A PHASE CONTROL STRATEGY FOR OWC DEVICES IN IRREGULAR SEAS

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A device for extracting energy from ocean waves (the Oscillating Water Column, OWC) is based on the exchange of energy from the water into the air that is trapped above the water free-surface by an air chamber open at the bottom. The air chamber, which we assumed to be formed by a fixed, partly submerged structure, is connected with the atmosphere by a duct containing an air turbine. As a result of an incident, in general irregular, wave field, the internal free-surface as well as the air pressure are made to oscillate, and a flux of air is driven back and forth through the turbine. This paper deals with the OWC device control in irregular seas. The control problem is to find out the instantaneous flow across the turbine that maximizes energy absorption. In the following, linear water wave theory and isentropic flow of air across the turbine are assumed.

Under the assumption of small compressibility effects, the mass balance of air in the chamber is shown [1] to be given by

$$q_t(t) = q_d(t) + q_r(t) - V_0\,(\gamma P_a)^{-1}\,\frac{dp}{dt}. \qquad (1)$$

Here V_0 is the volume of air in the chamber for undisturbed conditions, $\gamma = c_p/c_v$ is the ratio of specific heats of air, $q_t(t)$ is the air flow rate across the turbine and $p(t)$ is the air pressure in the chamber relative to the atmospheric pressure, P_a.

For a given device geometry, the diffaction flow rate, $q_d(t)$, depends only on the incident wave field, and so it can be taken as a forcing term.

The radiation flow rate, $q_r(t)$, is dealt with by means of a convolution integral involving the time-dependent air-pressure and the OWC impulse response function, $h_r(t)$. Taking into account causality (i.e. the fact that no air flow occurs prior to the forcing pressure action), the convolution integral which expresses the radiation flow turns out to be given by

$$q_r(t) = \int_{-\infty}^{t} h_r(t-\tau)\; p(\tau)\; d\tau\,. \qquad (2)$$

The amount of energy absorbed by the OWC device during a time interval T is expressed as

$$E(T) = \int_0^T p(t)\, q_t(t)\ dt. \quad (3)$$

In order to specify the control strategy that maximizes (3),we apply Parcifal theorem to equation (3) and get

$$E(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P^*(i\omega)\ Q_t(i\omega)\ d\omega, \quad (4)$$

where $P(i\omega)$ and $Q_t(i\omega)$ are the Fourier transforms (FT) of the air pressure and flow rate across the turbine, and * represents complex conjugate. By applying the Fourier transform to equation (1), it turns out that

$$Q_t(i\omega) = Q_d(i\omega) + [\ H_r(i\omega) - i\omega \frac{V_o}{\gamma P_a}\ ]\ P(i\omega)\ , \quad (5)$$

where $H_r(i\omega) = -[B(\omega) + i\omega\ C(\omega)] = FT[h_r(t)]$. Substituting (5) into (4), we get after some algebra

$$E(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|Q_d(i\omega)|^2}{4B(\omega)} - B(\omega) |\ P(i\omega) - \frac{Q_d(i\omega)}{2B(\omega)} |^2\ d\omega\ . \quad (6)$$

This expression shows that maximum energy absorption is attained when

$$P(i\omega) = \frac{Q_d(i\omega)}{2B(\omega)}\ . \quad (7)$$

This result has been obtained previously by [2] for monochromatic waves. Substituting (7) into (6) it cames out that

$$E(T)_{max} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|Q_d(i\omega)|^2}{4B(\omega)}\ d\omega\ . \quad (8)$$

Introducing (7) into (5), and applying the inverse Fourier transform (IFT) one gets the following expression for the optimum flow rate across the turbine

$$q^{\varepsilon}(t) = \int_{-\infty}^{\infty} h_1(t-\tau)\, d\tau\ -\ \frac{V_o}{\gamma P_a} \frac{dp}{dt}\ , \quad (9)$$

where $h_1(t) = IFT[B(\omega) - i\omega C(\omega)] = -IFT[H_r(i\omega)] = -h_r(-t)$. Noting that $h_r(t<0) = 0$ it turns out that

$$q^{\varepsilon}(t) = -\int_{t}^{\infty} h_r(\tau - t)\, p(\tau)\, d(\tau) - \frac{V_o}{\gamma P_a}\frac{dp}{dt} \,. \qquad (10)$$

The first term in the right hand side of (10) is intrinsically anti-causal; this means that in order to achieve maximum energy absorption it is required that the air pressure be known in advance, theoretically infinitely far in the future. Obviuosly this information is unavailable because of the randomness of the diffraction flow rate, and because future values of the pressure are affected by previous control decisions. For practical reasons, then, turbine flow control must be based only on present and/or past air pressure values. Of course, under these conditions, it will be impossible to obtain the theoretical maximum energy absorption, as expressed by (8).

