On the solution of the radiation and diffraction problems for a floating body with a small forward speed

Jan Nossen, Enok Palm and John Grue

University of Oslo

The radiation and diffraction problems for a floating body with a small forward speed are most fundamental topics in marine hydrodynamics and their solutions are of practical relevance in wave analysis of many offshore constructions. Examples are prediction of wave drift forces and wave drift damping of the resonant induced low-frequency motions of a tension-leg platform or a moored ship. In the present analysis we adopt Green's theorem to formulate the radiation and diffraction problems. Solution of the problems is obtained by a panel method, which is applicable to bodies of arbitrary shape. The forward speed Green function is expanded in Taylor series to the first order in the forward speed (Huijsmans and Hermans 1985) close to the body. In the far-field the exact Green function is applied, since the local expansion diverges. Close to the body nonlinear coupling between the oscillatory motion (the waves) and the steady flow is accounted for. The present method only requires unknowns on the body surface.

The underlying assumptions are that the incoming waves as well as the body motions superposed on the steady forward translation of the body have small amplitudes. Viscous forces are neglected, the fluid is assumed incompressible and the motion irrotational so that we can apply potential theory.

1 Formulation of the problem

We consider the fluid motion in the frame of reference $Oxyz$ fixed to the steady translation with speed U of the body. For given frequency of oscillation σ of the body or the incoming wave, the potential governing the fluid flow may be decomposed into an oscillatory part

$$\text{Re}\,(\phi(x,y,z)e^{i\sigma t}) \tag{1}$$

where ϕ denotes the potential in the diffraction or the radiation problems, or the sum of them, and a stationary part

$$\phi_s(x,y,z) = U(\chi(x,y,z) - x) \tag{2}$$

due to the steady translation of the body. For small speed U, ϕ_s is approximated by the double body flow, hence we consider flow regimes where the lee waves are absent.

The free surface condition for ϕ (see Zhao et al. 1988) reads for small U

$$-\sigma^2\phi + 2i\sigma\nabla_1\phi_s\cdot\nabla_1\phi + i\sigma\phi\nabla_1^2\phi_s + g\frac{\partial\phi}{\partial z} = 0 \text{ at } z = 0 \tag{3}$$

where ∇_1 denotes the horizontal gradient and g the acceleration of gravity. Appropriate radiation conditions (outgoing waves at infinity), body boundary condition and condition of vanishing velocity in the infinitely deep water close the boundary value problem for the Laplacian potential ϕ. We now apply Green's theorem and obtain

$$\iint_{S_B}\left(\phi\frac{\partial G}{\partial n} - G\frac{\partial\phi}{\partial n}\right)dS - 2i\tau\iint_{S_F}\phi(\nabla_1 G\cdot\nabla_1\chi + \frac{1}{2}G\nabla_1^2\chi)dS - 4\pi\phi_0 = \left\{\begin{array}{l} -4\pi\phi(\mathbf{x}) \\ -2\pi\phi(\mathbf{x}) \end{array}\right. \tag{4}$$

where S_B denotes the mean wetted surface of the body and S_F the mean free surface. G is the forward speed oscillatory Green function and

$$\tau = \frac{U\sigma}{g} \tag{5}$$

ϕ_0 denotes the incoming wave potential, $-4\pi\phi$ applies when the field point is in the fluid and $-2\pi\phi$ when (x, y, z) is on S_B.

2 Solution of the problem

For (x, y, z) on S_B we introduce the singular expansions

$$\phi = \phi^0 + \tau\phi^1 + \ldots \tag{6}$$

$$G = G^0 + \tau G^1 + \ldots \tag{7}$$

into (4) and obtain for zeroth and first order in τ :

$$\left\{\begin{array}{c} 4\pi\phi^0 \\ 2\pi\phi^0 \end{array}\right\} + \iint_{S_B}\phi^0\frac{\partial G^0}{\partial n}dS = 4\pi\phi_0 \tag{8}$$

$$\left\{\begin{array}{c} 4\pi\phi^1 \\ 2\pi\phi^1 \end{array}\right\} + \iint_{S_B}\phi^1\frac{\partial G^0}{\partial n}dS = 2i\iint_{S_F}\phi^0(\nabla_1 G^0\cdot\nabla_1\chi + \frac{1}{2}G^0\nabla_1^2\chi)dS - \iint_{S_B}\phi^0\frac{\partial G^1}{\partial n}dS \tag{9}$$

for the diffraction problem. We obtain corresponding equations for the radiation problem.

To solve the integral equations we adopt a panel method, with ϕ^0 and ϕ^1 being constant on each panel. The left hand sides of (8) and (9) then involve identically the same integrals over a panel on the body surface . Now, from Huijsmans and Hermans (1985) G^1 may be given by

$$G^1 = 2i\frac{\partial^2 G^0}{\partial\nu\partial x} \tag{10}$$

where $\nu = \sigma^2/g$. Thus, the right hand side of (3) is determined by zero speed quantities.

Having obtained the potential on the body, the potential elsewhere in the fluid, for example the far-field flow, is obtained by the original equation (the singular expansions diverge in the far-field).

3 Acknowledgements

Financial support for this work was provided by the Royal Norwegian Council for Science and the Humanities and by Saga Petroleum. Computations were performed by means of the WAMIT version of Veritas Sesam Systems.

References

[1] HUIJSMANS, R.H.M. AND HERMANS, A.J., A fast algorithm for computation of 3-D ship motions at moderate forward speed. *4th Internat. Confer. on Numerical Ship Hydrodynamics*, 1985.

[2] ZHAO, R., FALTINSEN, O.M., KROKSTAD, J.R. AND AANESLAND, V., Wave-current Interaction Effects on Large-Volume Structures. *Proc. of the International Confer. on the Behaviour of Offshore Structures*, 1988.

DISCUSSION

Miloh: It is well-known that linearized theory breaks down at $\tau=1/4$. Nevertheless, since we know the type of singularity, logarithmic and square root for 2-D and 3-D flows I believe that in certain situation the drift force will be finite even at $\tau=1/4$ provided the integration is done in a proper manner. Did you try in your calculations to approach this critical frequency from below without facing out numerical problems?

Nossen, Palm & Grue: Due to the neglection of U^2 in the free surface condition and the perturbation expansion with respect to τ used, our theory is not good at τ values greater than about 0.1. Therefore the $\tau=1/4$ problem is not relevant to our method.