ON THE ADDED MASS AND DAMPING OF POROUS OR SLOTTED CYLINDERS

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In order to decrease their first natural frequencies, compliant structures such as ETPM's "ROSEAU" tower are fitted with a so-called stabilizer, which is nothing but a box, 55 x 55 m wide, and 65 m high, its top being immersed at 70 m under the mean free surface. Only the 4 vertical sides are materialized and they consist of 13 plates, 55 m long and 4.5 m high, separated by slots of about 0.5 m (see fig. 1). Typical range of the horizontal motion is 0 to 2 meters.

The problem considered here is the effect of these slots on the hydrodynamic characteristics of the stabilizer : added mass and damping, and the resulting motion of the complete structure under regular waves. The following drastic idealizations and assumptions are made :
1. the square box is turned into a circular cylinder ;
2. potential flow is assumed inside and outside ;
3. a quadratic discharge law is assumed across the slots : $\rho V_{nr}|V_{nr}| = 2\mu(p_i - p_e)$ (1)
where p_i is the internal pressure, p_e the external pressure, μ a discharge coefficient ($\mu \simeq 0.7$ from preliminary experiments), and V_{nr} the normal component of the relative velocity.

Hypotheses 1 and 2 allow analytic expansions to be used for the interior and exterior velocity potentials :

$$\Phi_{i,e} = Re\{\varphi_{i,e}\, e^{-i\omega t}\} \qquad \varphi_{i,e} = \sum_{m=0}^{\infty} cos m\theta \sum_{n=0}^{\infty} a_{mn}\, f_{mn}^{i,e}(R)\, g_{mn}^{i,e}(z)$$

R,θ,z being the cylindrical coordinates. Here z=0 coincides with the real or a fictitious bottom, and z=h with the free surface, or a fictitious horizontal plane in the infinite fluid case.

Part of the nonlinearity in equation (1) is overcome through Lorentz linearization :

$$Re\{ae^{-i\omega t}\}\;|Re\{ae^{-i\omega t}\}| = \frac{8}{3\pi}||a||\; Re\{ae^{-i\omega t}\}$$

which is applied to $cos\theta|cos\theta|$ as well : $cos\theta|cos\theta| = \frac{8}{3\pi}cos\theta$

As a result, for the forced oscillation problem, only the $cos\theta$ components of φ_i and φ_e need to be considered.

2D POROUS CYLINDER

The general method and the trends of the obtained results are best illustrated in this simple case. Porosity can be considered as the limiting case of the slots when their number goes to infinity with a constant total area. Equation (1) in this case takes the form :

$$\rho V_{nr}|V_{nr}| = 2\mu\tau^2(p_i - p_e)$$

τ being the porosity, defined as the ratio of the slots area to the total area. If $Re\{i\; a\; e^{-i\omega t}\}$ is the forced motion, the pressure drop and the radial velocity on the cylinder can be written as :

$$\varphi_i - \varphi_e = 2\, R_0\, a\, \omega\, (1+b)\, cos\theta \qquad\qquad \varphi_R = a\, \omega\, (1+b)\, cos\theta$$

R_0 being the radius of the cylinder, and b an unknown determined through satisfying the quadratic equation, which takes the form :

$$b\, \|b\| = i\, (\frac{3\pi}{4})^2\, \tau^2\, \mu\, \frac{R_0}{a}\, (1+b)$$

from which the added mass and damping coefficients can be obtained :

$$C_m = 2\, -\, C\, (\sqrt{C^2+4} - C) \qquad\qquad C_a = \sqrt{\frac{C}{2}}\, (\sqrt{C^2+4} - C)^{\frac{3}{2}}$$

Figure 2 shows these coefficients as functions of the parameter $C = (\frac{3\pi}{4})^2\, \tau^2\, \mu\, \frac{R_0}{a}$. It is interesting to note that the damping coefficient takes a peak value of 1 for $C = \sqrt{2}/2$.

3D POROUS CYLINDER

We now assume the cylinder to be of finite extent in the z direction. Due to its submergence infinite fluid is assumed, that means here the fluid domain is limited by two horizontal planes, at z=0 and z=h, sufficiently far away from the cylinder that no wall effect may occur. The planes are situated symmetrically with respect to the cylinder, which extends from z_b to z_t. The following analytical expressions for φ_i andf φ_e may then be used (with $k_n = 2n\pi/$ h) :

$$\varphi_i = \{A_0 \frac{R}{R_0}\ +\ \sum_{n=1}^{\infty} A_n\, cosk_n z\, I_1(k_n R)\} cos\theta$$

$$\varphi_e = \{B_0 \frac{R_0}{R}\ +\ \sum_{n=1}^{\infty} B_n\, cosk_n z\, K_1(k_n R)\} cos\theta$$

