RUNUP AND REFLECTION OF A SOLITARY WAVE ON STEEP SLOPES IN A NUMERICAL WAVE TANK * (by Stéphan Grilli & Ib A. Svendsen †)

1 Introduction

Over the last decade numerical solution of the exact nonlinear equations for water waves using a Boundary Integral Equation (BIE) description has become an extremely successful method for analyzing particularly the overturning of breaking waves. Practically all previous contributions, however, utilize space periodicity of the waves and most of them also are based on a conformal mapping of physical space onto a plane in which the equivalent of the free surface is the only part left of the boundary. The Boundary Integral Equation is then solved in the transformed plane. A few contributions use some other complex variable-dependent techniques. There are many advantages of this method which has been explored extensively by many authors. Particularly noteworthy for significant steps to the development are contributions by Longuet-Higgins & Cokelet (1976) and Dold & Peregrine (1984).

The procedure, however, also has some important limitations which are associated with the use of complex variables and with the assumption of waves that are periodic in space. The present study aims at developing an equally efficient and accurate method without utilizing those two crucial assumptions and at demonstrating its application to a problem for which the above-mentioned version is not well suited, namely the generation of a solitary wave by a wave maker, and its subsequent propagation, runup on and reflection from a steep slope. The computations are compared with recent experimental results by Losada, et al. (1986) (LVN).

The runup of solitary waves has been extensively analyzed using long wave theory or Boussinesq theory. The special case of reflection from a vertical wall is equivalent to the problem of collision between two opposing solitary waves of the same height and several studies have been published utilizing that analogy. A first order solution can be obtained by the Inverse Scattering Technique developed in the 1960's and 70's , and a third order analysis was developed by Su & Mirie (1980) (S&M). Fenton & Rienecker (1982) (F&R) used a Fourier Method to obtain very accurate results for the same problem. The runup on a slope was analyzed numerically on the basis of the Boussinesq equations by Pedersen & Gjevik (1983), and Synolakis (1987) generalized an earlier approach, based on nonlinear shallow water solutions, to apply to the situation also considered in this study with a constant depth region in front of the slope. Kim, et al. (1983) used a BIE and an iterative implicit time integration procedure to study the same problem.

2 Mathematical and numerical formulation

We consider the wave motion in the physical space $\Omega(t)$ (Figure 1) and describe it using a free space Green's function $G(\mathbf{x},\mathbf{x}_l)$ to transform the Laplace differential equation into an integral equation that involves only values of the velocity potential and its normal derivative along the physical boundary $\Gamma(t)$,

$$\alpha(\mathbf{x}_l)\phi(\mathbf{x}_l) = \int_\Gamma [\frac{\partial \phi}{\partial n} G(\mathbf{x},\mathbf{x}_l) - \phi \frac{\partial G(\mathbf{x},\mathbf{x}_l)}{\partial n}]\, d\Gamma \qquad (1)$$

The procedure used to solve (1) is a Boundary Element Method (BEM) using second order elements for the present computations. Collocation nodes are distributed along the entire boundary to describe the variation of boundary geometry as well as boundary conditions and the unknown functions of the problem. Between the collocation points, the variation of all quantities is described by means of shape functions, and for this purpose, the boundary is divided into elements each of which contain three nodes. The integrals in (1) are computed within each element by Gaussian quadrature.

The nonlinear kinematic and dynamic boundary conditions are imposed on the free surface $\Gamma_f(t)$. The solitary waves are generated by simulating a piston-type wavemaker motion on the boundary $\Gamma_{r1}(t)$ and by

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imposing the continuity of its normal velocity. Along the stationary bottom Γ_b and the slope Γ_{r2}, a zero normal velocity is imposed.

