ON SMALL-TIME EXPANSION OF NON-LINEAR FREE SURFACE PROBLEMS.

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During the latest years time simulation of non-linear free surface problems, formulated according to the semi-Lagrangian scheme introduced by Longuet-Higgins & Cokelet /1/, has been applied to several problems. The subject of the present discussion will not be the time simulation procedure as such, but rather some of the problems one may encounter when selecting the initial condition for these simulations. The discussion will, further more, be limited to the problems which may be caused by possible singularities at the intersection point between the free surface and the surface piercing body for Taylor series expansions for small times.

It is established in the literature (Refs. /2/, /3/) that a log(z) singularity in the complex velocity will occur at the intersection point between the wavemaker and the free surface for the 2-D wavemaker problem when started impulsively from rest.
It has been shown (Ref./3/) that the singularity, as should be expected, becomes increasingly worse for the higher derivatives in time of the displacement vector. The development in Ref./3/ is based on a non-linear Lagrangian expansion, and these results can thus not be rejected on the basis that the singularities will be situated outside the fluid domain after a finite time has elapsed. There exist problems which show a much worse behaviour from the singularity point of view than this one, the entry of a cone, which has been treated by Greenhow & Lin /4/, is one, the exit/entry of a circular cylinder, initially less than half submerged, is another. The last problem has been approached numerically by Greenhow /5/; the initial velocity distribution is found, among other places, in Ref./6/.

The occurrence of these singularities, and the fact that they grow increasingly worse for the higher derivatives in time when approaching the intersection point, indicates that this expansion is an outer expansion, and has to be matched to a an inner expansion to form the total expansion. Roberts /7/ and Joo & al. /8/ have treated the corresponding linear problem, computing the solution for the wavemaker problem for small times and small velocities. They have shown that the singularity disappears, and that the corresponding linear small time expansion from Refs./2/ and /3/ indeed form the outer expansion of the problem. In /8/ it is shown that introduction of surface tension makes the solution smooth close to the intersection point, eliminating the rapid oscillations of the free surface elevation experienced in Ref./7/.

In addition to the wavemaker problem we have a class of problems which at first sight seem quite innocent, showing an uniformly valid solution for the initial velocity, with a finite value at the intersection point. The present paper will be treating some of these. One is the problem of a 2-D circular cylinder moving vertically and starting impulsively from rest, from the position of being half submerged. This is the classical test case used in Refs./9/ and /10/. The geometry is shown in Fig.1, together with the boundary conditions specifying the problems for the initial values of the higher time derivatives of the potential. It is clear that the initial velocity distribution will be given by a dipole.

The problem determining the partial derivative of the complex potential with respect to time is defined in Fig.2. The solution of this problem can be found as:

$$
\begin{aligned}
w_t(z,t) = & -iV_t a^2/z - V^2 a^4/2z^4 \\
& - (iV^2/\pi)\{(z/a)^2 - (a/z)^2 + \tfrac{1}{2}(z/a)^4 - \tfrac{1}{2}(a/z)^4\}\ln[(z-a)/(z+a)] \\
& - (iV^2/\pi)\{7z/3a - 7a/3z + (z/a)^3 - (a/z)^3\}
\end{aligned}
$$

where $w(z,t)$ is the complex potential and the index "t" indicates partial differentiation with respect to t. The corresponding value for the complex acceleration is found as:

$$
\begin{aligned}
d^2Z^*(x,y,0)/dt^2 = Dw(x,y,0)/Dt = & \; iV_t a^2/z^2 - 2V^2 a^4(z^2 - z^{*2})/z^3|z|^2 \\
& -(2iV^2/\pi z)[(z/a) + (a/z)][(z/a)^3 + (a/z)^3]\ln[(z-a)/(z+a)] \\
& -(4iV^2/\pi z)[(z/a) + (a/z)][(z/a)^2 + 1/3 + (a/z)^3]
\end{aligned}
$$

where $Z = (X + iY)$ is the complex (Lagrangian) displacement and the asterisk denotes complex conjugation. For $t=0$ the Lagrangian and the Eulerian coordinates coincide. This expression clearly shows a log(z-a) singularity close to z=a. This singularity is caused by the inconsistency in the horizontal acceleration close to this point, showing different values when referring to the free surface and to the cylinder. The formulation, and the also the solution, correspond to those found for a wavemaker starting from rest with a finite acceleration (Ref./3/). The correspondence between the Eulerian solution and the Lagrangian solution clearly shows that the singularity will not be removed when changing to Lagrangian coordinates. It is relatively easily shown that the singularity grows stronger and stronger for the higher derivatives in time, becoming of order $(z-a)^{-\mu}$, where μ is an integer for the partial derivative of the complex potential of order $\mu+2$ with respect to time.

Another, related, problem is the one introduced by Grosenbauch & Yeung /11/, with a half submerged circular cylinder in a current. The initial condition is the one introduced in Ref./11/ with the free surface acting as a streamline. In this case the log(z-a) singularity will appear for a much higher time derivative, the fifth to be more precise, but is unavoidable here as well.

The seemingly inevitability that singularities will occur for the innocent problems discussed, raises the question if the small time expansion in general will lead to singularities for some higher order terms, independent of the problem. This question can not be answered from the present analysis. If this is the case it becomes even more interesting to develop the inner expansion, or even better, the uniformly valid non-linear expansion.

