Eigenfunction Expansion Techniques Applied in Open and Confined Waters *

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This presentation is concerned with the coupling of Eigenfunction expansion techniques and the Rankine source based Boundary Integral Equation technique to analyse different fluid-structure interaction problems. The 3D linearised free-surface models are applied to situations involving rigid floating bodies in open and confined waters.

Introduction

Hybrid methods of analysis describe those methods that employ different solution techniques in two or more sub-regions of the solution domain. The general approach is to exploit available analytic solutions in flow regions where the geometry is very simple. These special solutions generally satisfy, implicitly, the necessary radiation condition. Computations based on this approach have been undertaken by Garrett(1971) to predict the wave forces acting on vertical cylindrical structures. In sub-regions where the geometry is complicated the finite element or the boundary integral methods may be applied, as illustrated by Bettess & Zienkiewicz(1977), Garrison(1978) and Hearn et al.(1987). The solutions generated in the different sub-regions must then be properly matched across common boundaries by ensuring continuity of both the pressure and the normal velocity components. The hybrid approach has previously been investigated by Bai & Yeung(1974), Yue et al.(1978) and Eatock Taylor & Zietsman(1981).

The method applied in this study is similar to that used by Yeung(1975) and Bai(1981). The flow region is divided into an "inner" and "outer" domain using a fictitious continuous cylindrical boundary for the "open" sea problems, and appropriate "up-stream" and "down-stream" control surfaces for the "confined" or

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"canal" flow problems. The velocity potential in the inner domain is calculated using the boundary integral method based on a simple and frequency-independent Rankine fluid singularity as the kernel of the associated integral equation. Replacing the radiation condition, on the indicated control surfaces, by the stated matching conditions, we use a single asymptotic eigenfunction expansion to represent the solution in the outer domain for the open sea problems. For the canal problems two sets of trial functions are required to represent the up-stream and down-stream fluid flows. This approach leads to a significant reduction in computational effort because the simple source velocity potential is frequency independent and thus the influence coefficient matrix of the inner problem can be computed once. Furthermore, by appropriate selection of the location of the matching boundary the velocity potential and their derivatives need only be evaluated once per frequency for all panels considered. The resulting gains in the computational effort, in comparison to conventional free surface Green function methods, is significant. This is because the Green function method treats each frequency and panel independently.

Numerical Algorithm and Results

In generating the geometric model, flat triangular and quadrilateral panels are used. Over each panel the velocity potential is assumed invariant. The boundary S consists of the body wetted-surface,S_w, the free surface, S_f, the seabed and side-wall surface, S_b, and the matching surface, S_r. For the two classes of problems under consideration see Figures 1 & 2 for typical discretisations of the solution domains.

Applying Green's second identity, the integral equation governing flow in the inner domain can be written as

$$\phi(p) = \frac{1}{2\pi}\int_S \left[\frac{1}{r}\frac{\partial\phi(q)}{\partial n_q} - \phi(q)\frac{\partial}{\partial n_q}(\frac{1}{r})\right] dS(q) \quad \ldots \quad S = S_w + S_f + S_b + S_r$$

where, r denotes the distance between the field point p and the boundary surface located source point q and ϕ is the unknown velocity potential to be evaluated.

The eigenfunction expansion used in the outer domain for the open sea problem is

$$\phi(r,\theta,z) = \sum_{l=-\infty}^{\infty} \left\{ A_{l0} \frac{H_l(m_0 r)}{H_l(m_0 R)} \cosh m_0(z+h) + \sum_{j=1}^{\infty} A_{lj} \frac{K_l(m_j r)}{K_l(m_j R)} \cos m_j(z+h) \right\} e^{i(l\theta)},$$

see Chau & Yuen(1986), whereas

$$\phi(x,y,z) = \sum_{l=0}^{\infty} \left\{ A_{l0} e^{\mp i(D_{l0}x)} \cosh m_0(z+h) + \sum_{j=1}^{\infty} A_{lj} e^{\pm(D_{lj}x)} \cos m_j(z+h) \right\} \cos[(\frac{l\pi}{W})\ (y-\frac{W}{2})]$$

