THE SLOW-DRIFT WAVE DAMPING OF FLOATING BODIES

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Ships and offshore structures operating in waves and constrained by weak restoring forces may undergo large amplitude low-frequency oscillations excited predominantly by nonlinear wave forces. This slow motion is damped by viscous as well as ideal fluid effects both contributing a significant component to the damping force.

This abstract outlines a new theory for the evaluation of the ideal component of the damping force, often called "slow-drift wave damping". It is valid for three-dimensional structures of general geometry floating in waves of arbitrary heading and frequency. Explicit "slow-drift" Green functions are derived which allow the solution of the problem without the discretization of the free surface. The theoretical framework is similar to that developed in Sclavounos (1988) for second-order wave-body interactions.

The Boundary Value Problem

Consider a coordinate system (x, y, z) fixed on a body which translates with a small speed U in the positive x−direction while it interacts with monochromatic plane progressive waves of amplitude A, frequency ω_0, wavenumber ν and direction β off the positive x−axis. With respect to the translating frame, the flow oscillates at the frequency of encounter

$$\omega = \omega_0 - U\nu \cos\beta. \tag{1}$$

Omitting throughout the complex time factor $e^{i\omega t}$, the deep-water incident-wave potential far from the body is given by

$$\varphi_I = \frac{igA}{\omega_0} e^{\nu(z - ix\cos\beta - iy\sin\beta)}. \tag{2}$$

where g is the acceleration of gravity. At small speeds U, the steady-state flow past the body is to leading-order described by the double-body velocity potential $U(-x + \bar{\phi})$, and the radiation/diffraction potential φ is subject to the linearized free-surface condition

$$\varphi_z - (\omega^2/g)\varphi + 2i(\omega U/g)(\nabla\varphi \cdot \nabla\bar{\phi} - \varphi_x) - i(\omega U/g)\varphi\bar{\phi}_{zz} = 0, \tag{3}$$

enforced on the $z = 0$ plane and containing errors of $O(U^2)$. Equation (3) also governs the modified incident wave potential $\bar{\varphi}_I$ which unlike (2) is convected by the double-body flow. On the mean position of the body wetted surface S_B, the diffraction and radiation potentials are subject to

$$\varphi_{D_n} = -\bar{\varphi}_{I_n}, \qquad \varphi_{j_n} = i\omega n_j + U m_j, \quad j = 1, \dots, 6 \tag{4}$$

where n_j are the unit normal vectors corresponding to the six rigid body oscillatory modes and m_j depend on gradients of the double-body potential $-Ux + \bar{\phi}$. The details of the derivation of equations (3) and (4) are given in Newman (1978).

The slow-drift wave damping is related to the U-slope of the drift force experienced by the body, in the limit as $U \to 0$. The conditions for this interpretation to be valid are discussed by Faltinsen (1987). It is therefore sufficient to determine the leading-order forward-speed correction to the velocity potential φ. Assume a perturbation expansion of the form

$$\varphi = \varphi_0 + \tau_0 \varphi_1 + \cdots \tag{5}$$

where the selection of the small parameter $\tau_0 = \omega_0 U/g$ follows from the form of the free surface condition (3) and φ_0 represents the zero-speed incident, radiation or diffraction potentials. Upon substitution of (1) and (5) in (4) the potential $\psi = \varphi_1$ can be shown to satisfy the non-homogeneous free-surface condition

$$\psi_z - (\omega_0^2/g)\psi = 2i\varphi_{0x} - 2\nu \cos\beta\, \varphi_0 - 2i\nabla\bar{\phi} \cdot \nabla\varphi_0 + i\bar{\phi}_{zz}\varphi_0 \tag{6}$$

and the body boundary conditions

$$\psi_{Dn} = -\psi_{In}, \qquad \psi_{jn} = -i\omega_0 \cos\beta\, n_j + (g/\omega_0)m_j \tag{7}$$

for the diffraction and radiation problems respectively. The potential ψ_I represents the correction to the far-field incident wave (2) arising from its convection by the double-body flow. It will be determined from the solution of the Laplace equation subject to (6) on the $z=0$ plane with φ_0 set equal to φ_I.

