TITLE : COMPUTATION OF TRANSIENT LINEARIZED GRAVITY WAVES

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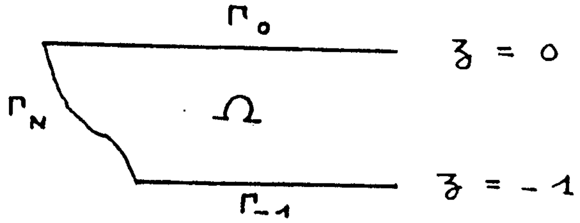
We are interested in the 2D and 3D water waves created by small motions of the submarine ground. The problem is treated in infinitesimal potentiel theory (see /1/). We are thus led to solve :

$$\Delta\phi = 0 \quad \text{in } \Omega$$
$$\partial_n \phi = h(t) \quad \text{on } \Gamma_N$$
$$\partial_z \phi = 0 \quad \text{on } \Gamma_{-1}$$
$$\partial_t^2 \phi + \partial_z \phi = 0 \quad \text{on } \Gamma_0$$



where $\phi(t)$ is the velocity potential, Γ_0 the linearized free surface and Γ_{-1} the flat part of the bottom.
Let us denote by ϕ^p an approximation of $\phi(p\Delta t)$ where $\Delta t > 0$ is the timestep and p is an integer. We introduce the following finite difference scheme with respect to time :

$$\frac{\phi^{p+2} - 2\phi^{p+1} + \phi^p}{t^2} + \partial_z \left(\frac{\phi^{p+2} + 2\phi^{p+1} + \phi^p}{4}\right) = 0 \quad \text{on } \Gamma_0$$

Let us set :

$$(1) \qquad \chi^{p+2} = \frac{\phi^{p+2} + 2\phi^{p+1} + \phi^p}{4}$$

and :

$$g^{p+2} = \frac{h^{p+2} + 2h^{p+1} + h^p}{4}$$

At each time-step, the following elliptic problem is solved :

$$\Delta\chi^{p+2} = 0 \quad \text{in } \Omega$$
$$\partial_n \chi^{p+2} = g^{p+2} \quad \text{on } \Gamma_N$$
$$\partial_z \chi^{p+2} = 0 \quad \text{on } \Gamma_{-1}$$
$$\partial_z \chi^{p+2} + \frac{4}{\Delta t^2}\chi^{p+2} = \frac{4}{\Delta t^2}\phi^{p+1} \quad \text{on } \Gamma_0$$

It must be noticed that for fixed t, only the second member is modified from one time-step to another and that no radiation condition is required due to the positivity of the factor $\frac{4}{\Delta t^2}$.

A priori ϕ^{p+1} is going to spread towards infinity while g^{p+2} keeps localized in the vicinity of the source. Taking these two points into account, we divide our calculation in two stages .

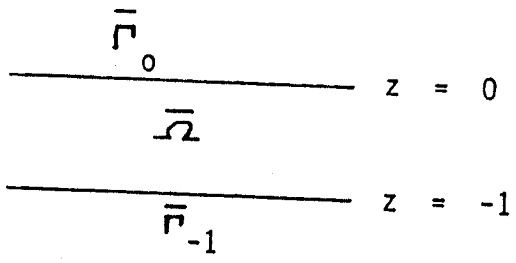
STAGE 1 :

We calculate a harmonic extension of $\phi^{p+1} \mid \Gamma_o$ by solving the following problem :

$$\Delta \chi_o^{p+2} = 0 \text{ in } \bar{\Omega}$$

$$\partial_z \chi_o^{p+2} = 0 \text{ on } \bar{\Gamma}_{-1}$$

$$\partial_z \chi_o^{p+2} + \frac{4}{\Delta t^2} \chi_o^{p+2} = \frac{4}{\Delta t^2} \quad {}^{p+1} \text{ on } \bar{\Gamma}_o$$



These equations are exactly solved by means of the continuous Fourier Transform which leads to make use of the Fast Fourier Transform algorithm in the numerical resolution .

