LOW-FREQUENCY WAVE FORCES ON MULTI-ELEMENT STRUCTURES

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The aim of this work is to clarify certain aspects of the limiting behaviour for low-frequency of first-order wave diffraction forces on a structure with more than one element or, equivalently, of wave forces on a number of structures in close proximity to each other. The forces to be considered are due to the potential flow only.

Consider the scattering of a plane wave train by N fixed bodies in an arbitrary configuration. For the purposes of the present discussion, attention is focussed on the horizontal components of the first-order exciting forces on the bodies. As a result of hydrodynamic interactions within this "array" of bodies, the forces on each body will, in general, differ from those measured when the body is in isolation. As a measure of the interaction effects it is sometimes convenient to use a "force ratio" for each body. This is defined as the magnitude of a force component on a given body when within the array, divided by the magnitude of the force it would experience in isolation. Thus, if there were no hydrodynamic interactions within the array the force ratios for each body would be identically equal to unity. Typically, however, a force ratio will oscillate about this value as frequency is varied. The size of the oscillations will depend on the size and spacing of the bodies, roughly speaking small, widely-separated bodies will be weakly interacting and a force ratio will be close to one. Thus, three dimensional parameters are important when considering interactions of this type; the wave frequency ω, the typical body dimension a and the typical body spacing l. From these, three non-dimensional parameters may be defined; these are a/l, Ka and Kl, where $K = \omega^2/g$ and g is the acceleration due to gravity. Here it will be assumed throughout that the fluid is infinitely deep and that $a/l \ll 1$, although lifting this restriction probably doesn't affect the conclusions.

The scattering process might be envisaged as follows. The incident plane wave is diffracted by one of the array members to give a scattered wave which propagates away in all horizontal directions. The scattered wave in turn is incident upon the other members of the array and is diffracted to give further scattered waves, and so on. It is the propagation of these scattered waves within the array that give rise to the hydrodynamic interactions that result in force ratios that differ from unity. Now, for a single, isolated body in low-frequency waves ($Ka \ll 1$), so that the waves are very much longer than the typical body dimension, the far-field amplitude of the scattered waves is typically $O((Ka)^2)$ relative to the amplitude of the incident wave train. Thus in very long waves, as Ka approaches zero, the scattered waves in the far field become negligibly small relative to the incident wave field. It might be tempting to conclude from this that, for an array of bodies, hydrodynamic interactions must also be negligible and so a force ratio must necessarily be very close to one. In particular, this line of reasoning leads to the conclusion that a force ratio must tend to one in the low-frequency limit. This particular limiting behaviour appears to have been assumed by a number of authors over recent years when displaying the results of force ratio calculations. However, in general, this is *not* the correct limiting behaviour of a force ratio. Hydrodynamic interaction effects can be present even in the low-frequency limit so that a force ratio differs from unity.

The above reasoning is at fault because it considers only the relationship between the wavelength and the body dimension a and ignores that between the wavelength and the spacing l. For moderate wavelengths ($Ka = O(1)$, for example) and large spacings ($a/l \ll 1$), the above description of the scattering process within the array is quite reasonable. However, for wave lengths longer than the spacing l, then wave-like motion is no longer discernible within the array and the description breaks down. For waves much longer than the spacing ($Kl \ll 1$), it is perhaps more appropriate to think of the array as a single compound body of complicated geometry. Thus, an analysis of the low-frequency/long-wave limit must allow $Kl \ll 1$ as well as $Ka \ll 1$. The conclusions of the previous paragraph *are* correct for $Ka \to 0$ provided $Kl = O(1)$ or larger so that the picture of waves within the array is still appropriate. This latter limit process is equivalent to a tending to zero for fixed wavelength and spacing l (the "point-absorber" limit of wave-energy theory).

The conclusions of the above discussion have been confirmed mathematically by analysing the scattering problem using the method of matched asymptotic expansions. Solutions for the velocity potential have been constructed for long-wave scattering in arrays for three different geometries; these are vertical cylinders extending throughout the depth, half-immersed spheres on deep water and submerged horizontal circular cylinders in deep water. A more general analysis will be attempted in the future.

For the "point-absorber" limit the problem is treated under the assumptions $Ka \ll 1$ and $Kl = O(1)$ (which together imply $a/l \ll 1$). There are two basic length scales in the problem, a and K^{-1}, and so for the purposes of the analysis the fluid domain is divided into N inner regions close to each body, where the length scale is a, and an outer region within and surrounding the array, where the length scale is $K^{-1}$and the motion is wave-like. From the solutions obtained the force ratios may be calculated and shown to approach one as $Ka \to 0$.

To determine the genuine low-frequency limit for fixed geometry, the scattering problem is analysed under the assumptions $Kl \ll 1$ and $a/l \ll 1$ (which together imply $Ka \ll 1$). There are now three different length scales, a, l and K^{-1}, and it is therefore necessary to introduce a further division of the fluid domain. Thus, there are N inner regions close to each body, where the length scale is a, an intermediate region within the array, where the length scale is l, and an outer region of wave-like motion surrounding the array, where the length scale is K^{-1}. Again force ratios may be calculated and, now, the limit determined for $Kl \to 0$ with a/l fixed. For the horizontal forces under discussion, the limiting value of a force ratio differs from one by a quantity of $O((a/l)^2)$.

Clearly, the limits that may be calculated are not of direct practical use as the horizontal wave forces (rather than the force ratio) are zero in the long wave limit. However, the analysis should give a wider understanding of how long waves interact with multi-element structures and the limits themselves can be used as a check on numerical calculations.

DISCUSSION

Eatock Taylor: Our interest in the force ratio for interacting bodies was stimulated by considering the drift forces on multi-body structures. This leads us to examine the problem in an intuitive manner, based on the far field behaviour in terms of Kochin-functions. Would there be any value in attempting to use this approach in a systematic manner to examine the influence of body geometry (by considering the far field wave dependence)?

McIver: The approach described in my talk is able to yield the far field behaviour in terms of body geometry under the stated restrictions on wavelength. The solutions were obtained for specific geometries of individual bodies but it should be possible to arrive at more general results (e.g. in terms of the far field response to a uniform flow).