The perturbation expansion (5) is not uniformly valid in the entire fluid domain because the velocity potential $\psi = \varphi_1$ grows in magnitude at large horizontal distances from the body. At a fixed position near the body, on the other hand, the second term in (5) may always be rendered small relative to the first for a sufficiently small τ_0. Formally, this nonuniformity can be treated by the asymptotic matching of the "inner" solution (5) to an "outer" solution subject to (3). It can be shown that this matching will not change the character of the inner solution by the addition of a homogeneous component. This is here illustrated for a wave disturbance generated by a point submerged time-harmonic source potential subject to the free-surface condition

$$G_z - (\omega_0^2/g)G - 2i\tau_0 U G_x = 0 \tag{8}$$

which accepts the uniformly valid solution

$$G(\vec{x};\vec{\xi}) = \frac{1}{r} - \frac{1}{r'} + \frac{1}{\pi}\int_0^\infty k\,dk \int_0^{2\pi} d\theta\, \frac{e^{k(z+\zeta)-ik[(x-\xi)\cos\theta+(y-\eta)\sin\theta]}}{k(1-2\tau_0\cos\theta) - (\omega_0 - i\epsilon)^2/g} \tag{9}$$

where r, r' are the radial distances from the source located at $\vec{\xi}$ and its image above the free surface to the field point $\vec{x}$, and ϵ is the Rayleigh viscosity. The formal expansion of (9) for small τ_0 gives

$$G = G_0 + \tau_0 G_1 + \cdots, \quad G_1 = \frac{2}{\pi}\int_0^\infty k^2\,dk \int_0^{2\pi} d\theta \cos\theta\, \frac{e^{k(z+\zeta)-ik[(x-\xi)\cos\theta+(y-\eta)\sin\theta]}}{[k-(\omega_0 - i\epsilon)^2/g]^2}, \tag{10}$$

where G_0 is the zero-speed wave source potential. The Green function G_1 has been derived and studied by Huijsmans and Hermans (1985), it is easy to verify that it satisfies the free-surface condition

$$G_{1z} - (\omega_0^2/g)G_1 = 2iG_{0x} \tag{11}$$

and that it represents a wave disturbance growing in the far field. This growth is associated with the residue from the double pole in (10). The absence of a homogeneous component in the "inner" potential ψ is also supported by the asymptotic analysis in Ogilvie and Tuck (1969) on the related forward-speed problem for slender ships.

The "Slow-Drift" Green Functions

Three new Green functions will be derived for the solution of the boundary-value problem for ψ. Let $\bar{G}(\vec{x};\vec{\xi}) = 1/r + 1/r'$. The first slow-drift Green function D_I satisfies the Laplace equation in the entire fluid domain and the free-surface condition

$$D_{Iz} - (\omega_0^2/g) D_I = [i\bar{G}_{xz} - 2\nu(\cos\beta\,\bar{G}_x + \sin\beta\,\bar{G}_y)] e^{-i\nu(x\cos\beta + y\sin\beta)}, \tag{12}$$

on $z = 0$. Using the techniques of Fourier transforms, we obtain

$$D_I(\vec{x};\vec{\xi}) = i\frac{e^{-i\nu(x\cos\beta + y\sin\beta)}}{\pi} \int_0^\infty k\,dk \int_0^{2\pi} d\theta\, \frac{e^{\ell z + k\zeta - ik[(x-\xi)\cos\theta + (y-\eta)\sin\theta]}}{\ell - (\omega_0 - i\epsilon)^2/g} [k + 2\nu\cos(\theta - \beta)], \tag{13}$$

where $\ell^2 = k^2 + 2k\nu\cos(\theta - \beta) + \nu^2$. The second slow-drift Green function D_A is subject to the free-surface condition

$$D_{Az} - (\omega_0^2/g) D_A = 2iG_{0x}(\vec{x};\vec{\xi}) - 2\nu\cos\beta\, G_0(\vec{x};\vec{\xi}) \tag{14}$$

where G_0 is defined in (10). The solution for D_A is

$$D_A(\vec{x};\vec{\xi}) = \frac{2}{\pi} \int_0^\infty k\,dk \int_0^{2\pi} d\theta\, \frac{e^{k(z+\zeta) - ik[(x-\xi)\cos\theta + (y-\eta)\sin\theta]}}{[k(\omega_0 - i\epsilon)^2/g]^2} (k\cos\theta - \nu\cos\beta). \tag{15}$$

