Evaluation of the Green Function for Ship Motions at Forward Speed and Application to Radiation and Diffraction Problems.

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Summary

A numerical evaluation scheme, including a complementary method ensuring accuracy when both the source and field points are close to the free surface, is developed of a new compact analytical expression of the Green function of ship motions at non-zero forward speed. This expression has a genuine single integral form whose integrands are elementary functions;it should permit straightforward computation and reduction of computer time.

Radiation and diffraction problems of 3-D bodies are solved with the panel method evaluating the Green function by this scheme. Convergence of the solutions with increase of the number of panels on the bodies is investigated numerically.

Comparison with measured hydrodynamic forces might not be strict examination confirming the superiority of sophisticated panel method to traditional strip theory, for the forces are not free from integration effects. Hence diffraction wave pattern around a body is measured in model experiment and compared with the computed.

Evaluation of the Green function.

Green function $G(x,y,z;x',y',z')\cdot\exp(i\omega t)$ is written in a single integral form (Bessho 1977),

$$G(x,y,z;x',y',z')=(1/r_1 - 1/r_2)/4\pi - iT(x-x',y-y',z+z')/2\pi \tag{1}$$

where

$$T(X,Y,Z)=\int_{\alpha-\pi}^{-\pi/2+\beta} \frac{[k_2\exp(k_2 P)-\mathrm{sgn}(\cos\theta)k_1\exp(k_1 P)]\,d\theta}{\sqrt{1+4\tau\cos\theta}} \tag{2}$$

$$\begin{aligned} r_{2,1} &= \sqrt{(x-x')^2+(y-y')^2+(z\pm z')^2} \\ P &= Z+(X\cos\theta+Y\sin\theta) \\ k_{1,2} &= K_0/2\,(1+2\tau\cos\theta\pm\sqrt{1+4\tau\cos\theta})\sec^2\theta \end{aligned} \tag{3}$$

and

$$\alpha = \cos^{-1}(1/4\tau) \tag{4}$$

$$\beta = \cos^{-1}(X/\sqrt{X^2+Y^2}) - i \cdot \sinh^{-1}(|Z|/\sqrt{X^2+Y^2}) \tag{5}$$

K_2 denotes g/U^2 and $\tau = U\omega/g$.

Eq.(2) is a genuine single integral expression of the Green function in the sense that its integrands are elementary functions (integral whose integrand is exponential integral is sometimes claimed to be single integral), while integration has to be done in the complex plane. Notice that α given by eq.(4) is an imaginary number when τ is less than 0.25 and β a complex number.

Numerical integration of the first term of eq.(2), we refer to as k_2-integral, is straightforward. Singularity at $\theta = \alpha - \pi$ is not essential unless τ is just equal to 0.25. The second term, k_1-integral, requires special consideration; integrand of this term oscillates rapidly without limit and becomes infinite in magnitude as θ approaches to $\pm\pi/2$. Introduction of new variable of integration $m = k_1\cos\theta$ transforms the k_1-integral between $\theta = -\pi/2 + \beta$ and $-\pi/2$ into

$$\int_{\delta}^{\infty} \exp[\phi(m)] / \sqrt{1 - [K_0 m/ m - K_0\tau)]^2}\, dm \tag{6}$$

where

$$\phi(m) = Z(m - K_0\tau)^2 / K_0 + i \cdot [Xm - y\sqrt{(m - K_0\tau)^4 - (K_0 m)^2} / K_0] \tag{7}$$

$$\delta = k_1 \cos(-\pi/2 + \beta) \tag{8}$$

δ is a complex number dependent on X, Y and Z. Integration of eq.(6) may be done along a contour in the complex plane going into the direction

$$-\pi/4 - 1/2 \cdot \tan^{-1}(Y/|Z|) \leqq \arg(m) \leqq \pi/4 - 1/2 \cdot \tan^{-1}(Y/|Z|) \tag{9}$$

as long as Z is negative or Y is not zero.

Our method of numerical evaluation of the contour integral (6) is:

(1) Find a deformed contour starting from δ on which the imaginary part of $\phi(m)$ is constant and the real part descends steepest; this is done integrating differential equation of the contour step by step.

(2) Integrate eq.(6) numerically along this deformed contour until a point beyond which contribution from the remaining part of the contour is less than a specified error limit; actually convergence of this integration is very fast.

In this computation the contour must not cross the branch cuts of $\phi(m)$ and if the starting point δ is located left of one of the zeroes of $\phi'(m)$, Re[$\phi(m)$] will ascend at the first stage of the contour until reaching to an appropriate location.

When both Z and Y are close to zero compared with X^2, our numerical steepest descent integration becomes slower in convergence; a complementary method must be introduced. Integral (6) is divided into two parts: one until a certain value of m for the numerical steepest descent integration and the other beyond this m large enough for the square root of eq.(7) to be approximated by only the first term. Contribution from the second part can be evaluated effectively noting that the corresponding integral is transformed into an integral known as Dawson's integral and using Newman's result (Newman 1987)).