The time integration of the free surface conditions corresponds to the explicit scheme, based on a truncated Taylor expansion, developed by Dold & Peregrine (1984), which has been modified to using the shape functions of the BEM. A version correct to second order in Δt ($n = 2$) has been developed for the computations presented here,

$$\mathbf{r}(t + \Delta t) = \mathbf{r}(t) + \sum_{k=1}^{n} \frac{(\Delta t)^k}{k!} \frac{D^k \mathbf{r}(t)}{Dt^k} + O[(\Delta t)^{n+1}] \tag{2}$$

Notice that no sawtooth instabilities occured during the computations. A detailed account of the application of this numerical method to water waves is given in Grilli, Skourup & Svendsen (1989).

3 Numerical experiments

3.1 Wave generation

Since the computations are made in physical space, but only covering a limited region of an infinitely long wave tank, the waves to be studied have to be generated at the outer edge of the computational area. One of the main purposes of this study has been to compare the numerical method to the measurements of LVN who generated waves by moving a piston wavemaker according to Goring's (1978) solution, to create a first-order solitary wave. We have reproduced that process as closely as possible in the numerical experiments.

Within the frame of Boussinesq approximation, the inverse scattering theory will predict that, unless it corresponds to an exact solitary wave, a distributed elevation of the free surface will disintegrate into one or more solitons and a tail of disturbances. Thus, a generated first-order solitary wave of appreciable height may be expected to exhibit disturbances and modifications of its profile while it propagates down the wave tank. In fact Goring (1978) found that the limit for an accurate solitary wave reproduction by his method was $\frac{H}{d} = 0.2$. In accordance with this, it is found that the higher the waves, the more pronounced is the amplitude A (Figure 1) of the tail of disturbances they are shedding.

3.2 Reflection from a vertical wall

We first analyze the simple reflection from a vertical wall and compare with the results of F&R. The maximum runup R_u at the wall is compared to the third order results by S&M and to the F&R's computations for different wave amplitudes (Figure 2). For most of the propagation of a wave of $\frac{H}{d}$=0.456, the total energy remains constant to within 0.01 %. In the brief interval of the rapid surface movements at the wall during runup, however, the total energy increases by about 0.06 %. This increase occurs simultaneously with a relative change in volume which creates potential energy. During this short interval, the (fixed) time step used in the computations seems too large, so that the surface movement is not predicted with quite the same accuracy during that part of the process.

F&R also compute total maximum forces and moments of forces acting on the wall, to which our BEM computations agree to a fraction of a percent. A somewhat surprising result is a double maximum found in the time variation of the total force for high waves.

3.3 Comparison with experimental results by Losada, et al. (1986)

The computational region has the dimensions shown in Figure 1. With a depth of 0.3 m, the distance from our wave maker to the still water level intersection with the slope is 10 m or 33 times the depth. To save computational effort, this is only about one fifth the length of Losada's wave basin. The minor inaccuracies this leads to are almost entirely concentrated in the tail following the wave as was discussed in Section 3.1: In our computations the tail meets the reflected wave somewhat before it does in the experiments. Hence, the major effect of the reduced computational region is a limitation in the time over which we can meaningfully compare our results with those of LVN. However, this does not constitute a major problem since most of the action takes place before that time. It is also found that a new trailing system of high frequency oscillations is generated by the runup and reflection process. Computations, for slope angles 45° and 70° and heights $\frac{H}{d}$=0.259 and 0.457, compare very favorably with the measurements even for those high frequency parts of the wave motion. Figures 3 shows, for instance, how a wave of $\frac{H}{d}$=0.259 reflects from a 70° slope. The computations (solid lines) are compared with the surface profiles measured by LVN at four different times (dashed curves a to d), of which the second is the instant of maximum runup used for synchronization. The symbols mark original experimental data points.

The number of nodes in the computations was considerably higher, and the time step smaller, than required for a plain solitary wave (see Grilli et al. (1989)), in order to obtain sufficient resolution to describe also the higher frequency oscillations in the tail created both by the wave generation, and after the wave reflection. Due to the high degree of vectorization of the computations, however, the CPU-time per time step was found to increase less than proportional to the square of the number of nodes.

