VECTORIZED COMPUTATION OF THE TIME-DOMAIN GREEN FUNCTION

by
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In the time-domain solution of the three-dimensional unsteady ship motions problem, the computation of the free surface term of the Green function accounts for the largest portion of the CPU time. In order to proceed with realistic computations, fast techniques for evaluating the Green function are needed since the body-nonlinear problem requires computation of the Green function over the past history of the motion at each time step. An increase of speed.of about ten times over previously available techniques for calculating these terms has been achieved through the use of vectorized interpolation and asymptotic routines on the CRAY X-MP/48. The new Green function techniques are being applied to determine the hydrodynamic forces on a body of arbitrary geometry undergoing large-amplitude motions in the presence of a free surface.

In the present work, the body boundary condition is being applied on the exact time-dependent three-dimensional body surface and the free surface is linearized. This method corresponds to the classic Neumann-Kelvin approximation for steady flow, and success has been achieved in solving the unsteady linearized radiation and diffraction problems using the mean body position (c.f., King, Beck, and Magee, 1988, Magee and Beck, 1988a, and Korsmeyer, 1989). Other researchers are also working on body-nonlinear computations. Ferrant (1989) has solved for the radiation forces and wave enegy dissipation due to translational motion of axisymmetric submerged bodies in both the time and frequency domains using the body-nonlinear formulation. Work on this problem is also proceeding outside academic circles.

The problem is formulated as follows. The fluid domain is bounded by the free surface, S_f, the body surface, S_h, and a surface at infinity, S_∞. An inertial coordinate system is chosen with the z-axis upwards and the origin fixed at the calm water surface. Ideal irrotational flow is assumed so that the velocity potential satisfies Laplace's equation

$$\nabla^2\phi = 0 \tag{1}$$

On the plane z = 0 a linearized free surface boundary condition is applied such that:

$$[\,(\partial/\partial t)^2 + g\,\partial/\partial z\,]\,\phi = 0 \qquad \text{on } z=0 \tag{2}$$

The body boundary condition is the no penetration condition applied on the *exact* moving body surface, S_h (t). Thus,

$$\partial\phi/\partial n = (\mathbf{U} + \omega \times \mathbf{r}) \bullet \mathbf{n} \qquad \text{on } S_h(t) \tag{3}$$

where,

U = the vector of the body translational velocity

ω = the body rotation vector

n = the unit normal to the body surface out of the fluid

The condition at infinity is that

$$\nabla\phi \rightarrow 0 \qquad \text{as } r \rightarrow \infty \tag{4}$$

and an initial value problem is posed such that

$$\phi,\ \partial\phi/\partial t \rightarrow 0 \qquad \text{as } t \rightarrow -\infty \tag{5}$$

The appropriate time-domain Green function is given by

$$G(P, Q, t, \tau) = (1/r - 1/r')\,\delta(t-\tau) + H(t-\tau)\,\tilde{G}(P, Q, t, \tau) \tag{6}$$

$$\tilde{G}(P, Q, t, \tau) = \sqrt{(g/r'^3)}\,\hat{G}(\mu,\beta)$$

$$\hat{G}(\mu,\beta) = \int_0^\infty d\lambda\,\sqrt{\lambda}\,\sin(\beta\sqrt{\lambda})\,e^{-\lambda\mu}\,J_0(\lambda\sqrt{(1-\mu^2)}) \tag{7}$$

where,

$P = (x, y, z)$

$Q = (\xi, \eta, \zeta)$

$\lambda = k\,r'$

$\mu = -(z+\zeta)/r'$

$\beta = \sqrt{(g/r')}\,(t-\tau)$

$r' = [(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2]^{1/2}$

The parameter μ relates the vertical to horizontal distance between source and field points, and β is time like and relates to the phase of the generated waves. The function is oscillatory for large β and is sharply peaked (though not singular) for μ near 0.

