FREE SURFACE PANEL METHODS
FOR UNSTEADY FORWARD SPEED FLOWS

by
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(Abstract for the 4th Int. Workshop, Øystese, Norway, 7-10 May 1989)

Free surface panel methods appeared more than a decade ago having as sole objective the solution of the steady ship wave problem. Their use over the years remained an art, rather than a science, while being primarily limited to the steady problem.

The purpose of this study is to derive a systematic methodology for the analysis of their convergence properties, and show that they can be successfully employed for the solution of unsteady free surface flows.

For the sake of simplicity, the two-dimensional time harmonic wave radiation of a submerged body, that oscillates while advancing at a constant speed beneath the free surface, is considered. Three-dimensionality is a straightforward, yet cumbersome, extension, while the design of non-reflecting "radiation" conditions is the additional complexity of the transient problems.

Statement of the Problem

The Boundary Value Problem, that governs the flow under consideration, is linearized about the uniform incoming flow and Green's 2nd identity is applied between the disturbance potential $\phi(\vec{x})$ and the Rankine source, $G(\vec{x};\vec{\xi}) = \log\|\vec{x}-\vec{\xi}\|/\log\|\vec{\xi}\|$. This yields a linear Boundary Integral Equation for ϕ over the free surface (FS) and the hull (B), as follows :

$$\mathcal{W}\phi = \mathcal{R} \quad : \quad -\pi\phi(x) + \int_{(FS)} \left\{-\Omega^2\,\phi(\xi) + 2\,i\,\Omega\,F\,\frac{\partial\phi(\xi)}{\partial\xi} + F^2\,\frac{\partial^2\phi(\xi)}{\partial\xi^2}\right\} G(x;\xi)\,d\xi$$

$$+ \int_{(FS)+(B)} \phi(\xi)\,\frac{\partial G(x;\xi)}{\partial n_\xi}\,d\xi \;=\; \mathcal{R}(x)\;, \qquad (1)$$

where x and ξ lie on the boundary, $\Omega = \omega\sqrt{L/g}$ is the non-dimensional frequency and $F = U/\sqrt{gL}$ the Froude number, while the known forcing function $\mathcal{R}(x)$ depends on the hull geometry and the mode of motion.

The discretization process starts with the approximation of ϕ in terms of some basis function $B(x)$ and concludes with collocation at a sufficient number of points x_i. In particular, the Quadratic Spline scheme (Sclavounos and Nakos (1988)), selects the quadratic B-spline as the basis function and the centroids of the panels as the collocation points. The discrete problem for the finite number of degrees of freedom a_j reads as follows:

$$W\phi_h = R \quad : \qquad \phi_h(x) = \sum_j a_j B_j(x)\;, \qquad (2a)$$

$$\sum_j a_j\,\{-\pi B_{ij} - \Omega^2\,S_{ij}^{(0)} + 2i\,\Omega\,F\,S_{ij}^{(1)} + F^2\,S_{ij}^{(2)} + D_{ij}\} \;=\; R_i\;, \qquad (2b)$$

where $B_{ij} = B_j(x_i)$ and the influence coefficients $S_{ij}^{(0)}$, $S_{ij}^{(1)}$, $S_{ij}^{(2)}$, D_{ij} are given by closed-form expressions. The system of linear equations $(2b)$ is solved for a_j and the velocity potential is obtained by $(2a)$

Stability Analysis

In order to isolate the errors due to the discretization of the free surface, the dipole terms of (1) can be neglected when the collocation point is on the free surface. Both the continuous and discrete formulations are of convolution type and, consequently, continuous and discrete Fourier techniques may be used in analyzing the behavior of the corresponding solutions. The potential on the free surface can be written as :

$$\phi(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} du \, \frac{\tilde{\mathcal{R}}(u)}{\tilde{\mathcal{W}}(u\,;\,\Omega, F)} \, e^{-iux} \,, \tag{3}$$

$$\phi_h(x) = \frac{1}{2\pi} \int_{-\pi/h}^{\pi/h} du \, \frac{\hat{R}(u;h)}{\hat{W}(u\,;\,\Omega, F\,;\,h)} \, e^{-iux} \,, \tag{4}$$

where $\tilde{\mathcal{W}}$, $\tilde{\mathcal{R}}$ and $\hat{W}$, $\hat{R}$ are respectively the continuous and discrete Fourier Transforms of the operator and forcing of (1) and (2).

