A NEW EXPRESSION FOR THE STEADY WAVE SPECTRUM OF A SHIP

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Calculations of the free-wave spectrum of a ship in steady motion are required for wave-resistance predictions and remote sensing of ship wakes. In particular, the latter practical application requires the ability of determining the short divergent waves in the spectrum having wavelengths between 5 cm and 40 cm associated with Bragg scattering of the electromagnetic waves in typical SAR systems used in remote sensing of ship wakes. No meaningful predictions of such short waves can be obtained on the basis of currently available numerical methods. More generally, numerical predictions of the steady wave pattern at large and moderate distances behind a ship are notoriously unreliable, as was recently made clear at the Workshop on Kelvin Wake Computations (1988). Ship wave-resistance calculations are also known to be unreliable.

It is shown in the present study that a major cause of the notorious unreliability of numerical predictions of ship waves mentioned above is mathematical in nature. More precisely, the known mathematical expression for the wave-spectrum function which form the basis of existing calculation methods is quite ill suited for accurate numerical evaluation because the wave spectrum is defined by the sum of integrals around the ship waterline and over the ship hull surface that very nearly cancel out one another. The errors which inevitably occur in the numerical evaluation of the waterline and hull integrals in the expression for the wave spectrum cause imperfect numerical cancellations between these components and corresponding large errors in their sum. Numerical errors in the sum of the waterline and hull integrals are especially difficult to control because the errors associated with the numerical evaluation of the waterline and hull integrals are not necessarily comparable due to differences in the errors corresponding to numerical integration over waterline segments and hull panels.

The fundamental difficulty noted in the foregoing is resolved in this study by means of a modified mathematical expression for the wave spectrum function. The new expression for the wave spectrum is mathematically equivalent to the usual known expression, from which it has been obtained by means of several applications of Stokes' theorem for combining the waterline and hull integrals. However, the previously-noted cancellations occurring between these waterline and hull integrals are automatically and exactly accounted for via a mathematical transformation in the new expression for the wave-spectrum function, which involves modified waterline and hull integrals. This new expression is considerably better suited for numerical evaluation, as is now shown.

The problem considered in this study is that of evaluating the steady wave spectrum and the wave pattern of a ship in terms of the near-field flow at the hull surface. The near-field flow thus is assumed known for the purpose of the present study, which is specifically concerned with the prediction of the steady wave spectrum and the wave pattern within the Neumann-Kelvin theoretical framework. This theory expresses the wave potential $\phi_W(\xi,\eta,\zeta)$ at any point (ξ,η,ζ) behind the ship stern in terms of a well-known Fourier representation which may be expressed as follows:

$$\phi_W(\xi,\eta,\zeta) = (2/\pi)\int_0^\infty \exp(\nu^2\zeta p^2)\cos(\nu^2\eta pt)\,\mathrm{Im}\,\exp(i\nu^2\xi p)\,K(t)\,dt\ ,$$

where $\nu = 1/F$ is the inverse of the Froude number F, p is defined in terms of the Fourier variable t by the relation $p = (1+t^2)^{1/2}$ and K(t) is the wave-spectrum function. The wave potential ϕ_W thus is expressed in terms of a familiar Fourier superposition of elementary plane waves propagating at angles θ from the ship track given by $\tan\theta = t$. The wave-spectrum function K(t) evidently contains essential information directly relevant to a ship's wave pattern and wave resistance. In particular, the wave resistance R experienced by the ship is defined in terms of the wave-spectrum function by means of the well-known Havelock formula

$$\pi R/(\rho U^2 L^2) = \int_0^\infty [K(t)]^2 p\, dt .$$

It is convenient and useful to express the wave-spectrum function K(t) as the sum of two terms, as follows:

$$K(t) = K_0(t) + K_\phi(t) ,$$

where K_0 represents the (zeroth-order) slender-ship approximation, which is defined explicitly in terms of the Froude number and the hull shape, and K_ϕ the Neumann-Kelvin correction term in the Neumann-Kelvin approximation $K_0 + K_\phi$. Both the terms K_0 and K_ϕ are defined as sums of integrals along the ship waterline and over the hull surface. These waterline and hull integrals nearly cancel out in the expressions for both K_0 and K_ϕ and modified expressions for both of these terms have been obtained. The cancellations between the waterline and the hull integrals are more important for the Neumann-Kelvin term K_ϕ than the slender-ship approximation K_0. Only the term K_ϕ is considered here for brevity.