An integral equation is developed by applying Green's theorem to the fluid domain and integrating with respect to time. The final form is found to be

$$\phi(P,t) + 1/2\pi \int_{S_h(t)} (\phi(Q,t)\,\partial/\partial n_Q(1/r - 1/r')\,dS = -1/2\pi \int_{S_h(t)} (1/r - 1/r')\,\partial/\partial n_Q\,\phi(Q,t))\,dS$$

$$- 1/2\pi \int_{-\infty}^{t} \int_{S_h(\tau)} (\phi(Q,\tau)\,\partial/\partial n_Q\,\tilde{G}(P,Q,t,\tau) - \tilde{G}(P,Q,t,\tau)\,\partial/\partial n_Q\,\phi(Q,\tau))\,dS\,d\tau \tag{8}$$

The dynamic pressure is found from Bernoulli's equation, and the hydrodynamic forces determined by integrating the pressure over the body.

Because of the history dependence in the time domain, over 90 percent of CPU time in the calculation is spent evaluating the free surface term of the Green function (7). A very fast method for computing this function is needed for body-nonlinear calculations to be practical. Interpolation seemed a logical alternative to the semi-analytical methods which have previously been applied. However, the simple bilinear interpolation described by Ferrant (1988), while very fast, requires an enormous amount of data for sufficient accuracy. On the CRAY virtual memory is not available, and the cost of reading this data from disk is prohibitive. To reduce the size of the data file, bicubic interpolation is being employed.

Given the restrictions imposed by limited core memory, it was also necessary to compute $\tilde{G}$ in two parts. A simple analytical approximation is made, and this is subtracted off to obtain a smoother function which can be more easily calculated by the interpolation scheme. Wehausen and Laitone (1960) have shown that when both source and field points lie on the free surface (that is, for μ=0), the Green function reduces to the following form in terms of Bessel functions of the first kind:

$$\hat{G}(0,\beta) = \pi\,\beta^3/(8\sqrt{2})\,(J_{1/4}(\beta^2/8) * J_{-1/4}(\beta^2/8) + J_{3/4}(\beta^2/8) * J_{-3/4}(\beta^2/8)) \tag{8}$$

The form of the asymptotic expansion (see King, 1987) suggests a dependence on $\exp(-\beta^2 * \mu/4)$ so that the Green function can be calculated as

$$\hat{G}(\mu,\beta) = \exp(-\beta^2 * \mu/4) * \hat{G}(0,\beta) + G_i(\mu,\beta) \tag{9}$$

where $G_i(\mu,\beta)$ is the part which must be interpolated over the grid. The two functions $\hat{G}(\mu,\beta)$ and $G_i(\mu,\beta)$ are shown in Figures 1 and 2 respectively. The function $\hat{G}(0,\beta)$ is precomputed and stored for one-dimensional interpolation. Decomposing the function into these two parts allows a much larger grid spacing to be used than would have been possible without the separation. However, except for $\mu = 0$ (where $G_i = 0$), the analytic approximation is not perfect, and G_i is still oscillatory. A nonuniform grid spacing with a simple dependence on μ and β was needed to further reduce the data requirements. The nonuniform cell boundaries are easily computed, thus avoiding the need to search through a table of grid values.

The function G_i was computed at 25 points per grid cell, and the 16 coefficients of the bicubic representation were found for each cell. The system was overspecified to reduce spurious numerical oscillations. The final data requirement is about 2 million words (1 word = 1-64 bit number), and hence the program and data fit under the maximum limit of 4MW on the CRAY. The coefficients are stored in a file which is read in at the beginning of the time-domain computation.

The interpolation routines are fast and accurate. They run at about 85 million floating point operations per second (MFLOPS). The X-MP has a maximum possible speed of 205 MFLOPS. Approximately 5.0E-06 CPU seconds are required for one evaluation of the Green function and its derivatives -- a factor of ten less than previously available methods (see Korsmeyer,1989), and sixteen times less than those used by King. Ferrant gives CPU times only for his entire computation, which include large amounts of unvectorized convolutions, since these were performed on a VAX 8700. Overall, the CPU times for similar calculations are 40 to 60 times less on the CRAY, which seems reasonable when one compares the speeds of the two machines.

