Reflection Coefficients Due to Open Boundary Conditions

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In this note the effectiveness of the open boundaries sometimes proposed to simulate wave motion due to forced oscillation of a body in an infinite domain using only a finite solution domain is examined and some factors which influence their performance are determined. Such boundaries are desirable in order to reduce the size of the solution domain, and thus computation time and storage, as much as possible, while avoiding the effects of waves reflected from the boundary. The situation is analogous to the use of a beach in a wave tank to reduce or eliminate reflections. In three dimensional problems it is possible to consider matching an analytical solution on a boundary at a reasonable distance because radiated waves will decay rapidly with distance from the body generating them until at some point they can be considered to be linear. Similarly, in two dimensional linear problems matching is possible. However, in non-linear two dimensional problems this is not possible because the outgoing waves do not decay.

The geometry of the problem considered is shown in Figure 1. A rectangular fluid domain is bounded by a simple wavemaker on the left, the free surface FS on the top, a vertical boundary OB on the right on which either a wall or other boundary condition to be described is to be applied, and by the bottom at a depth h. The solution is carried out using a conventional 2D linear time domain wave model based on Green's theorem to solve the boundary value problem for the potential ϕ at each time step and integrating the resulting ϕ_t on the free surface obtained from the free surface boundary condition in time to obtain the potential problem at the next time interval. The linear problem was considered, because at the time of writing the application of open boundary conditions to the nonlinear solution was still under development. Another benefit is reduced computation time in running many cases. The present results should be a useful guide to effectiveness of open boundaries in the nonlinear problem.

An approach that has been used by a number of people (see Jagannathan [1] or Lee and Leonard [2] for surveys) is to apply a suitable condition on the outer boundary to simulate the effect of a semi-infinite domain adjacent to this boundary. These schemes are known by various names such as "open", "absorbing", and "radiation" boundaries. In the field of water waves Jagannathan [1] and later Lee and Leonard [2] adopted an idea of Orlanski [3] to the water wave situation. For regular waves, the Sommerfeld radiation condition, which can be written in the time domain as

$$\phi_t + c\phi_x = 0 \quad \text{as} \quad x \to \infty \tag{1}$$

where ϕ is the velocity potential and c the phase velocity, is required to be satisfied at the outer boundary. For steady state time-harmonic problems, c is known for plane waves at the appropriate frequency. For transient problems, however, it is not obvious what value of c to use. Orlanski [3] proposed determining the value numerically in terms of the solution. Jagannathan [1] applied this to the water wave problem by evaluating ϕ_t and ϕ_x on the free surface near the intersection of FS and OB, calculating

$$c = -\phi_t/\phi_x \tag{2}$$

at these points, setting

$$\phi_t = -c\phi_x \tag{3}$$

on OB, where ϕ_x can readily be evaluated, and then stepping ϕ on this boundary forward in time in a manner similar to that used on the free surface. Because $\phi_x = 0$ at some points (at which, for a linear regular wave, we also have $\phi_t = 0$), Jaganathan [1] took an average of the value of $-\phi_t/\phi_x$ at several suitable points near the right FS-OB intersection.

Lee and Leonard [2] developed an extrapolation method, in which c is calculated on the free surface at a point near the OB-FS intersection for a short period of time, in a manner similar to that of Jagannathan [1], and then an empirical formula for c as a function of time is applied which matches the calculated c at the start of the period of extrapolation and which decays exponentially to the correct steady-state value for large time.

In this note, the effect of simply using a constant value of c in the transient case is examined. Experience with open boundary conditions seemed to indicate the lack of a strong sensitivity to the exact value of c which was used, as long as it was not too unreasonable. The method used to investigate this question was to generate a group of N waves of period T using a modulated sine motion of a flat plate wavemaker

$$x_{wavemaker}(t) = \begin{cases} A\sin(\pi t/NT)\sin(2\pi t/T) & 0 \le t \le NT \\ 0 & t > NT \end{cases} \tag{4}$$

and allow them to interact with the boundary OB. The reflection coefficient K_r was then calculated as the ratio of the largest peak in the first reflected group to the largest peak in the original outgoing group, instead of the usual technique used in wave basins of running the wavemaker until a steady state is achieved and then analyzing the resulting wave pattern, which is not suitable for numerical solutions. A similar method was used by Naito, et al. [4] in a wave tank.