The usual expression for the correction term K_ϕ in the expression for the Neumann-Kelvin approximation $K_0 + K_\phi$ to the wave-spectrum function K takes the form

$$K_\phi(t) = K_W(t) + i\sigma K_W'(t) + \sigma^2 K_H'(t) \quad \text{with } \sigma = p/F^2 ,$$

where K_W and K_W' represent waterline integrals (these two integrals can of course be grouped together) and K_H' corresponds to a hull integral. In the foregoing expression σ is a real number that usually varies between about 10 and 1000 since we typically have $10 \le 1/F^2 \le 100$ and $1 \le p \le 10$ (except for very short waves corresponding to even larger values of p). The terms $i\sigma K_W'$ and $\sigma^2 K_H'$ may then be expected to be dominant and to nearly cancel out, as was already noted.

This phenomenon is depicted in the figure which represents the real and imaginary parts of the spectrum function K_ϕ in the lower right corner and its three components K_W , $i\sigma K_W'$ and $\sigma^2 K_H'$ on the left side. These functions are depicted for values of $t = \tan\theta$ between 0 and 10, corresponding to $0 \le \theta \le 84°$, for a particular hull form at a value of the Froude number equal to 0.15. The spectrum function K_ϕ, which is determined numerically by adding the three terms shown on the left side of the figure, can be seen to be considerably smaller than its three components. This is true at all Froude numbers and for all values of θ. The phenomenon is especially dramatic for large values of θ corresponding to the short waves in the spectrum. In particular, the function K_ϕ vanishes fairly rapidly as θ increases but the waterline integral K_W' and the hull integral K_H' do not vanish as $\theta \to 90°$.

The new expression for the spectrum function K_ϕ may be expressed in the form

$$K_\phi(t) = K_W^*(t) + K_H^*(t) ,$$

where K_W^* and K_H^* represent modified waterline and hull integrals, respectively.

These modified integrals, depicted on the right side of the figure, are comparable to the function K_ϕ and significantly smaller than the terms $\sigma^2 K_H'$, $i\sigma K_W'$ and K_W in the usual expression. The cancellations occurring among these three terms thus are automatically and exactly accounted for in the modified waterline and hull integrals K_W^* and K_H^* in the new expression. This modified expression for the spectrum function therefore is much better suited than the usual expression for accurate numerical evaluation.

Another interesting feature of the modified expression for the wave-spectrum function is that it only requires the tangential velocity at the hull, not the potential, whereas the usual expression requires the values of both the velocity potential and its gradient at the hull. The modified expression thus defines the wave-spectrum function in terms of the speed and the size of the ship, the shape of the mean-wetted hull surface, and the tangential velocity at the mean hull surface. This expression is suitable for use in conjunction with a boundary-integral-equation method based on a source distribution or any other numerical method in which the velocity vector (but not the potential) is determined at the mean hull surface. It provides a practical and reliable method for coupling a far-field Neumann-Kelvin flow representation and any near-field flow calculation method, including methods based on the use of Rankine sources or finite differences.

A detailed presentation of the modified expression for the wave-spectrum function is given in Noblesse and Lin (1988). This new expression has been obtained from the known usual expression by means of several applications of Stokes' theorem, as was already noted, and by making use of a basic formula from vector analysis and an asymptotic analysis of the short-wave limit. An approximate expression, defined in terms of a single integral along the ship waterline, suited for efficient and accurate numerical evaluation of the short waves in the wave spectrum has also been obtained by using the Laplace method for approximating the hull-surface integral. Finally, an explicit approximate analytical expression for the very short divergent waves in the spectrum has been obtained by using the method of stationary phase. This asymptotic approximation is of the form

$$K(\theta) \sim F \cos^3\theta \left[\sum K_0 + 2F \cos\theta \, (K_B - K_S) \right] \quad \text{as } \theta \to 90^\circ ,$$

where $K_{B,S}$ represent the contributions of the bow and the stern and $\sum K_0$ that of the point(s) of stationary phase on the waterline. The amplitudes of the waves emanating from the points of stationary phase, the bow and the stern are proportional to the vertical velocity $\partial\phi/\partial z$ at these points.

ACKNOWLEGMENTS

This study was supported by the Applied Hydrodynamics Research Program at the David Taylor Research Center. The numerical calculations presented in the figure were performed by Dr. Woei-Min Lin.