The interpolated values using the two part Green function calculation scheme described above agree with those of King's computations to within an absolute error of 1.0E-08 for the Green function and 1.0E-06 for the derivatives. The accuracy of Ferrant's interpolation scheme was not given, except that the final results showed no significant difference when compared to results obtained using analytic methods for the Green function evaluation. Since only a limited number of Green functions can be calculated in a given amount of CPU time, there is a tradeoff between the numerical error made in calculating the Green function and the discretization error made in using a finite number of panels and a given time step size. This tradeoff is presently under study.

For $\beta > 10$ the asymptotic expansion of the Green function (see King, 1987) was retained, and the special expansion was used for μ near 1.0, where the usual asymptotic expansion becomes singular. This routine is now vectorized and requires only slightly more CPU time than the interpolation scheme.

The calculation of the infinite fluid terms is performed using the method of Hess and Smith (1964). These routines have also been vectorized with a resulting speed up of eight times over the old versions. For the submerged body, the (1 / r) term of the Green function need only be calculated once at the beginning, but because the body position moves with respect to its image, the (1 / r ') term must be recomputed once at each time step. An LUD matrix solver is presently being used since this is the fastest available at the San Diego Supercomputer Center. The convolutions are fully vectorized, and the remainder of the calculations compute the geometry of the changing body position. The entire program now runs at approximately 83MFLOPS from start to finish for 100 panels.

The new fast Green function routines are being applied to determine the hydrodynamic forces on a body of arbitrary geometry undergoing large-amplitude motions. Forward speed effects can easily be included for the submerged body.

Acknowledgements.

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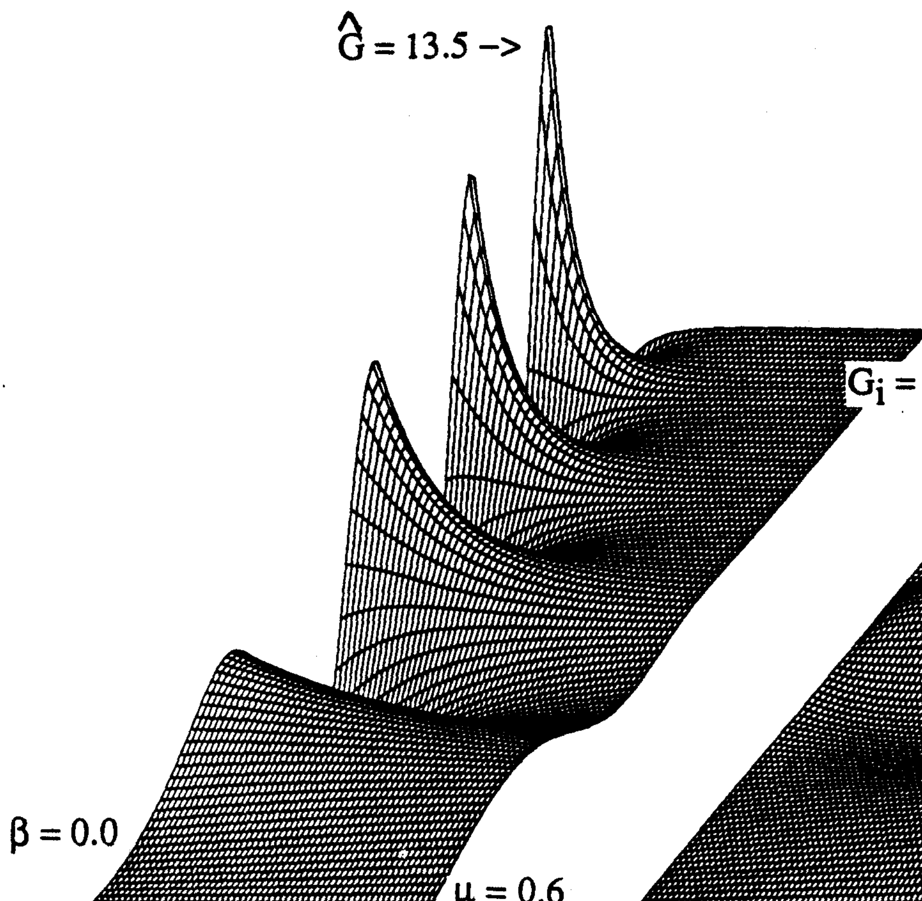

Fig. 1. $\hat{G}(\mu,\beta)$.

