Theoretical Prediction of Tank-Wall Effects on Hydrodynamic Forces Acting on an Oscillating and Translating Slender Ship

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INTRODUCTION

In carrying out experiments of a ship model in a towing tank with limited width, the effects of tank-wall interference will be expected, particularly when the forward velocity and oscillation frequency of a ship model are relatively small. We are required not only to judge whether the tank-wall effects are expected or not, but also to predict precisely the magnitude of the effects when those are present. Despite such requirements, few theoretical studies have been reported so far [1][2][3]; these studies are based upon the thin-ship theory or the strip theory. Nevertheless the proposed calculation methods seem to be fairly complicated, and may not be used when quantitatively precise prediction is necessary. Of course an "exact" solution will be obtained by the integral-equation method, if a markedly efficient numerical scheme is developed of calculating the forward-speed version of 3-D Green function. However even with the present super computer, it seems impossible to pursue this "exact" calculation method.

In the present paper, the slender-ship theory is applied to develop a new method of estimating with good accuracy the effects of tank-wall interference on hydrodynamic forces acting on a ship with forward and oscillatory motions. The theory is restricted to the radiation problem of heave and pitch, but the diffraction problem may be treated in a similar manner.

After the illustration of the theoretical framework, numerical results are presented for the added-mass and damping coefficients of a floating spheroid with zero speed, showing remarkable agreement with "exact" values by the 3-D panel method [4]. Computations for the case of nonzero forward velocity are now in progress, and will be accomplished in the foreseeable near future.

PROBLEM FORMULATION

With the assumption of irrotational motion in an inviscid fluid, we develop a linear slender-ship theory of forced heave and pitch motions. Following the method of matched asympotic expansions, the flow field around a ship is divided into inner and outer regions.

In the outer region, the governing equation and boundary conditions to be satisfied by the velocity potential ϕ_j (time-dependent term $e^{i\omega t}$ is factored out and mode index $j=3$ for heave, $j=5$ for pitch) can be written as

$$[L] \qquad \phi_{jxx} + \phi_{jyy} + \phi_{jzz} = 0 \qquad \text{for } z>0,\ |y|<B_T/2 \tag{1}$$

$$[F] \qquad \phi_{jz} + K\phi_j + i2\tau\phi_{jx} - \frac{1}{K_o}\phi_{jxx}$$

$$[R] \qquad - i\mu(K\phi_j + i\tau\phi_{jx}) = 0 \qquad \text{on } z=0 \tag{2}$$

$$\text{where} \quad K=\omega^2/g, \quad \tau=U\omega/g, \quad K_o=g/U^2$$

$$[W] \qquad \phi_{jy} = 0 \qquad \text{on } y=\pm B_T/2 \tag{3}$$

$$[B] \qquad \phi_{jz} \longrightarrow 0 \qquad\qquad \text{as } z \longrightarrow \infty \tag{4}$$

Here ω and U denotes the circular frequency of oscillation and constant forward speed of a ship, respectively. The positive x-axis is taken in the direction of ship's forward motion and the z-axis is positive downward. The tank breadth is denoted by B_T. Note that Rayleigh's artificial viscosity coefficient μ is introduced in (2) to impose the radiation condition at infinity, and that the boundary condition on the ship hull is absent.

The outer solution can be described by a line distribution of 3-D Green function along the ship's centerline on the free surface, in the form

$$\phi_j(x,y,z) = \int_L q_j(\xi) G(x-\xi,y,z)\,d\xi \tag{5}$$

with the Green function $G(x,y,z)$ being determined to satisfy all of the above conditions (1)-(4). To impose the tank-wall boundary condition [W], an infinite number of mirror-image singularities with respect to both of the side walls are considered, and then the summation of the resultant infinite series is analytically obtained in a closed form.

In (5), q_j denotes the source strength, which is unknown at this stage. However, this unknown can be settled by requiring the inner expansion of (5) to be compatible with the outer expansion of an appropriate inner solution. For this matching procedure, the inner expansion of the Green function must be sought. Assuming Ky, $Kz=O(\varepsilon)<<1$ and $\tau=o(1)$, the desired result can be obtained, in the Fourier-transformed domain, in the form

$$\begin{aligned} G^*(k;y,z) &= \int_{-\infty}^{\infty} G(x,y,z)e^{-ikKx}dx \\ &= G_{2D}(y,z) + \frac{1}{\pi}(1-Kz)\{\, f_O^*(k) + f_T^*(k)\,\} + O((Kr(1-\nu), K^2r^2) \end{aligned} \tag{6}$$

where

$$f_O^*(k) = \log\frac{2}{|k|} + \pi i - \left[\begin{array}{l} \dfrac{1}{\sqrt{1-k^2/\nu^2}}\{\, \cosh^{-1}(\frac{\nu}{|k|}) + \pi i\,\mathrm{sgn}(1+k\tau)\,\} \\ \dfrac{1}{\sqrt{k^2/\nu^2-1}}\{\, \cos^{-1}(\frac{\nu}{|k|}) - \pi\,\} \end{array}\right] \tag{7}$$