In addition to the well known dispersion relation of the continuous problem, $\tilde{\mathcal{W}} = 0$, its discrete counterpart is defined as $\hat{W} = 0$. The dispersion relation, corresponding to a particular grid size $h = 0.25U^2/g$, is plotted in figure 1 and compared to the continuous one. It should be noted that the dispersion relation of the discrete system is periodic in the wavenumber space, with period $2\pi/h$, a fact that allows the limits of the integration in (4) to freely be changed to $\pm\infty$.

In view of (4) the discrete solution can be interpreted as the response of a different linear system, characterized by the discrete dispersion relation. Consequently, the theory of stability of linear systems can become an extremely powerful tool in analyzing numerical instabilities. As seen in figure 1, there are real wavenumbers that correspond to complex frequencies ($u_3 < |u| < \pi/h$),

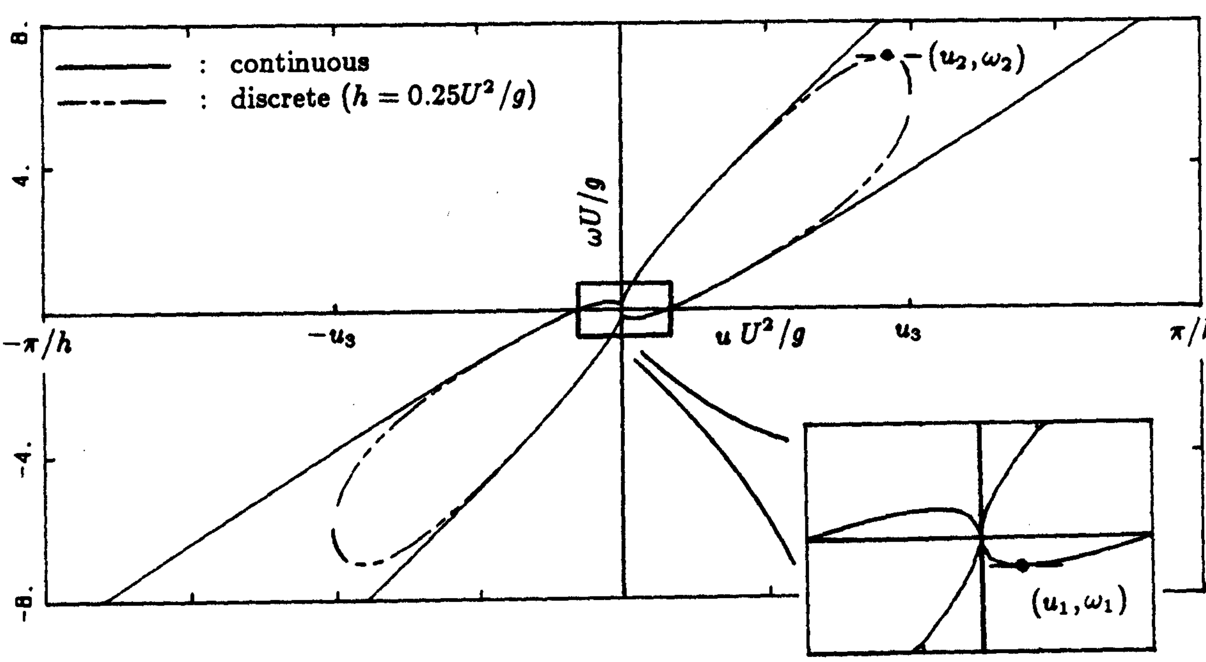

Figure 1 : Dispersion relations of continuous and discrete systems.

therefore the analytical properties of $\hat{W}$ for complex u and ω should be investigated in order to identify and classify instabilities. Following Bers (1983), and for the particular grid of figure 1, it can be shown that the interval $(u_3, \pi/h]$ results in convective instabilities which are of no concern in the steady-state limit. On the other hand, the two points (u_1, ω_1) and (u_2, ω_2) correspond to resonant behavior, of the same nature as the $\tau = 1/4$ point of the continuous system.

In the absence of instabilities the convergence of the scheme hinges upon the proximity of $\tilde{W} = 0$ and $\hat{W} = 0$ for real frequencies and wavenumbers. The rate at which these curves approach each other as $h \to 0$ is defined to be the order of the scheme, which for the Quadratic Spline scheme is qubic.