3.4 The internal velocity field

The examples in Section 3.3 show that it has been possible to reproduce the surface variations recorded in the experiments down to very small details and over quite extended periods of time relative to the time scale of the important events. This can only be possible if the computational solution actually represents the whole flow pattern to a high degree of accuracy. Thus it will be possible to use the computational solution to analyze other properties of the flow than those actually measured in the experiments and to have a reasonable confidence in the correctness of such predictions. As an example we have computed the internal velocity field at some points in the neighborhood of a 45° slope, for a solitary wave of $\frac{H}{d}$ = 0.457. To measure velocities in that detail in the physical experiments would be a tremendous task, whereas it only requires a limited extra effort computationally. Hence this example illustrates the usefulness of combining experiments and numerical computations.

Figure 4a-c shows the velocity field at three different stages . When the first part of the wave reaches the slope, the flow field looks almost like a rotation around a point well above the incoming wave crest. Then, as the wave crest approaches the slope, the upward movement almost looks like a jet rushing up along the slope (Fig. 4a). In Fig. 4b, there is still a jet-like up-rush of the tip of the wave, but a stagnation point has developed near the toe of the slope as an indication of the reflection taking effect. Later on, this stagnation point moves up along the slope. Fig. 4c shows a remarkably interesting feature in the combination of high velocities and large accelerations around the time of the lowest position. The steep shoreward slope of the free surface in this non-distorted figure suggests horizontal accelerations close to 1 g. In a porous slope, there would be a strong outward directed pressure gradient which would result in a high risk of units being pulled out of the slope. The damage to rubble mound structures observed during experiments actually often occurs at this stage of the process.

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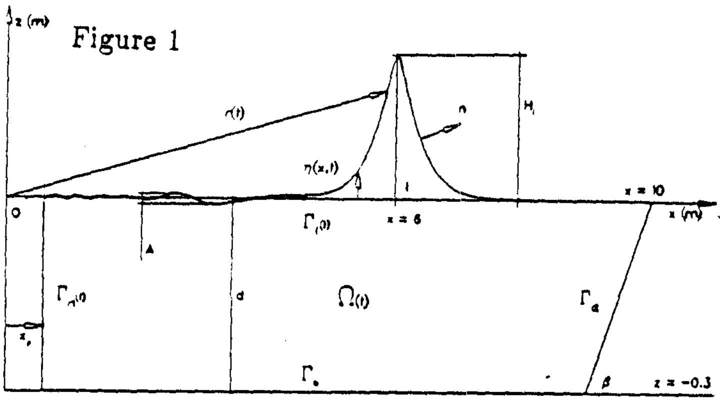