Equality of φ_{iR} and φ_{eR} at $R = R_0$ permits to get rid of one set of the A_n and B_n unknowns, but numerically the problem is better behaved by introducing as unknowns the a_n defined as :

$$\varphi_i - \varphi_e|_{R=R_0} = \{a_0\ +\ \sum_{n=1}^{N} a_n\, cosk_n z\ \} cos\theta$$

$$\varphi_R|_{R=R_0} = \{a_0\, \alpha_0\ +\ \sum_{n=1}^{N} a_n\, \alpha_n\, cosk_n z\ \} cos\theta$$

with $\alpha_0 = (2R_0)^{-1}$ $\qquad \alpha_n = k_n\, [\frac{I_1(k_n R_0)}{I_1'(k_n R_0)} - \frac{K_1(k_n R_0)}{K_1'(k_n R_0)}]^{-1}$

The following set of equations has to be satisfied :

$$a_0\ + \sum_{n=1}^{N} a_n\, cosk_n z = 0 \qquad\qquad 0 \le z \le z_b \quad z_t \le z \le h$$

$$(a_0\alpha_0 + \sum_{n=1}^{N} a_n \alpha_n cosk_n z - a\omega) \| a_0\alpha_0 + \sum_{n=1}^{N} a_n \alpha_n cosk_n z - a\omega \| = iC'(a_0 + \sum_{n=1}^{N} a_n\, cosk_n z) \quad z_b \le z \le z_t$$

(where $C' = (\frac{3\pi}{8})^2 2\tau^2 \mu\omega$)

Linearity is recovered by defining the iterative process :

$$a_0^{(j)} + \sum_{n=1}^{N} a_n^{(j)} \cos k_n z = 0 \qquad 0 \leq z \leq z_b \quad z_t \leq z \leq h$$

$$(a_0^{(j)}\alpha_0 + \sum_{n=1}^{N} a_n^{(j)}\alpha_n \cos k_n z - a\omega)\|a_0^{(j-1)}\alpha_0 + \sum_{n=1}^{N} a_n^{(j-1)}\alpha_n \cos k_n z - a\omega\| = iC'(a_0^{(j)} + \sum_{n=1}^{N} a_n^{(j)} \cos k_n z)$$

or :

$$a_0^{(j)} + \sum_{n=1}^{N} a_n^{(j)} \cos k_n z = 0 \qquad 0 \leq z \leq z_b \quad z_t \leq z \leq h$$

$$(1 - f^{(j-1)}(z)\alpha_0)\, a_0^{(j)} + \sum_{n=1}^{N}(1 - f^{(j-1)}(z)\alpha_n)\, a_n^{(j)} \cos k_n z = -a\omega\, f^{(j-1)}(z) \qquad z_b \leq z \leq z_t$$

Multiplying the first equation by $\cos k_m z$, the second one by $\cos k_m z/(1 - f^{(j-1)}(z)\alpha_m)$, integrating them both in z upon their domains of validity, and adding them up, for m=0,...N, gives a linear system which is solved by a standard Gauss method. This process, for the values of τ, R_0, and a of interest, converges in a few iterations (see figure 3). (For small values of $\tau^2 R_0/a$ a simpler and more obvious iterative process may be derived).

3D SLOTTED CYLINDER

Because of the small motion amplitude of the stabilizer as compared to the slots spacing, the porous approach yields results in limited agreement with the experimental ones. Assuming equation (1) to hold on the slots (with p_i, p_e, and V_{nr} functions of z !), it is quite easy to extend the "porous" method, the only difference being that different equations have to be satisfied on the slots and on the plates :

$$a_0 + \sum_{n=1}^{N} a_n \cos k_n z = 0 \qquad 0 \leq z \leq z_b \quad z_t \leq z \leq h$$

$$a_0\alpha_0 + \sum_{n=1}^{N} a_n\alpha_n \cos k_n z = a\omega \qquad \text{plates}$$

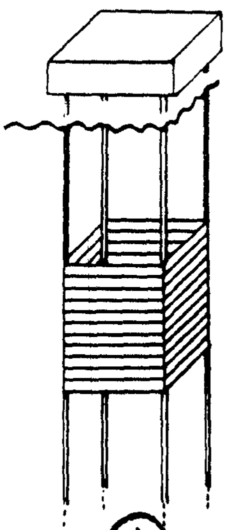
$$(1 - f(z)\alpha_0)a_0 + \sum_{n=1}^{N}(1 - f(z)\alpha_n)a_n \cos k_n z = -a\omega\, f(z) \qquad \text{slots}$$

with $f(z) = -\frac{i}{2\mu\omega}(\frac{8}{3\pi})^2\|a_0\alpha_0 + \sum_{n=1}^{N} a_n\alpha_n \cos k_n z - a\omega\|$ calculated at the previous iteration.

In order to properly represent the flow through the slots a large number of terms in the a_n expansion must be considered (typically a few hundreds). Figure 4 shows some typical results for the real part of the pressure drop on the lower half of the cylinder (damping component).