The baseline case considered corresponds to an example of Lee and Leonard [2], with tank depth $h = 3$ m, wave period $T = 2$ seconds, boundary element size $ds = 0.75$ m on all sides, and time step $dt = 0.1$ second. A tank length of 18 m was adopted except as noted. In Figures 2 and 3 the results of a run with a wall condition $\phi_n = 0$ on OB are shown. In the first of these figures, four time histories are given: wavemaker displacement and the calculated wave elevations at the wavemaker, at the FS-OB intersection, and at the midpoint of the tank. The resulting K_r is 0.96. The second figure shows the wave profile in the tank at 2 second intervals. The persistent motion of the fluid is obvious. In Figures 4 and 5 similar results for a typical run with open boundary conditions is shown, with the chosen c corresponding to the correct value for a linear regular wave at the 2 second period of excitation. The reflection coefficient in this case is 0.35.

Figure 6 presents the reflection coefficient for the 2 second wave as a function of the assumed value of c used in Equation 3. Note that 3.107 is the correct value for a 2 second wave in water of this depth and 5.424 is the long wave length limit for this depth.

Figure 7 shows K_r as a function of incident wave period T for a constant value of $c = 3.107$. Also shown is the effect of reducing the boundary element size ds by a factor of one half to 0.375 m, first on OB only with the result $K_r = 0.22$, and then on the entire boundary with the result $K_r = 0.19$. For a 2 second period, $\lambda/ds = 8.28$ and 16.57, respectively, for the two panel sizes. The former is rather crude. Results for the smaller ds were not shown for 3.0 and 3.5 second periods because the reflected wave in these cases was quite indistinct and was swamped in low amplitude noise. It is clear in these cases, however, that K_r is improved, as shown in Figures 8 and 9 which present time histories and wave profiles for the 3.5 second period case. The 2 second wave with $ds = 0.75$ was also run in a tank 9 m long, with essentially no change in K_r from that obtained in the 18 m tank.

In conclusion, open boundary conditions with a constant value of c can be quite effective in reducing wave reflections at the boundary of the solution domain. The effectiveness is quite sensitive to accuracy of the potential solution, and less sensitive to the specific value of c chosen. Proper selection of solution parameters can result in open boundaries which are nearly as effective as the best beaches used in wave tanks, which can have values of K_r as low as 0.02 [5]. It is further encouraging that for a given set of solution parameters, the open boundary is more effective for longer waves, which travel faster, and thus travel back to the body more rapidly, to contaminate the local solution earlier.

References

[1] Jagannathan, S., Non-linear free surface flows and the application of the Orlanski boundary condition, Int. Jl. Num. Meth. Fluids, **8**, 1051-1071 (1988)

[2] Lee, J. F., and J. W. Leonard, A time-dependent radiation condition for transient wave-structure interactions, Ocean Engng., **14**, 469-488 (1987)

[3] Orlanski, I., A Simple Boundary Condition for Unbounded Hyberbolic Flows, J. Comp. Phys., **21**, 251-269 (1976)

[4] Naito, S., J. Huang, and S. Nakamura, Research on the absorb-making of Irregular waves with the terminal device, J. Kansai Soc N.A., No 207 (1987)

[5] Ouellet, Y. and I. Datta, A survey of wave absorbers, J. Hydraulic Res., 24, 265-277 (1986)