is used in the canal problems, see Bai(1981). Here h is water depth, W is the width of the canal, and A_{l0} & A_{lj} are the unknown coefficients to be determined by satisfying the matching conditions. The parameter m_0 associated with the Hankel function of the first kind, H_l, and the parameters m_j associated with the modified Bessel function of the second kind, K_l, are the real roots of

$$m_0 \tanh(m_0 h) = \frac{\omega^2}{g} \qquad \text{and} \qquad m_j \tan(m_j h) = -\frac{\omega^2}{g}$$

respectively. For the canal problem the parameters D_{l0} and D_{lj} satisfy

$$D_{l0} = [m_0^2 - (\frac{l\pi}{W})^2]^{1/2} \qquad \text{and} \qquad D_{lj} = [m_j^2 + (\frac{l\pi}{W})^2]^{1/2}.$$

The discretisations for the open and the confined waters examples are illustrated in Figures 1 & 2. In Figures 3 & 4 the reactive hydrodynamic coefficients for surge and sway are presented for the hemisphere in open water and the floating barge in a canal respectively. These results together with the corresponding results for first order excitation forces and moments, the response amplitude operators for each degree of freedom and the second order drift forces have been shown to agree well with other published results. The indicated comparative studies are sufficient to demonstrate the value and applicability of the eigenfunction expansion techniques outlined.

Acknowledgements

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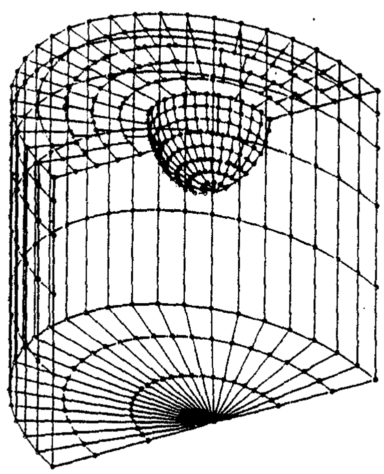
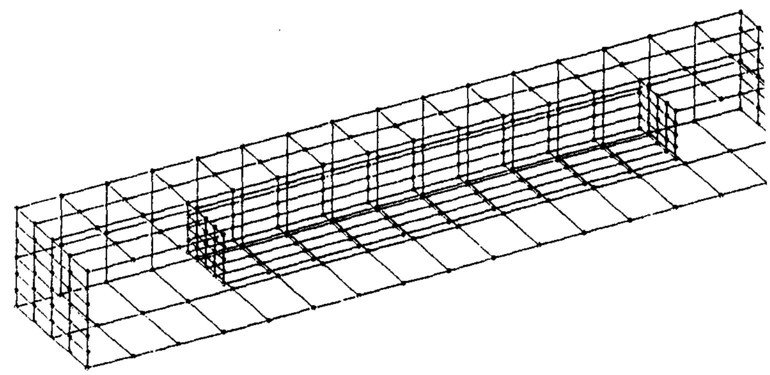
1. SURFACE DISCRETISATION FOR 3D PROGRAM
A FLOATING HEMISPHERE