For consistent and stable schemes convergence is guaranteed as long as the forcing function is sufficiently smooth. The smoothness of $\mathcal{R}(x)$ is directly related to the rate of decay of its Fourier Transform for large wavenumbers, and it becomes an issue of significant concern in the case of free surface piercing bodies.

Radiation Condition

The radiation condition of the Boundary Value Problem has not been incorporated in (1), therefore the solution of (2) does not have to satisfy it. This condition is treated as an essential boundary condition, implemented through the end-conditions of the spline. A straightforward extension of the related analysis for the steady problem, given by Sclavounos and Nakos (1988), leads to the following conditions for overcritical frequencies ($\tau = \omega U/g > 1/4$) :

$$(i\,\Omega + F\,\frac{\partial}{\partial x})\phi = (i\,\Omega + F\,\frac{\partial}{\partial x})^2\phi = 0 \ , \qquad (5)$$

both to be applied at the most upstream point of the computational domain. The extension, however, to undercritical frequencies ($0 < \tau < 1/4$) is not trivial, and is still under investigation.

The Submerged Elliptical cylinder

The wave radiation due to an elliptical cylinder which is advancing at a constant speed beneath the free surface is solved as a numerical checking of the predictions of the stability analysis. The same problem is also solved using the wave source as the Green function (see eg. Grue and Palm (1985)), in which case the enforcement of the free surface and radiation conditions is assured. Consequently, comparison of these results to the results of the Quadratic Spline scheme will show the convergence properties of the later with respect to stability, numerical dispersion, dissipation and satisfaction of the radiation condition.

The resulting wave fields for steady and time harmonic motion at $F = 0.5$ are plotted in figure 2. For the frequencies considered ($\tau = 0, 0.75, 1.25, 1.75$) the Quadratic Spline scheme with grid size $h = 0.25U^2/g$ is, evidently, stable and free of numerical damping, while the numerical dispersion becomes signifficant when the smallest wavelength becomes too short to be resolved by the number of panels used. Moreover the numerical implementation of the radiation condition is shown to be excellent.

In conclusion it should be noted that, no matter how efficient the numerical scheme is, any attempt to resolve the short scales will ultimately make the computations prohibitively intense. It is true however that the shorter the wave it is, the less energy it carries, and therefore the less its global importance. An effective filtering of these short scales has been devised and it has been analysed by studying its effect on the added mass and damping coefficients.

This study has been supported by the Applied Hydromechanics Research Programm (contract no N00167-86-K-0010), administered by the ONR and DTRC.