STAGE 2 :

We solve :

$$\Delta \chi_1^{p+2} = 0 \text{ in } \Omega$$

$$\partial_n \chi_1^{p+2} = g^{p+2} - \partial_n \chi_o^{p+2} \text{ on } \Gamma_N$$

$$\partial_z \chi_1^{p+2} = 0 \text{ on } \Gamma_{-1}$$

$$\partial_z \chi_1^{p+2} + \frac{4}{t^2} \chi_1^{p+2} = 0 \text{ on } \Gamma_o$$

The numerical solution of this last problem has been performed by differents methods :

- the localized finite element method for 2D cases (see /2/)
- the coupling between finite elements and integral representation in 3D cases (see /3/)

This is in fact a matter of convenience. Other methods devoted to the solution of linear exterior problems might have been used such as the boundary element method.

We then put $\chi^{p+2} = \chi_o^{p+2} + \chi_1^{p+2}$ from which we deduce ϕ^{p+2} by (1). The second member of the free surface condition at the next time-steps follows.

Firstly, we have studied the wave propagation along a rectilinear coast when a localized perturbation is applied on the bottom. The wave guide phenomenon studied by garipov has been observed (see /4/).

Figure 1 shows the free surface contour lines after 300 s of propagation

The results ensure that our method is able to take the surface wave propagation into account thanks to the small size of the calculus domain.

We have wanted to study more precisely the initial hollow generation by comparison with other codes working in axisymetric cases .

SOLAVOF : solves the free surface non linear Navier - Stokes equations

PISCES : solves the conservation laws of continuous media mechanics

MELINA : computes the present method

the test problem is the filling of an empty cylinder of 100 meters height set under an ocean of constant depth H .

Figure 2 shows the free surface profiles given by the three codes at the moment of maximum hollow.

One can observe that the initial hollow is underestimated by our calculus when the linearity criterion ($\frac{\eta}{h} < .1$) is violated.

Further 3D calculus in the vicinity of an island are being carried out. They will be presented during the talk.

BIBLIOGRAPHY

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FIGURE 1
Propagation along a rectilinear coast
Contour lines of the free surface after 300s of propagation

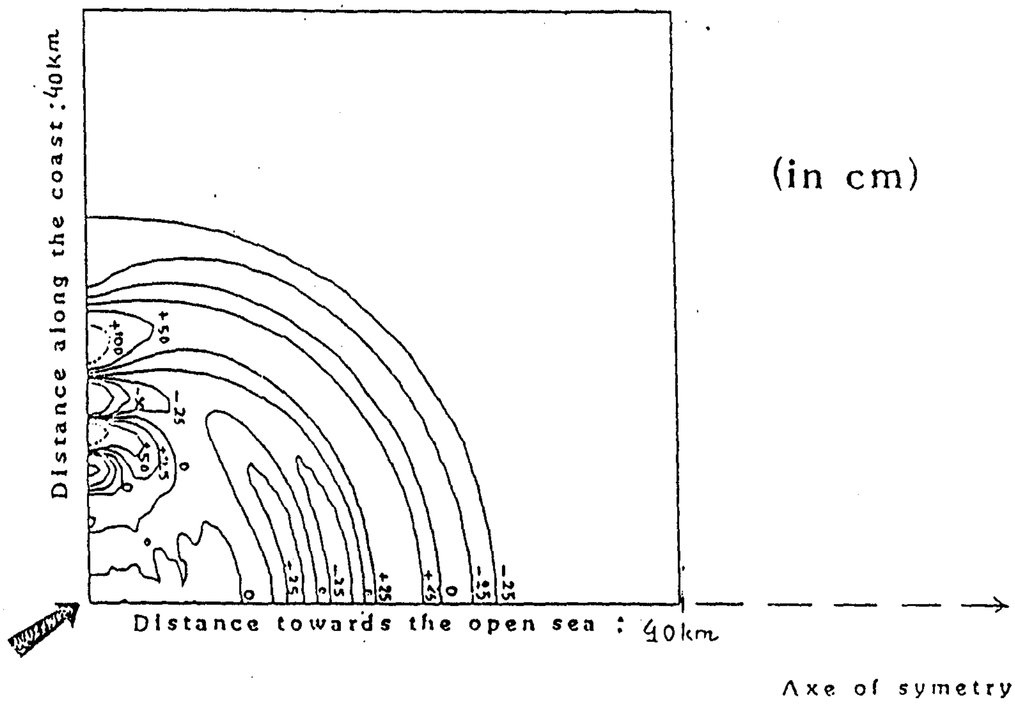

FIGURE 2
Filling an empty cylinder of 100m height
Initial hollow profile