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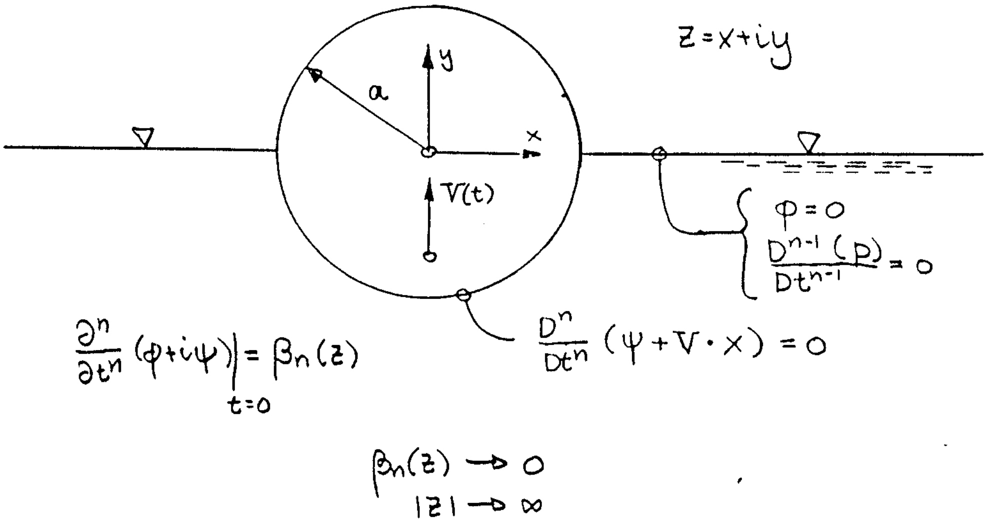

FIGURE 1.

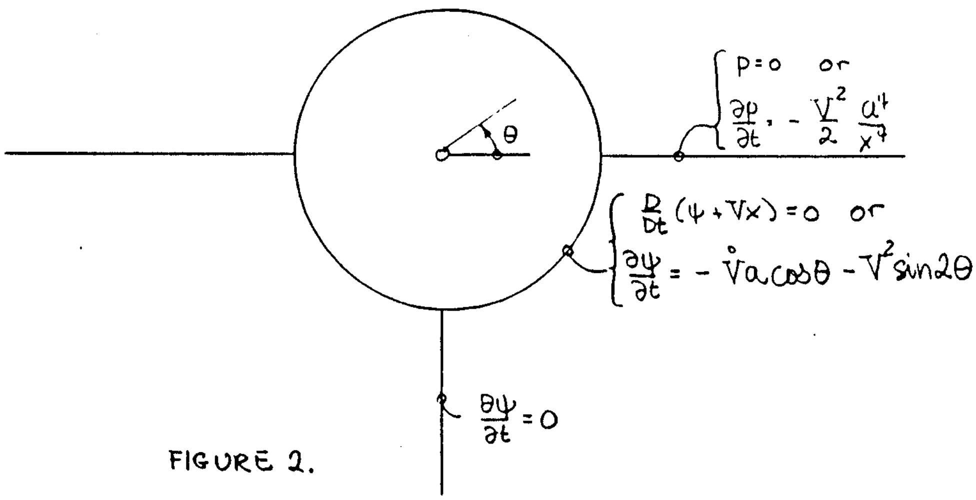

FIGURE 2.

Discussion

Grilli: What do you think are the practical consequences of these new results from a computational point of view? Do you think, in particular, there would be some problems if the motion starts very "gently"? The non-uniqueness of the horizontal acceleration, for instance, vanishes like V^2/a and is "numerically" small, in the small time considered here, V is very small.

Vinje: First of all I would like to point out that the present results are not "new" in the sense that such behaviour has not been found before. In this context I would like to point to Refs./2/, /3/ and /4/ on the wavemaker problem. We have to realize that the solution only forms the <u>outer</u> expansion, and is strictly speaking not valid at, or close to, the intersection point. The occurrence of the singularity in this outer solution does not <u>necessarily</u> introduce serious problems for the inner solution. Refs./7/ and /8/ clearly indicates that. The fact that no problems concerning singularities in numerical solutions of this problem have been observed so far might be interpreted the same way. In this context it must be mentioned that singularites, like the one described here, have not been expected, and thus not been looked for in the numerical computation of the heaving cylinder.

If the motion is started very "gently" (i.e. with a zero velocity and a non-zero acceleration) the singularity will appear in the second derivative, in stead of in the first, but will not disappear all together. The question is still open if this would affect the numerical solution, but a smoother development in time would be <u>expected</u> though.

Yeung: I commend the author for his elaborate analysis on the singular point properties of this type of problem. For the impulsive starting wavemaker, we arrived at a zlog(z) type singularity by a simplified procedure discussed in the Bristol workshop (authors: Wu & Yeung). The Grosenbaugh & Yeung (1988) initial condition apparently gives a much weaker singularity (actually the velocities are all regular). I was wondering if this more-favorable initial condition for their problem may be reinterpreted as a much "smoother" start.

Vinje: As I mentioned in the presentation, you do not avoid the singularity in the outer solution by applying the rigid free-surface condition initially, but I agree with you: it is definitely more preferable to have it appearing in the fifth derivative with respect to time than in the zeroth. The $\varphi=0$ initial condition for the swaying cylinder does, as you are well aware of, correspond to the wavemaker problem, which reflects the zlog(z) behaviour rather close to the intersection point, as shown in Ref./12/. The fact that you avoid this behaviour in itself shows that you get a "smoother" start.

The smoothness of the start, alone, is no criterion for the choice of initial condition though. We have to think a bit about our physical problem as well, and how the initial condition relates to that. I do not think that choosing the rigid free-surface condition for the heaving cylinder problem makes much sense, even if it would lead to a less singular solution than the $\varphi=0$ condition.