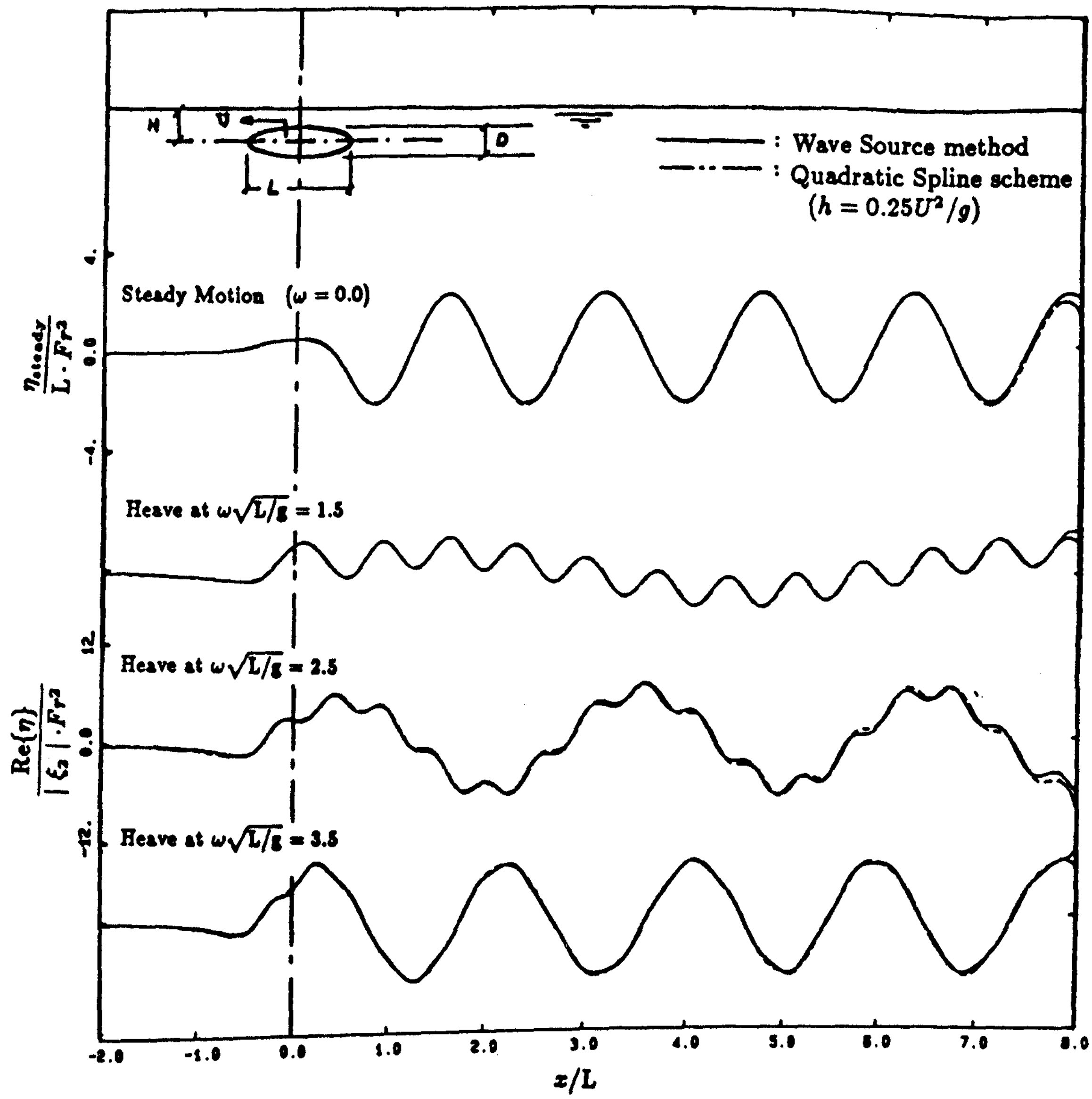

Figure 2 : Free surface wave elevations due to a submerged elliptical cylinder advancing at $Fr = U/\sqrt{gL} = 0.5$, while oscillating in heave at frequencies $\omega\sqrt{L/g} = 0, 1.5, 2.5, 3.5$. The cross section of the cylinder has diameter to length ratio $D/L = 0.2$ and mean submergence to length ratio $H/L = 0.2$.

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DISCUSSION

Bertram: How do you enforce "open boundary" condition in the unsteady case? Especially, what do you do for $0<\tau<0.25$? Can you give formulas?

Nakos: The conditions of zero value and slope of the wave elevation, at the upstream truncation line, have been shown to perform perfectly as long as no upstream waves exist ($\tau=0$, $\tau>0.27$). They also work quite satisfactorily for small values of the reduced frequency ($\tau<0.1$) basically because the upstream waves in this case are of very small amplitude. For the regime of $\tau \in [0.1, 0.27]$ further investigation is necessary.

Yeung: You indicated the superiority of using the spline functions for free-surface problem. This fact was pointed out and realized in a paper of mine back in 1977 (Yeung & Bouger, 2nd Int.Conf.Numer. Ship Hydrodynamics, see also Int.J.Numer.methods in Engrg., 1978). In this work a full spline was used and the radiation condition was satisfied by matching using basically unsymmetrical and conditions by having an unconventional distribution of collocation points. The author is apparently unaware of this work. I want to comment your fine numerical analysis that has undoubtedly shed much light on the "Equivalence" of the analytical and numerical radiation condition.

Nakos: The author is aware of the work mentioned by prof. Yeung and wants to comment the following:

1) The stability analysis of the present paper suggests that the use of quadractic splines on the free surface does not improve the order or numerical dispersion over the proposed quadratic spline scheme, although it increases significantly the computational effort for the influence coefficients (see ref. (3)).

2) The employment of the radiation condition through matching to an eigenfunction expansion, as proposed by Yeung & Bouger (77) is not trivial to be extended to three dimensions. And this unlike the upstream conditions proposed in the present paper which remains identical in 2-D and 3-D.

The author also wants to thank prof. R.W. Yeung for giving him the opportunity to add these comments on the abovementioned paper.