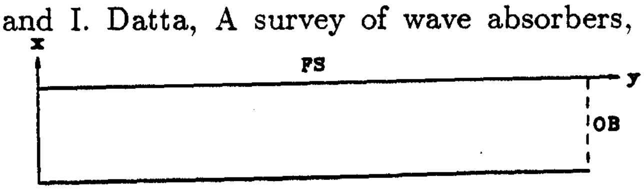

Figure 1 - Problem domain

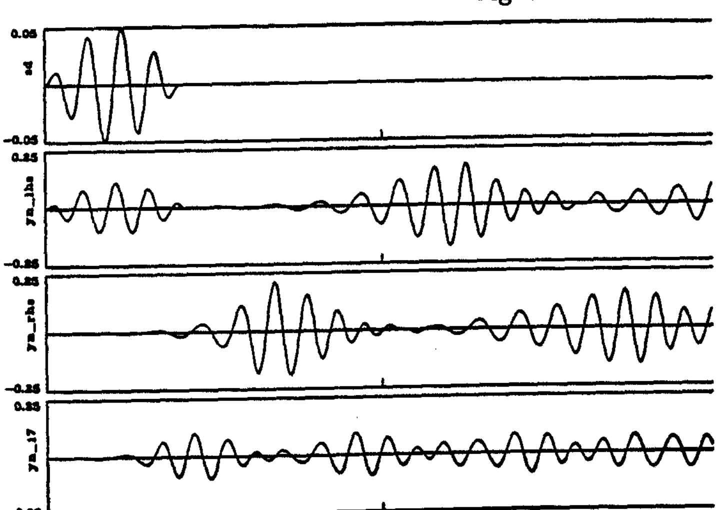

Figure 2 - Time histories for $T = 2.0$ sec, wall condition

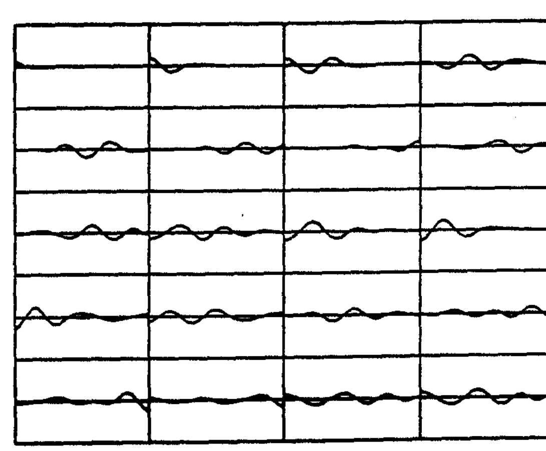

Figure 3 - Wave profiles at 2 sec intervals for $T = 2.0$ sec, wall condition

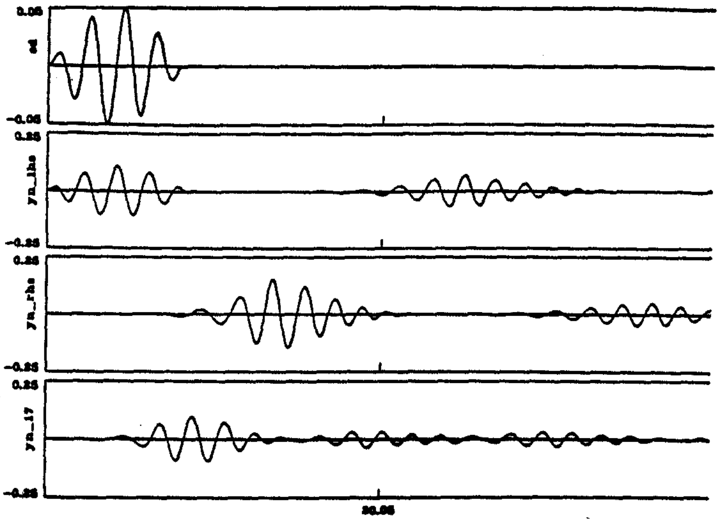

Figure 4 - Time histories for $T = 2.0$ sec, open BC with $c = 3.107$

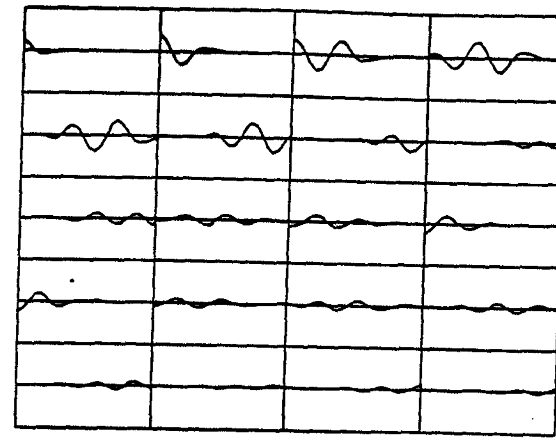