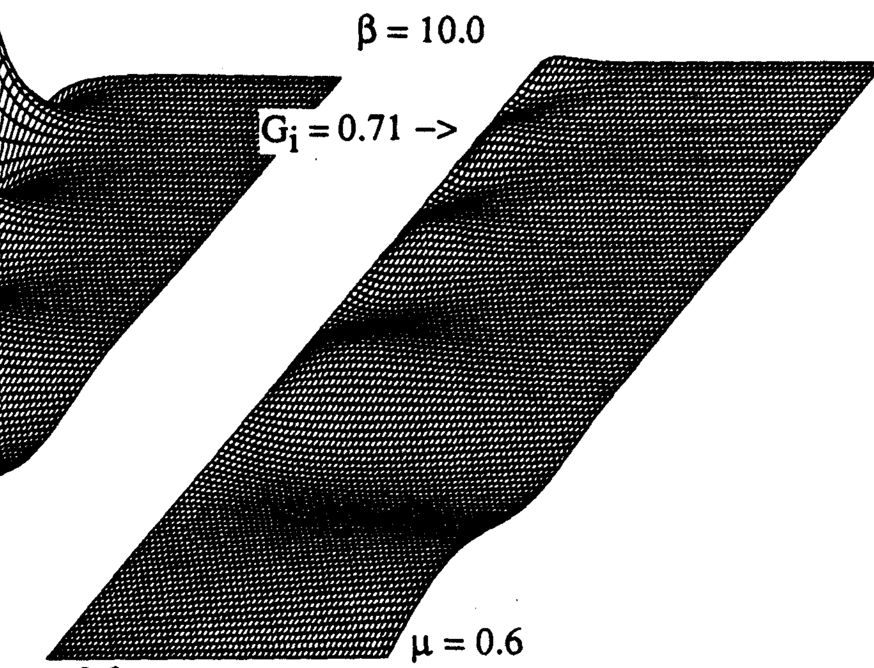

Fig. 2. $G_i(\mu,\beta)$.

DISCUSSION

Grilli: Did you use in your program any pre-vectorized CRAY routines that are not transportable? I think, in particular about procedures like GATHER-SCATTER, permitting partially vectorized IF-statements.

Magee & Beck: Conditional Vector Merge functions are used for simple IF-statements. While the simple IF's can be vectorized, these functions, which compute both results and then write out only one answer, perform slightly faster. Similar routines are available on many other vector machines such as Alliant. Other CRAY optimized routines include FFT and matrix solvers, but these are also widely available for various machines.

Yue: You have carried out your work specifically for the CRAY X-MP, a very popular machine, yet one which has its special architecture and data handling and computational characteristics. Please comment on the general applicability of your work or possible modifications for other machines.

Magee & Beck: We plan to port the linear time-domain code with the vectorized Green functions calculations to a Stellar-1000. Very few modifications should be required. The 32MB of RAM will be adequate to avoid virtual addressing for up to approximately 500 panels. In addition, since each Green function evaluation is independent, this method is ideally suited for such a parallel-vector machine. The inner DO-loop (say over field points) would still be vectorized, and the outer DO-loop (then over source points) could be processed in parallel.

The program is also upwardly compatible with such machines as the CRAY-2 and Y-MP. The availability of larger RAM on these machines would permit a larger region to be covered by the interpolation scheme, reducing the number of asymptotic evaluations, and thus producing a moderate speed-up. It is also possible that quadratic instead of cubic interpolation could be used if enough memory was on hand.

While local fast memory registers are arranged differently on the CRAY-2, there are still enough data locations to avoid any speed degradation. The special requirements for optimal code (such as the minimum vector length for top speed) may be different on this machine, but the method we have employed is sufficiently simple that it should perform very well here too.