The last slow-drift Green function D_B satisfies the free-surface condition

$$D_{Bz} - (\omega_0^2/g) D_B = iG_{zz}(\vec{x};\vec{\xi}_1)\, G_0(\vec{x};\vec{\xi}_2) - 2i\nabla_x G(\vec{x};\vec{\xi}_1) \cdot \nabla_x G_0(\vec{x};\vec{\xi}_2) \tag{16}$$

and is given by the expression

$$D_B(\vec{x};\vec{\xi}) = \frac{1}{\pi^2} \int_0^\infty k\,dk \int_0^{2\pi} d\theta\, \frac{e^{kz - ik[(x-\xi_1)\cos\theta + (y-\eta_1)\sin\theta]}}{k - (\omega_0 - i\epsilon)^2/g} Q(k,\theta) \tag{17}$$

$$Q(k,\theta) = i \int_0^\infty \ell\,d\ell \int_0^{2\pi} d\vartheta\, \frac{e^{\mu\zeta_1 + \ell\zeta_2 + i\ell[(\xi_1 - \xi_2)\cos\vartheta + (\eta_1 - \eta_2)\sin\vartheta]}}{\ell - (\omega_0 - i\epsilon)^2/g} \frac{k^2 - \ell^2}{\mu}, \tag{18}$$

where $\mu^2 = k^2 - 2k\ell\cos(\theta - \vartheta) + \ell^2$. The derivation of (13), (15) and (17)-(18) is similar to that presented in Sclavounos (1988) for an analogous set of Green functions arising in second-order wave body interactions.

The Solution for the Potential ψ

Let the double-body and zero-speed radiation/diffraction potentials $\bar{\phi}, \varphi$ be defined as a distribution of sources on the body boundary,

$$\bar{\phi}(\vec{x}) = \iint_{S_B} \bar{\sigma}(\xi)\, \bar{G}(\vec{x};\vec{\xi})\, d\xi \tag{19}$$

$$\varphi_0(\vec{x}) = \iint_{S_B} \sigma(\xi)\, G_0(\vec{x};\vec{\xi})\, d\xi. \tag{20}$$

Setting φ_0 equal to φ_I in the free surface condition (6) and combining (12) with (19), we obtain the solution for the potential ψ_I in the form

$$\psi_I(\vec{x}) = (igA/\omega_0) \iint_{S_B} \bar{\sigma}(\xi)\, D_I(\vec{x};\vec{\xi})\, d\xi \tag{21}$$

which can be shown to satisfy (6).

A *particular* solution ψ_P for the radiation/diffraction problem which satisfies (6), but not the body boundary conditions (7), can be obtained in an analogous manner by combining the slow-drift Green functions $D_{A,B}$ with (20). The result is

$$\psi_P(\vec{x}) = \iint_{S_B} \sigma(\xi)[D_A(\vec{x};\vec{\xi}) + D_B(\vec{x};\vec{\xi})]\, d\xi. \tag{22}$$

It follows from the definitions of the slow-drift Green functions [eq. (13), (15) and (17)] that the potentials ψ_I, ψ_P defined by (21) and (22) are regular in the fluid domain, including the interior of the body but develop a singularity at the body waterline. Given the source strenghts $\bar{\sigma}, \sigma$, they can be evaluated together with their gradients by quadrature.

The solution of the radiation/diffraction problem is completed by enforcing the body boundary condition with the addition of a potential ψ_H which satisfies the homogeneous free-surface condition. Define the diffraction and radiation potentials respectively as follows

$$\psi_D = \psi_P + \psi_H, \qquad \psi_j = \psi_{Pj} + \psi_{Hj}. \tag{23}$$

By virtue of (7) the *homogeneous* components ψ_H are subject to the body boundary conditions

$$\psi_{Hn} = -\psi_{In} - \psi_{Pn} \qquad \psi_{Hjn} = -i\omega_0 \cos\beta\, n_j + (g/\omega_0) m_j - \psi_{jPn}. \tag{24}$$

and can be evaluated by standard zero-speed boundary integral methods.

Finally, the slow-drift wave damping is obtained from the substitution of (5) in the Bernoulli equation and the evaluation of the component of the steady-state drift force that depends linearly on the forward velocity U.

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