$$\begin{aligned} f_T^*(k) = &\int_0^{\infty} \frac{n^2}{n^2+\nu^2}\{\, -1 + \coth(\frac{KB_T}{2}\sqrt{n^2+k^2})\,\}\frac{dn}{\sqrt{n^2+k^2}} \\ &+ \left[\begin{array}{l} \dfrac{-\pi i\,\mathrm{sgn}(1+k\tau)}{\sqrt{1-k^2/\nu^2}}\{\, \frac{2\pi}{KB_T}\,\delta(\sqrt{\nu^2-k^2}, \frac{2\pi}{KB_T}) - 1 \\ \qquad\qquad - i\,\mathrm{sgn}(1+k\tau)\cot(\frac{\kappa B_T}{2}\sqrt{1-k^2/\nu^2})\,\} \\ \dfrac{\pi}{\sqrt{k^2/\nu^2-1}}\{\, -1 + \coth(\frac{\kappa B_T}{2}\sqrt{k^2/\nu^2-1})\,\} \end{array}\right] \end{aligned} \tag{8}$$

and

$$\nu = (1+k\tau)^2, \quad \kappa = K\nu \tag{9}$$

Here the upper and lower expressions in brackets are to be taken according as $|k|<\nu$ and $|k|>\nu$, respectively. G_{2D} in (6) is the 2-D Green function of zero speed. $f_O^*(k)$ given by (7) is the 3-D longitudinal interaction function in the absence of tank walls, and identical to that provided in Newman's unified theory [5]. Therefore $f_T^*(k)$ in (8) represents the 3-D effects of tank-wall interference. The function $\delta(x, 2\pi/KB_T)$ appearing in (8) denotes the infinite series of

Dirac's delta function defined by

$$\delta(x, \frac{2\pi}{KB_T}) = \sum_{m=-\infty}^{\infty} \delta(x - \frac{2\pi}{KB_T} m) \tag{10}$$

and therefore contributes only when $\sqrt{\nu^2-k^2} = 2\pi m/KB_T$ ($m=0,1,2,\cdots$). At these particular points, the inverse Fourier transform of cotangent function in (8) should be interpreted as Cauchy's principal-value integral.

By comparison, in the inner region close to the ship hull, the tank-wall boundary condition [W] and the radiation condition [R] are absent, and a coordinate stretching argument can be applied. The resulting boundary-value problem can be summarized as

$$[L] \qquad \varphi_{jyy} + \varphi_{jzz} = 0 \qquad \text{for } z>0 \tag{11}$$

$$[F] \qquad \varphi_{jz} + K\varphi_j = 0 \qquad \text{on } z=0 \tag{12}$$

$$[H] \qquad \varphi_{jN} = N_j + Um_j/i\omega \qquad \text{on ship hull} \tag{13}$$

These are the same as conventional 2-D formulation except that the radiation condition is absent. Therefore the inner solution can be identified with Newman's unified slender-ship-theory solution [5]; composed of a particular solution given by the strip theory plus a homogeneous solution multiplied by a 3-D interaction coefficient. To be more specific,

$$\varphi_j(x;y,z) = \varphi_j^P(y,z) + \frac{U}{i\omega}\hat{\varphi}_j^P(y,z) + C_j(x)\,\varphi_j^H(y,z) \tag{14}$$

$$\varphi_j^H(y,z) = \varphi_j^P(y,z) - \overline{\varphi_j^P(y,z)} \tag{15}$$

where the overbar denotes the conjugate of complex quantity. The coefficient of homogeneous solution C_j in (14) is indeterminate in the inner region, but can be settled by matching (14) with the outer solution in an overlap region between inner and outer fields. In this overlap region, (14) reduces to

$$\varphi_j(x;y,z) \sim [\ \sigma_j + \frac{U}{i\omega}\hat{\sigma}_j + C_j(\sigma_j - \bar{\sigma}_j)\ ]G_{2D}(y,z) - (1-Kz)2iC_j\bar{\sigma}_j \tag{16}$$

Here σ_j and $\hat{\sigma}_j$ are the 2-D effective source strengths in the transverse y-z plane; these are given by the numerical implementation of the strip theory.

Comparing (16) with the inner expansion of outer solution given by the combination of (5) and (6), we can easily find out the matching conditions. From those we get the following equations, being able to settle the 3-D source strength q_j and the interaction coefficient C_j:

$$q_j(x) - \frac{i}{2\pi}(\sigma_j/\bar{\sigma}_j - 1)\int_L q_j(\xi)[f_o(x-\xi) + f_T(x-\xi)]d\xi = \sigma_j(x) + \frac{U}{i\omega}\hat{\sigma}_j(x) \tag{17}$$

$$C_j(x) = [q_j - (\sigma_j + \frac{U}{i\omega}\hat{\sigma}_j)]/(\sigma_j - \bar{\sigma}_j) \tag{18}$$

Eq. (17) is an integral equation for the 3-D source strength q_j, and in the case of no tank-wall effects, i.e. $f_T(x)=0$, this equation reduces to the corresponding one in Newman's unified slender-ship theory [5]. The inner solution (14) appears formally to be invariable regardless of whether the tank walls are present or not, but it should be noted that the effects of tank-wall interference are implicitly incorporated through the coefficient of homogeneous solution.