Solution of radiation and diffraction problems.

Our method is the simple version of panel method: the contribution of the Rankine terms in eq.(1) for the surface integral on each panel is calculated using the method of Hess and Smith; contribution of the wave term is obtained by representing a panel by a concentrated source at its centroid. The centroid of the field panel is selected as the field point at which body boundary condition is posed. We remark that although our numerical results here are for submerged ellipsoids of revolution we discovered recently the expression (2) is readily integrated analytically to provide a simpler analytical expression for constant source strength distributed over a line segment or a panel. This is more appropriate than the present monopole approximation for extending our computations to the surface piercing bodies. Several computations are being carried out and will be reported presently.

A measure for satisfaction of the body boundary condition is $\varepsilon_{ii} = | B_{ii} - b_{ii} | /(B_{ii} + b_{ii})/2$ for radiation problem (the corresponding one is also given for diffraction problem) where B_{ii} is damping coefficient of the i-mode motion obtained from far-field velocity potential and b_{ii} that obtained from pressure integration on the body. Fig.1 shows convergence of numerical solutions of radiation problem for an submerged ellipsoid of revolution(L/B=2, d/B=0.75, d:depth of the body center) with the number of panels in terms of ε_{ii} ($4 \times N_w^2$ is the total number of panels). CPU time for $N_w=8$, for instance, is approximately 5 minutes on FACOM M780/20. Although convergence is fast, N_w more than 15 appears to be required for achieving 99% accuracy.

Vertical wave force on an ellipsoid of revolution (L/B=5, d/B=0.75) restrained in head waves is compared with the measured in Fig.2

Comparison of diffraction wave pattern

Fig.3 is a comparison of computed and measured diffraction wave contour at a time instant around the same model of Fig.2 in head waves of $\lambda/L=1.0$ and at Fn=0.2. This contour is depicted with the scale as five times expanded into the y direction as the actual scale; ship model is located between X=-1 and 1 with its centerline on y=0. Good agreement of the wave contour implies that prediction by the panel method of added resistance by diffration waves is supposed to be accurate.

References.

1. Bessho,M.(1977):Fundamental singularity in the theory of ship motions in a seaway, Memoirs of the Defense Academy Japan, vol.17, No.8.

2. Newman,J.N.(1987):Evaluation of the wave resistance Green function :Part 2-the simple integral on the centerplane, J.Ship Research, Vol.3, No.3.