Figure 5 - Wave profiles at 2 sec intervals for $T = 2.0$ sec, open BC with $c = 3.107$

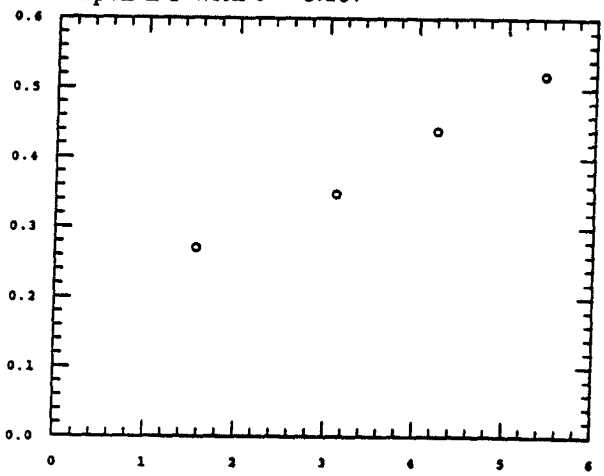

Figure 6 - Reflection coefficient for $T = 2.0$ as a function of c

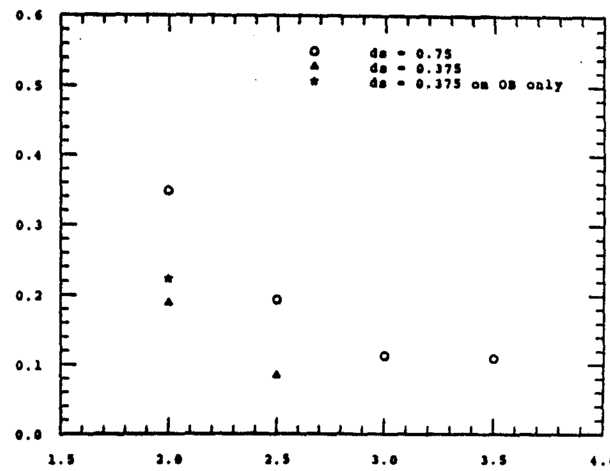

Figure 7 - Reflection coefficient as a function of T and ds for $c = 3.107$

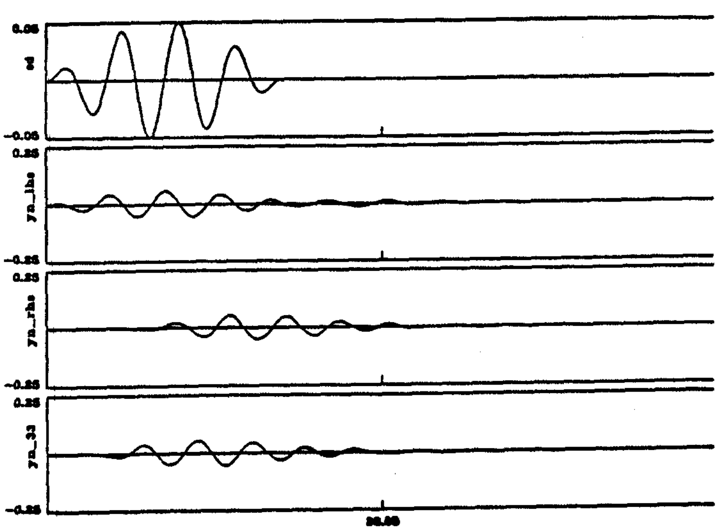

Figure 8 - Time histories for $T = 3.5$ sec, open BC with $c = 3.107$, $ds = 0.375$