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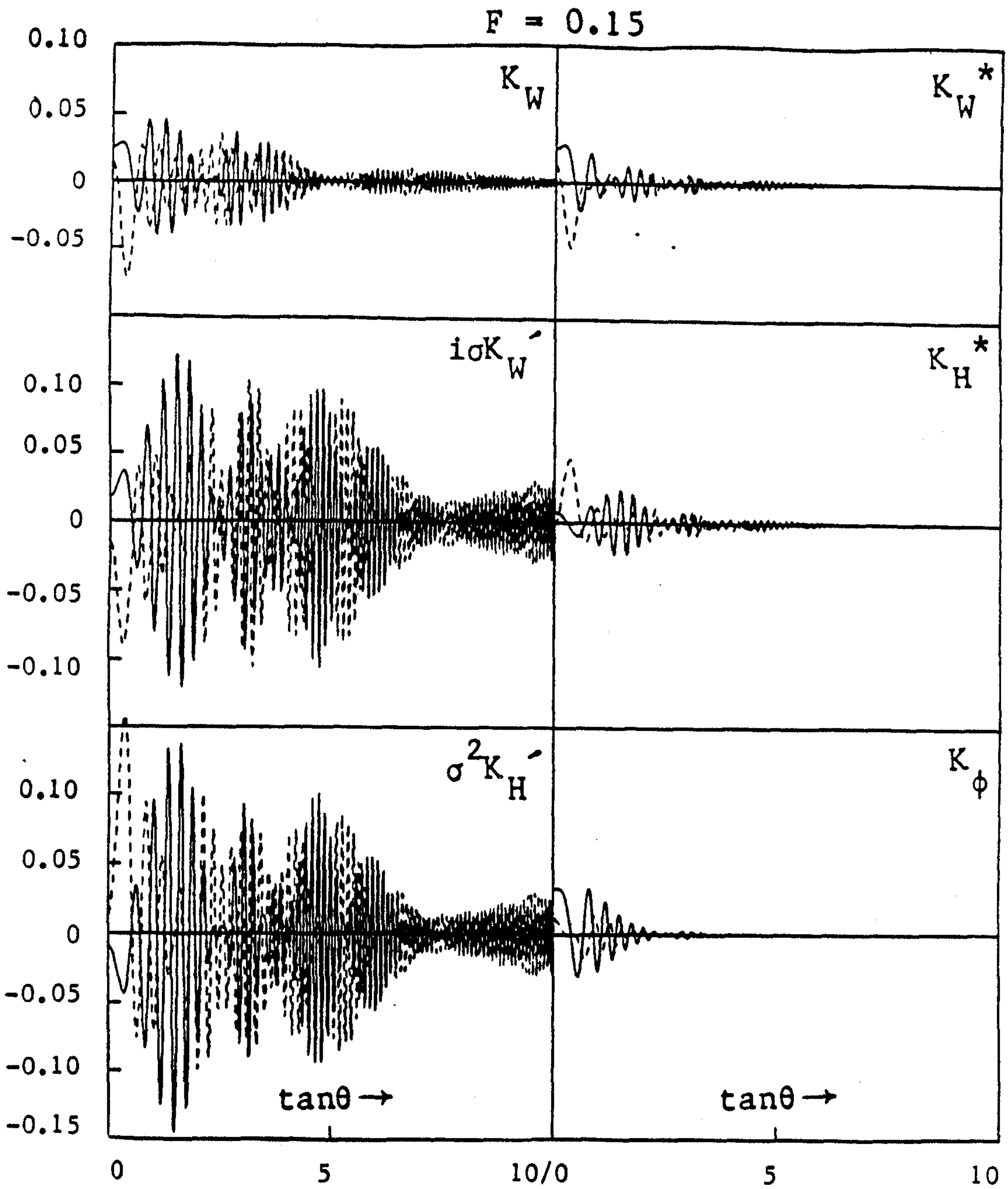

Comparison of the usual expression $K_\phi = K_W + i\sigma K_W' + \sigma^2 K_H'$ and the new expression $K_\phi = K_W^* + K_H^*$ for the Neumann-Kelvin correction term $K\phi$ in the expression for the wave-spectrum function $K(t)$ for $0 \leq t = \tan\theta \leq 10$. The real and imaginary parts of the functions K_W , $i\sigma K_W'$, $\sigma^2 K_H'$, K_W^* , K_H^* and K_ϕ are represented for a strut-like hull form at a Froude number equal to 0.15. The large cancellations occurring among the terms K_W , $i\sigma K_W'$ and $\sigma^2 K_H'$ in the usual expression are automatically and exactly accounted for via a mathematical transformation in the modified waterline and hull integrals K_W^* and K_H^* in the new expression, which is then much better suited for accurate numerical evaluation.