Figure 1

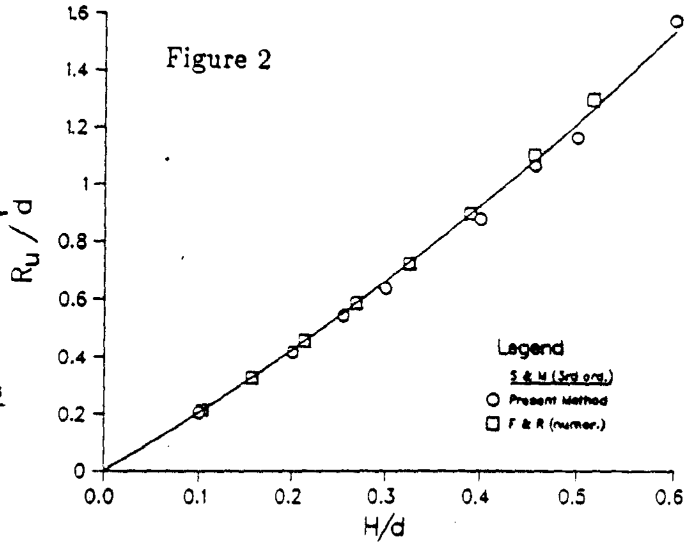

Figure 2

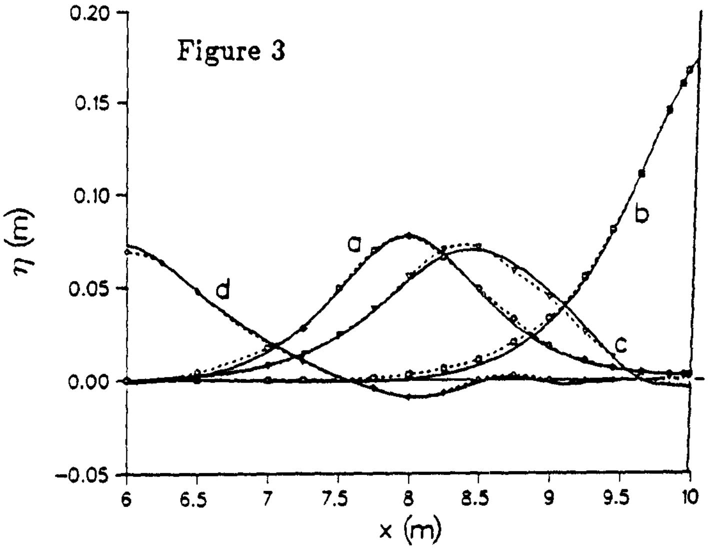

Figure 3

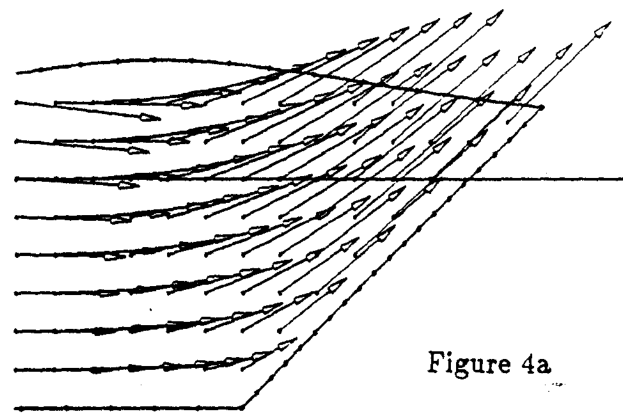

Figure 4a

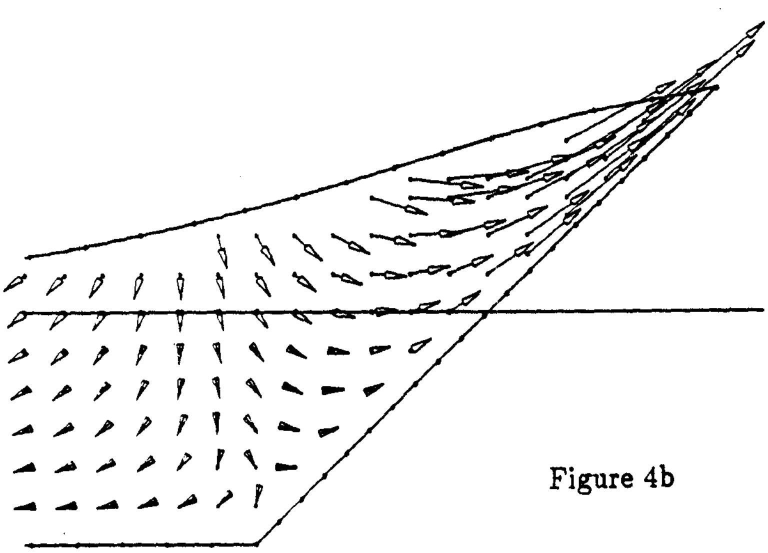

Figure 4b

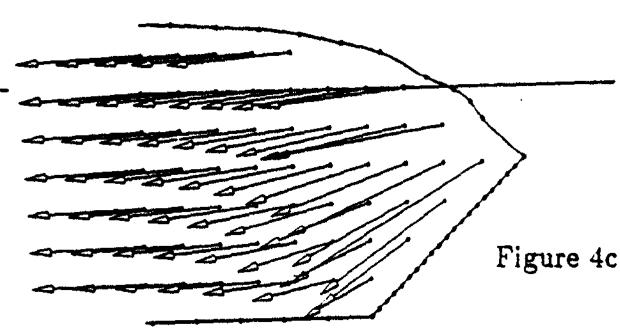

Figure 4c

DISCUSSION

Pedersen: On basis of comparison with experimens you suggested that preferably the fully dispersive and nonlinear equations should be used also for mild slopes. However for wavetanks of moderate depth the run up will be substantially influenced by frictional effects. We have in an earlier work found that the influence of friction is comparable to the total deviation between experiments and Boussinesq theory. (Pedersen & Gjevik, JFM, vol. 135, 1983).

Grilli & Svendsen: We know that friction becomes important for gentle slopes. This effect is neglected in both our fully nonlinear solution and in approximate solutions like KdV or the shallow water equations. Vertical accelarations and dispersive effect, however, are present in our solution and neglected in, e.g. the SWE. Our main purpose, as we said, is to deal with steep slopes. We found it interesting, however, to try to apply our method to gentle and very gentle slopes. From the few comparisons we have done with a 1:20 slope, we found that our computed result agrees with experiments by Synolakis within 5% whereas his SWE solution only agrees within 16%. Since the main neglected effect, in our solution, is friction, we tentatively interpreted the larger discrepancies of the SWE as an effect of the other effect it neglects on a gentle slope, i.e. dispersion. This needs however to be further investigated by also considering, for instance, some hypothesis made by Synolakis in his SWE solution (incident wave, patching condition,...).

Small water depths in the wave tank should probably increase the effect of friction, as you pointed out. The experimental results we considered, corresponded to a water depth of 30 cm. This is not very small, in our opinion, as confirmed by the good agreement of our results with the experiments.