RESPONSE UNDER WAVES

The method can readily be extended to this case, the only problem being increased computer cost. As the free surface (and the associated wave numbers) has to be introduced, the vertical symmetry is lost and twice as many wave numbers must be introduced at equal accuracy (with a fictitious bottom not too far away). Second, at the wave periods of interest, the first two or three orders in the *cosmθ* development of the velocity potential must be introduced since they are present in the incoming wave field. As the corresponding equations become coupled through equation (1), this means 4 to 6 times as many unknowns in the linear systems which must also include the equation of horizontal motion. Figure 5 shows some typical results for the pressure drop along different generating lines.



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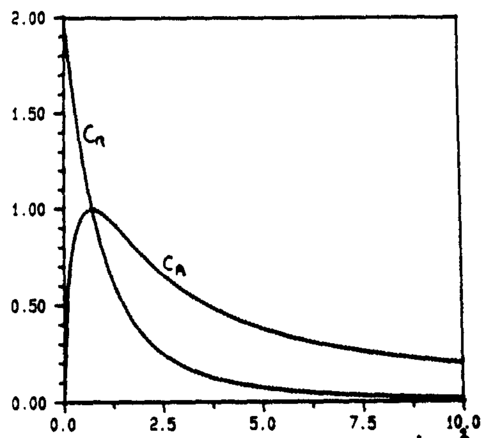

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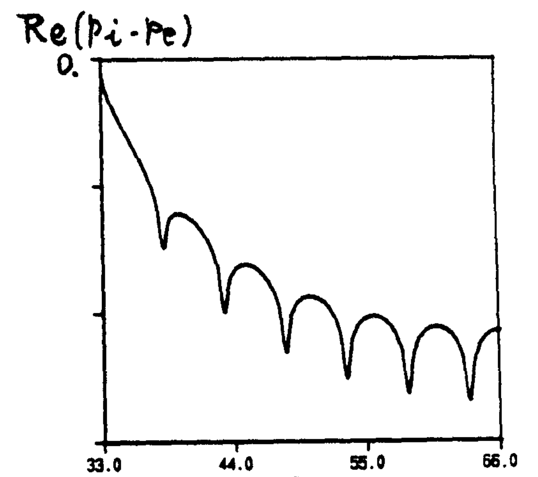

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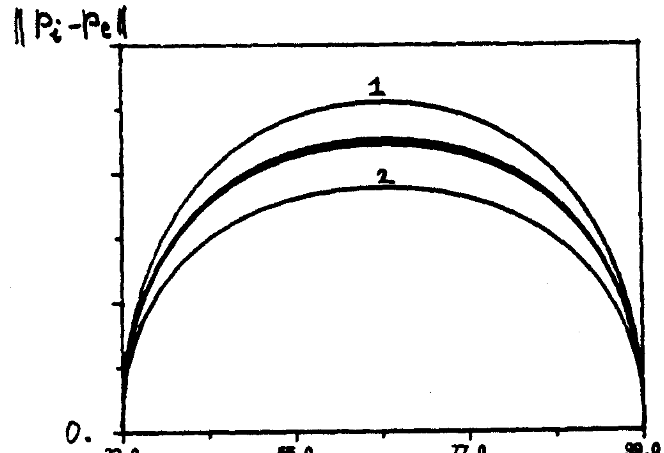

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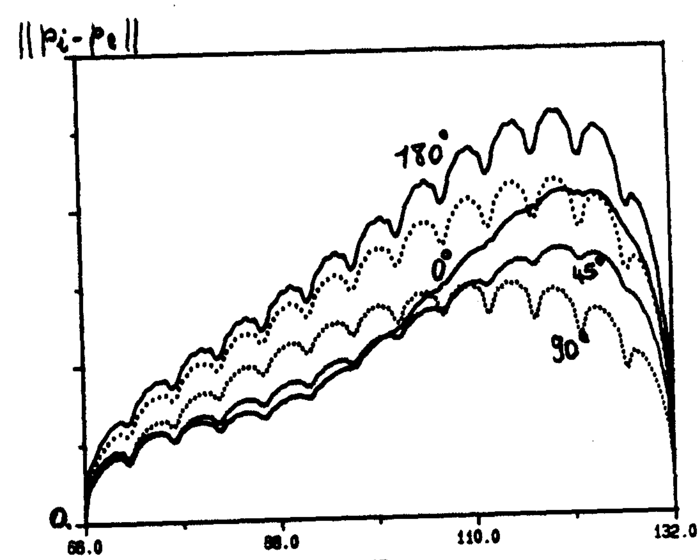

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DISCUSSION

Raven: From what sort of tests did you deduce the coefficient in the quadratic discharge low? It seems to be a tuning coefficient that must account for all viscous effects in the flow through the slots, and could, therefore, in fact be dependent on the amplitude of the motion and so on.

Molin: The discharge coefficient was obtained from tests performed in a tank with a slotted bottom. Different tests and different slots were tested. 0.7 is an averaged value, but there was little derivation from it. As the tests were performed in steady conditions (the water level was kept constant by filling in the tank as it emptied), the question that arises is whether the same coefficient may be used in unsteady conditions.