NUMERICAL RESULTS

In order to check the practical utility of the present theory, computations are firstly performed for a floating spheroid with zero forward speed. Numerical examples are presented in Figs.1 and 2 for the added-mass and damping coefficients of pitch motion, where the beam-to-length ratio of spheroid is 1/8 and the ratio of ship's beam to tank width is 1/16. Solid line shows the results of the present slender-ship theory including the effects of tank-wall interference. The "exact" solution calculated by the 3-D panel method [4] is also shown by open circles for comparison. The agreement between the results of 3-D panel method and of slender-ship theory is virtually perfect. Almost the same degree of agreement can be achieved for the case of narrower tank width, because the present theory takes into account the tank-wall effects also upon evanescent waves. Encouraged by these favorable results, we are now performing numerical calculations for the case of nonzero forward velocity.

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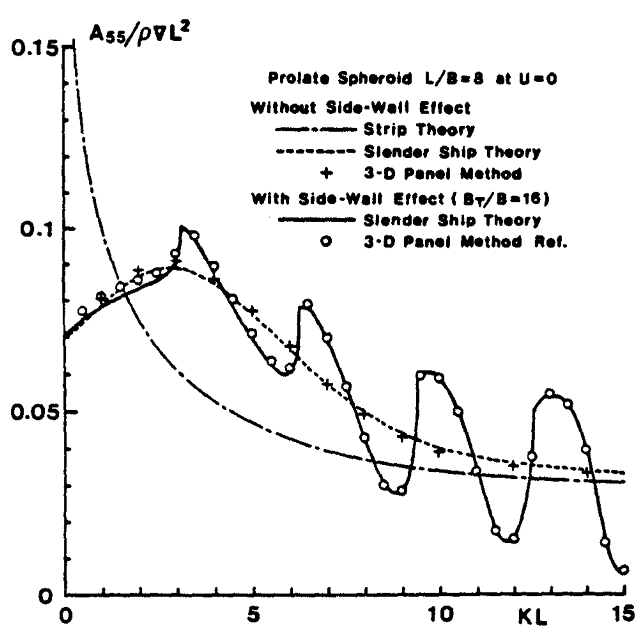

Fig.1 Pitch added moment of inertia of a prolate spheroid at U=0

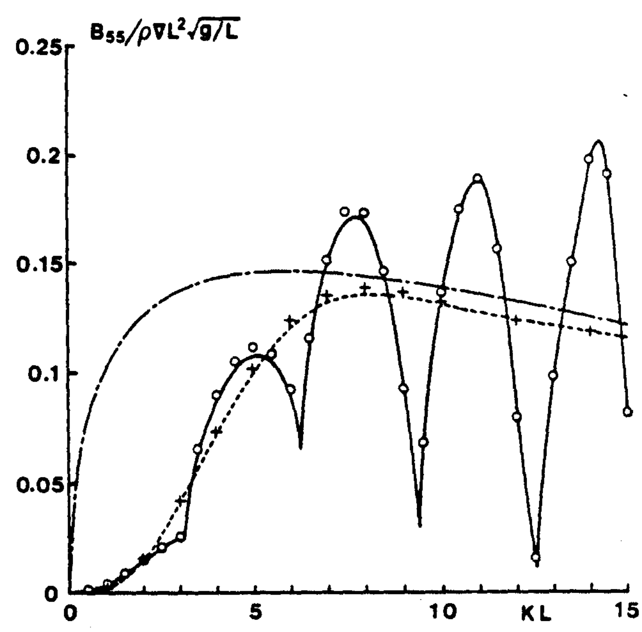

Fig.2 Pitch damping coefficient of a prolate spheroid at U=0

DISCUSSION

Hearn: Can you comment on the quality of the pitch damping coefficient B_{55} predictions and its dependece on forward speed? Since pitch motion is responsible for a large portion of the advancing ship's added resistance we have found (Hearn, Tong & Lau, PRADS'87) that prediction of B_{55} (with forward speed effects included) is important!

Kashiwagi: It's true that the hydrodynamic forces associated with pitch mode are greatly influenced by the ship's forward speed. In particular, when the tank-wall effects exist, the difference in pitch radiation forces between zero-speed and forward-speed cases is quite considerable. This fact may be attributed to that the force (or pressure) distribution in the ship's longitudinal direction will be easily changed by the presence of forward speed of a ship.