Cooker: I would like to comment on the apparent linearity of the maximum force F exerted by a solitary wave on a vertical wall. At Bristol Univ., England, we have found this near-linear relation F as a function of incident wave amplitude.

$$F(a) \cong 1/2\ \rho g(\text{depth})^2(1-2a).$$

Can you explain why F is so linear even for waves up to a=0.8?

Grilli & Svendsen: The near-linear relation F shown on the figure corresponds to the 3rd-order solution of Su & Mine. Our fully nonlinear results closely agree with it. The results obtained by Fenton & Rienecker, using a Fourier method, also agree with this. I have no real explanation of this except noticing that the sum up is not too far from being linear with H, the pressure diagrams also are reasonably linear with depth and increase to almost parallely to themselves with H. These trends are reproduced in F.

Yeung: I think it is worthwhile to mention that a variety of solitary waves/body interaction problems were quite successfully solved using the Green-Naghdi equations in the paper of Ertekin & Wehausen (1986), 16th Symposium of Naval Hydrodynamics. Perhaps some comparison of your results with some of those therein should be made.

Grilli & Svendsen: Thank you for your suggestion. I don't know this reference but I will get hold of it.

Greenhow: I consider this to be very interesting and important work which should be compared with similar calculations by G. Klopman at Delft. In the breaking region, is regridding possible, and is it possible to simulate and/or suppress breaking by a surface pressure distribution? Do you have any plans to try this method for slamming problems?

Grilli & Svendsen: Thank you for your nice comment. We have just been informed of the work by Klopman and intend to get it as soon as possible. Regridding is possible with our method though we have not tried it. The literature seems to consider this as a difficult problem. We suspect in particular that regridding would be difficult in our model when breaking occurs due to the slope. A surface pressure distribution can be simulated fairly easily in our model but has not been considered yet. We have no plans for trying to solve slamming problems.