2. SURFACE DISCRETISATION FOR 3D PROGRAM
FLOATING BARGE IN A CANAL



HYDRODYNAMIC COEFFICIENTS
FLOATING HEMISPHERE
100 body facets
depth/radius = 1.5

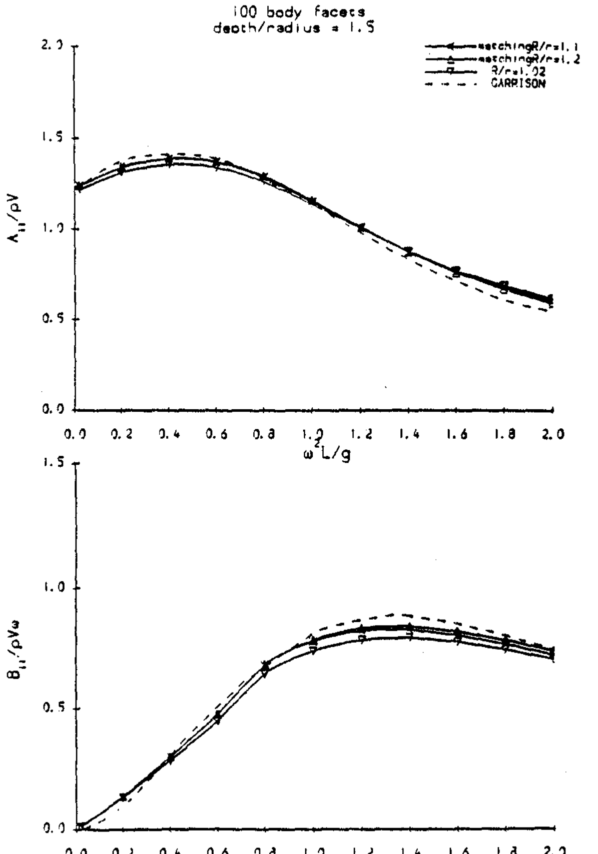


FIG.3

HYDRODYNAMIC COEFFICIENTS
FLOATING BARGE IN THE CANAL
added mass & damping
in sway

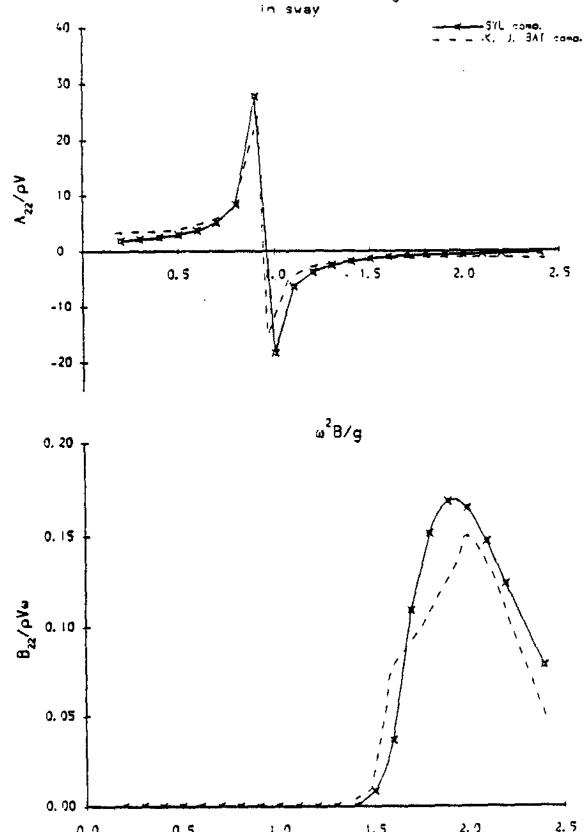


FIG.4

Discussion of Hearn and Liou's paper

Discusser S.Ando

Could you expand on the criteria you used to determine the location of the radiation boundary?

Authors' reply

Rather than fix the location of the radiation boundary, the matching boundary really, as a function of the wavelengths associated with the propagating or evanescent modes of the interaction problem we have undertaken various numerical experiments. Thus in practice one finds that as the matching boundary is brought closer to the wetted surface, so the number of facets representing the free-surface and the sea-bed may be reduced and thus more elements, and hence more coefficients can be used in the eigenfunction expansions. For the hemisphere in open water we selected $R/r = 1.2$, where R is the radius of the radiation boundary and r is the representative radius of the floating structure. However, as Figure 3 shows, the results coverge quite well for $R/r = 1.02$ for the same eigenfunction expansion, that is taking l from -10 to 10 and j from 1 to 6 in the associated outer domain series. For the canal problem the matching boundaries were set a distance 1.6L from the centre of the barge, where L is the length of the barge. Here expansions for $l = 1$ to 8 were used with j assigned the same values as for the open water problems.

Discusser R.Eatock Taylor

Grant, I don't think you have mentioned the matching between the inner and outer regions. Do you use a variational approach? Is the number of terms in the eigenseries related to the number of panels in the discretisation (as well as frequency)? Treatment of the number of terms is particularly critical in the channel problem. What procedures have you used to validate your results, e.g. comparisons with other published results for wave frequency and drift forces on a cylinder in a wave tank (Matsui, Eatock Taylor & Hung etc.)?