Note that for conventional Wells turbines, the flow rate and pressure difference are related to each other approximately by $q_t(t) = D\, p(t)$, where D is a real positive constant. Fourier transforming this equation yields $Q_t(i\omega) = D\, P(i\omega)$. Introducing this expression into (5), solving with respect to $P(i\omega)$, and substituting the result into (6) gives the following reduction of the energy absorption

$$\Delta E = \frac{1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\, |Q_d(i\omega)|^2 \left| \frac{1}{D + B(\omega) + i\omega[c(\omega) + \frac{V_o}{\gamma P_a}]} - \frac{1}{2B(\omega)} \right|^2 d\omega \,. \qquad (11)$$

This equation provides a means for determining the optimum values of turbine constant D and undisturbed volume V_o for a particular device (B and C fixed) and sea state (Q_d fixed). Their optimum values are those that minimizes $\Delta E(T)$.

A more powerfull means of reducing (11), and so of increasing energy absorption, is achieved if the air pressure, taken now as the control variable, is expressed as a causal function of the diffraction flow rate,

$$p(t) = \int_{-\infty}^{t} h_p(t - \tau)\, q_d(\tau)\, d\tau \qquad (12)$$

where $h_p(t<0)=0$. The use of this relation introduces two problems, the knowledge of both the diffraction flow rate q_d and the function h_p. The diffraction flow rate may be estimated by equations (1) and (2), provided the turbine flow rate and the air pressure are measured. The function h_p must be determined in order to minimize $\Delta E(T)$. Note that since $h_p(t<0)=0$, the real and the imaginary parts of $H_p(i\omega) = FT[h_p(t)]$ are pairs of Hilbert transforms (this is a way of expressing Kramers-Kronig relations). Substituting $P(i\omega) = H_p(i\omega)\, Q_d(i\omega)$ in (6), it turns out that the decrease in energy absorption becames

$$\Delta E = \frac{1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\, |Q_d(i\omega)|^2 \left| H_p(i\omega) - \frac{1}{2B(\omega)} \right|^2 d\omega \,. \qquad (13)$$

We now proceed in order to find out a function $H_p(i\omega)$ which minimizes (13), subject to the Hilbert transform condition referred to above. Recalling a well known theorem from system theory [3] we may state that if $H_p(s)$ (where s is a complex number) is an analytic fuction for $Re(s) \geq 0$ and if

$$\lim_{|s| \to \infty} H_p(s) = 0 \quad , \tag{14}$$

then $H_p(i\omega)$ is the transfer function of a causal system. These are the two conditions to be imposed on $H_p(i\omega)$ in order to garantee that $h_p(t<0) = 0$. Let us now assume

$$H_p(i\omega) = \frac{N(i\omega)}{D(i\omega)} \quad , \tag{15}$$

where $N(i\omega)$ and $D(i\omega)$ are polynomials of order n and m, respectively. Note that $m>n$ is required by (14). The analytic condition imposed on $H_p(i\omega)$ for $Re(s) \geq 0$, requires $D(i\omega)$ to have its roots on the 2nd and 3rd quadrants. This condition will impose certain constraints on the coefficients of $D(i\omega)$, depending on the order of the polynomial. No further progress can be done except if a particular form of $B(\omega)$ is assumed (and so a particular OWC geometry).

For two dimensional OWC of shallow draft in deep water, it was found that $m=3$ and $n=2$ were two good choices. In order to garantee the conditions imposed on the roots of the $D(i\omega)$, its coefficients must satisfy the following conditions

$$a_i > 0 \quad , \quad i=1,2,3 \tag{16}$$

and

$$a_3 < a_1 a_2 \ . \tag{17}$$

Results for this geometry are expected to be presented at the conference.

Note that this control strategy will require the use of a non-conventional turbine, capable of controlling the flow circulation around the blades (by changing the turbine geometry or by any other method).

The autors think that this kind of control technique may also be applied to other types of situations: for instance, if it is required to control the movement of a floating structure in irregular seas, subjected to some objective function.

References

[1] - Sarmento, A.J.N.A.; Gato, L.M.C. and Falcão, A.F. de O., *Wave Energy Absorption by an OWC Device with Blade-Pitch-Controlled Air-Turbine*, OMAE 1987, Vol. I, p.p. 465-474.

[2] - Evans, D.V., *Wave-Power Absorption by Systems of Oscillating Pressure Distribuitions*, Journal of Fluid Mechanics, Vol. 114, 1982, p.p. 481-499.

[3] - Papoulius, A., *Signal Analysis*, MCGraw-Hill, 1981.

DISCUSSION

Falnes: Your approach is a very interesting alternative to predicting the incoming wave. In a model experiment (ref. symp. Wave and Tidal Energy, Cambridge, 1981) we obtained, with a JONSWAP wave spectrum, 85% of the power we would have absorbed if we knew the future wave in advance. In comparison, how much power do you absorb in your numerical example as compared to the theoretical maximum?

Perdigão & Sarmento: In our example over 95% of the theoretical maximum efficiency is achieved. However, we think you cannot compare this result with the 85% energy efficiency you claim to obtain in your experiment. The reason is that you are not comparing with the theoretical maximum, but with the maximum you would obtain if your predictor was perfect. Note that your control strategy in the experiment is not similar to the optimum countrol strategy as stated by eq. (12) in my paper (which gives the theoretical maximum for energy extraction).