}

DISCUSSION

Palm: Wouldn't internal waves be important in your problem and therefore it is a need for a generalization of your expression for added mass and damping?

Greenhow: This depends on the density stratification but in principle yes. The internal wave generation drag is however more important since it is steady and will not phase cancel along the pipe bridge. I hope Miloh's work will help with this.

Vinje: A small comment only. I would like to point out that in addition to the internal waves problem another interesting aspect might appear in connection with "rørbruer", namely the effect of "rock slides" occasionally occurring in Norwegian fjords.

Greenhow: A small answer only. I agree!