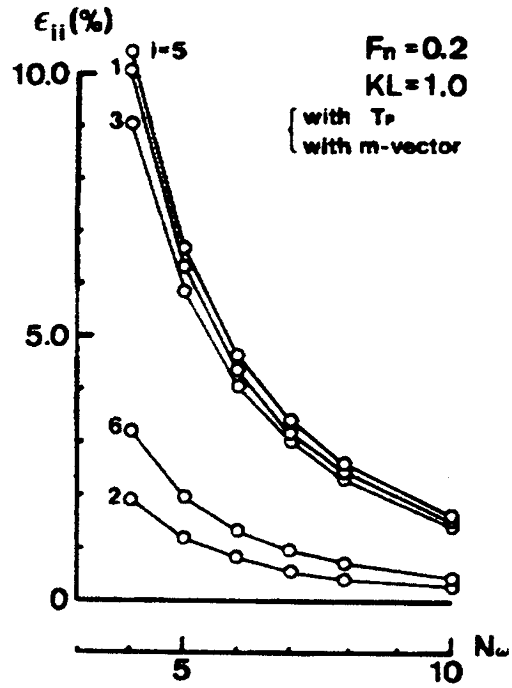

Fig.1 Convergence

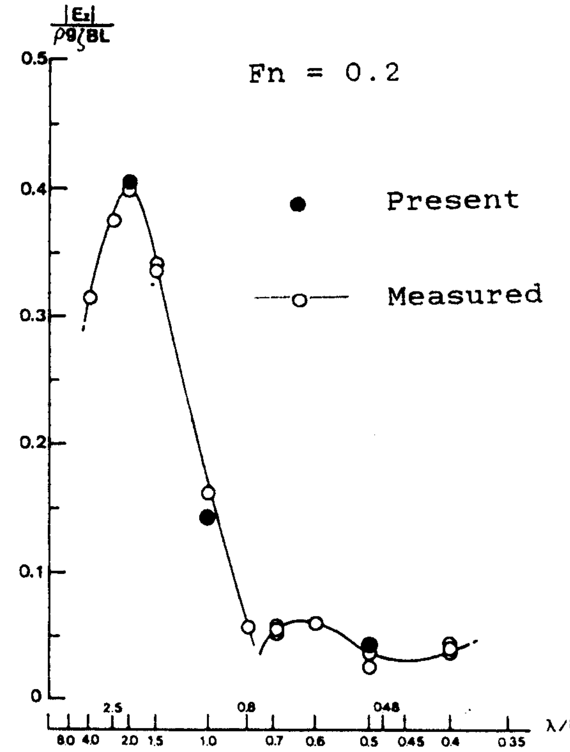

Fig.2 Vertical wave force

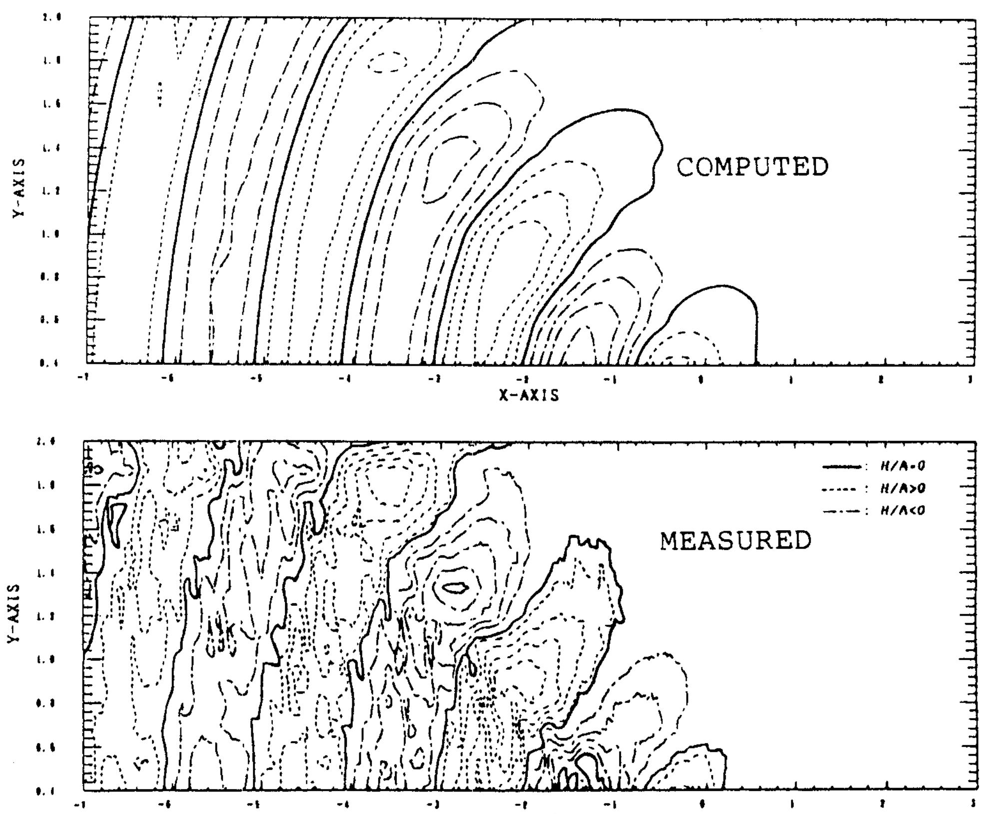

FIG.3 Diffraction wave pattern

DISCUSSION

Newman: First let me congratulate you on having the courage to numerically integrate this form of the Green function, in such an elegant manner. Secondly, have you compared your calculations for the submerged spheriod with the relatively simple "deeply submerged" approximation, e.g. my paper in J.S.R. circa 1965?

Ohkusu & Iwashita: I am afraid we did not compare your well-known results for submerged spheriod with our computation. This comparison, I like to do as soon as I can, must be useful to validate a part of our computation as well. I appreciate your suggestion.

Tuck: The explicit integral of G over a panel could be very useful since it could save some numerical work. However, to use it may require differences of the integral at the four corners of the panel, which may lose some accuracy.

Ohkusu & Iwashita: I share your view that uniform source distribution over a panel may not improve significantly convergence speed of computation with respect to number of panels, compared with distribution of a mono-pole source on each panel. However the explicit integral of the Green function over a panel will reduce substantially numerical work particularly when the source and field points are close to the free surface.

Sclavounos: We have found using a model 2-D problem that dropping the short-wave length system in the wave-source potential that the integrated forces are not affected substantially. Since this simplification will reduce substantially the computational effort, it would be interesting to report your experience in the three-dimensional case.

Ohkusu & Iwashita: Most cases for submerged bodies the short term component of the wave source potential does not seem to affect significantly the integrated forces. However, some examples for the steady force in following waves indicate that its effect is not to be ignored. I would like to remark that wave length varies continuously from the longer to the shorter wave component for 3-D at the propagation angle $\alpha_0=\cos^{-1}(1/4\tau)$, and we have no so clear distinction between them as in 2-D.