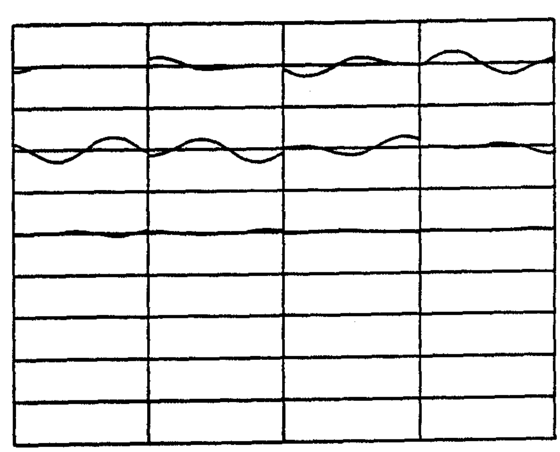

Figure 9 - Wave profiles at 2 sec intervals for $T = 3.5$ sec, open BC with $c = 3.107$, $ds = 0.375$

DISCUSSION

Newman: 1. What can you say regarding the 3D problem? 2. Perhaps the numerical community could learn from the experimental success of Salter's absorbing wavemaker, which appears to have quite low reflection - its most obvious feature vis-a-vis open boundary conditions is that the pressure is integrated vertically over the face of the wavemaker.

McCreight: 1. I have no experience with the 3-D problem. Absorbing wavemakers have been used in wave tanks successfully for this case. 2. Milgram, and also Jagannathan in his thesis, have obtained analytical solutions for absorbing wavemakers with good results. The simple scheme examined here is effectively a wavemaker with normal velocity proportional to the time derivative of the potential (see equation 3). Using the force provides another way to relate the normal velocity to the incident wave. It is not immediately obvious which is the better approach. The practical success of Salter's wave absorbing wavemaker is a strong argument in favour of that approach. I recall that a number of years ago in a discussion to a paper you made essentially the same suggestion, based on Milgram's work, although I don't recall which conference. This apparently fell on deaf ears. Perhaps there will be a better result this time.

Yeung: For credit to go where it is due, I feel I should point out that Bob Chen should be recognized as probably the first for using the Orlanski condition to treat the open boundary for water wave problems and that was in the mid 1970's. The problem with a condition like that, as I had pointed out in the discussion of the paper of Sen & Pawlowski in the Bristol Workshop, is that it fails when short waves ride on long waves, which happens often in many unsteady problems. There is also the problem with negative phase velocity, which also happens in transient flows (see Yeung, 1985).

Ref.: Yeung, R.W. "A comparative evaluation of numerical methods in Free-surface Hydrodynamics". IUTAM Symposium on Utilization of Ocean Wave Energy, Lisbon, Portugal, 1985.

McCreight: Thank you for your comment on Chen's earlier work. This was for the steady forward motion problem. Regarding the presence of two or more components of widely differing frequencies simultaneously at the outer boundary, one can by the present approach get some idea of how serious this may be in practice.

Schultz: In fig. 6 of your abstract you show a monotonic relationship between the reflection coefficient and the phase speed. It would appear that $c=0$ might give better results. Then the radiation boundary condition becomes $\phi=0$. Certainly it is not minimized at $c=3.107$. Can you comment on this result?

McCreight: This is a good observation, which I should of course have made myself. [Note: A few computations made after the Workshop indicate (1) there is a rise in K_r for $c<2.0$ and (2) The curve of K_r vs c changes considerably for smaller element size ds, so that Fig. 6 as it stands is not too useful a guide to anything!]