Authors' reply

Our method is based on the matching of an eigenfunction expansion of the outer domain solution and the inner boundary integral representation of the nearfield solution. The matching conditions imposed are those indicated in the abstract, namely

$$\phi_I = \phi_O$$

and

$$\frac{\partial \phi_I}{\partial n} = \frac{\partial \phi_O}{\partial n},$$

where the subscripts I and O indicate inner and outer solutions respectively.

The number of unknown coefficients in the eigenseries is equal to the number of panels used in the representation of the matching boundary. The choice of panel size on the wetted surface of the floating body and the free surface, in view of the invariance assumptions over the panels, is governed by the wavelength associated with the highest frequency of interest. These choices are independent of the location of the matching boundaries. However, the nearer the matching boundary is located to the floating body the greater the number of terms in the eigenseries. The number of times used in the tank problem is not particularly critical. What is critical is the proper assignment of meaningful values to the coefficients D_{lo}.

Validation of our predictions has been achieved by comparison with many other researchers predictions. As indicated in my opening remarks the particular objective of this study was to investigate how the presence of tank walls affected the predictions of second order forces, so as to understand the differences between predicted and measured values of low frequency damping. The validation has therefore consisted of validation of first order forces, predicted first order motions and drift forces in open water situations using the eigenfunction expansion technique. Thus for open sea problems we have used many different sources which provide experimental measurements or theoretical predictions or both. For the canal problem we have used Bai's 1981 results for comparison of first order quantities. Through comparison of open water barge drift force predictions and Pinkster's measurements we have validated barge drift forces and then used the canal related solutions to appreciate the affects of both finite depth and tank walls. By allowing the depth and width of the tank to increase we have also demonstrated that the canal solution readily converges to the open water solution. However we have yet to use the particular references you have cited.

Discusser R.W.Yeung

The eigenfunction expansion for the tank problem is not correct. The dummy index l associated with the propagating modes does not exist for $l >$ (M-1), where D_{Mo}, as defined by the authors, would become imaginary. Similarly the sum for the evanscent modes should start for l=M and go to ∞. This incorrect choice of eigenfunctions (Bai's expressions were correct, I believe) by the authors will lead to improper physics of the hydrodynamics.

Authors' Reply

Professor Yeung is thanked for identifying an important point which was neither discussed in the written form of the paper, nor in the presentation. The improper physics referred to arises from the inclusion of those terms in the canal problem eigenseries which have an imaginary value of D_{lo}. This occurs when $m_o < l\pi/W$. This point was also discussed by Bai (1981) and our implementation does in fact reflect the need to exclude unreal exponentially growing terms in the solution. Although the theoretical solution is presented in the form of an infinite series, we clearly use a finite truncated series in practice with the exponential terms associated with imaginary values of D_{lo} modified to correspond to decaying contributions only as $x \rightarrow \infty$.

Discusser T.Miloh

The expression for the velocity potential due to a wavemaker in a channel should be modified (see equation (6) of attached paper*). Basically it includes a finite sum $\sum_o^M$ of harmonic terms, a term $\sum_{M+1}^{\infty}$ of evanescent term and a double summation representing trapped modes.

*L.Shemer, E.Kit and T.Miloh (1987) Measurements of two-and three-dimensional waves in a channel, including the vicinity of cut-off frequencies, Experiments in Fluids, 5, pp.66-72.

Authors' Reply

The point being made by Professor Miloh is related to those points made by Professors Yeung and Eatock-Taylor earlier. In the paper kindly provided Professor Miloh and his colleagues deal with the imaginary roots in exactly the same manner as we have indicated in our reply to Ronald Yeung. In Professor Miloh's presentation the exponential terms related to our upstream and downstream flows are combined as a single hyperbolic term (a sinh function) and so a different unknown coefficient has to be determined in the corresponding eigenseries. Otherwise we appear to deal with the highlighted difficulty in the same manner.