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Solution of Nonlinear Water-Wave and Wave-Body Interaction Problems using a new Boundary-Fitted Coordinates Method

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In this paper, nonlinear time-dependent water-wave and wave-body interaction problems are studied using a finite difference formulation based on a new method of coordinates generation. Although the use of boundary integral methods is more economical and less time-consuming, for two-dimensional potential flow simulations, the finite difference method can provide a natural framework for solving unsteady viscous flow problems that are of importance in a number of naval and offshore engineering applications. In field discretization procedures such as the finite difference method, the accuracy of the solution depends on the implementation of the boundary conditions as well as on the properties of the mesh system. Generation of grids by the method of boundary-fitted curvilinear coordinates, with the extreme coordinates conforming to the boundaries, is an efficient means of discretizing a physical domain and implementing the boundary conditions. According to this method, the physical domain is mapped onto a computational domain which is usually uniform and rectangular. The governing equations and boundary conditions are also transformed and then solved in the computational space. An overview of some of the applications may be found in Yeung [1] and Thompson et al. [2]. Among the various grid-generation methods, the one introduced by Thompson et al. [3] which uses a set of Poisson equations to describe the transformations, has been quite popular in free-surface hydrodynamics (see e.g. Coleman and Haussling [4], Telste [5], Yeung and Wu [6]). However, this method is also known to cause problems such as grid-skewness and foldings when the free surface becomes steep or multivalued. In an attempt to overcome these problems, Ghia et al. [7] and Coleman and Haussling [4] suggested certain special treatments within the framework of the Thompson's method. They were able to model steep waves with limited success, but were unsuccessful in modelling overturning waves. Others like Miyata [8] resorted to the use of Lagrangian segments together with "irregular-stars" to track the free surface. Simulation of overturning waves was possible but there were inherent difficulties in applying such a scheme to flows where strong gradients exist in certain local regions.

In the present work, the grids that discretize the physical domain are generated using a variational formulation (see for e.g. Brackbill and Saltzman [9]) along with the notion of an intermediate reference space. The field equations for the grid-generation problem are derived as a product transformation of the physical space onto the computational space via a reference space. Results presented here show that our method can indeed cope with steep and overturning waves. Fig. 1 illustrates the transformations involved in this grid-generation procedure. In order to demonstrate its capability to treat highly nonlinear waves and wave-body interactions, two case studies are presented. In the first case, the overturning and breaking of a large-amplitude shallow-water wave are examined. The second case corresponds to the nonlinear flow about a slightly submerged cylinder. Both cases are tackled using a mixed Eulerian-Lagrangian formulation [10] with a finite-difference technique similar to [6]. Work-energy balance is used to monitor the accuracy and stability of the finite difference scheme.

For the first problem reported here, the solutions based on linear theory are used as initial conditions to start the calculations in shallow water. Periodicity conditions are imposed at the open boundaries. No saw-tooth type instability is experienced, which may be due to the numerical dissipation of the method. For a given time step, regridding based on equal arc-lengths is carried out once in about 50 time steps in order to satisfy the Courant condition. A spilling breaker is observed for a large-amplitude wave moving over a wavy bottom. Sample plots of the mesh and velocity vectors at a particular instant of time are given in Figures 2-a and 2-b. A time sequence plot of wave profiles is shown in Fig. 2-c. It shows that a cusp-like structure develops at the crest which overturns in time and eventually results in spilling. In the case of a larger amplitude wave propagating over a flat bottom, a plunging-type breaker is noticed. Typical grids and velocity vectors for this case are shown in Figures 3-a and 3-b. A parallel projection in time of the free-surface evolution is given in Fig. 3-c. Experimental observations [12] have confirmed that the free surface can remain smooth and rounded even after the overturning, which is a feature predicted by the calculations. The results of spilling and plunging breakers are also in agreement with those of Baker et al. [11], who modelled these problems using a boundary-integral method.

Next, the nonlinear free surface flow associated with the forced heaving of a slightly submerged two-dimensional cylinder is studied. The approximate open boundary condition of Grosenbaugh and Yeung [13], viz. Dp/Dt = 0 on a material open boundary, has been found to be effective before waves reaches the boundary during the first few oscillations of the cylinder. Several runs were made corresponding to a range of amplitude and frequency of the oscillation as well as mean-depth of submergence that are of practical interest. Close-up plots of the grids, velocity vectors and free-surface evolution in time corresponding to a high- and a low-frequency case are given in Figures (4-a, 4-b, 4-c) and (5-a, 5-b, 5-c) respectively. Splashing of waves near the centerline is observed in the case of low frequency and high amplitude oscillation. Results are also compared with experimental data and also with the results of Vinje et al. [14] who studied a similar problem using a boundary integral method.

The case studies presented in this paper thus show that the present method is capable of handling overturning waves and wave-body interaction problems that require grid clusterings to capture local flow details. These results represent the first successful finite-difference solution that can accurately and effectively handle bodies of a general shape in a wave field. Since the procedure is based on the solution of the field equation, the effects of viscosity can also be included more readily. Works in this direction are being pursued.

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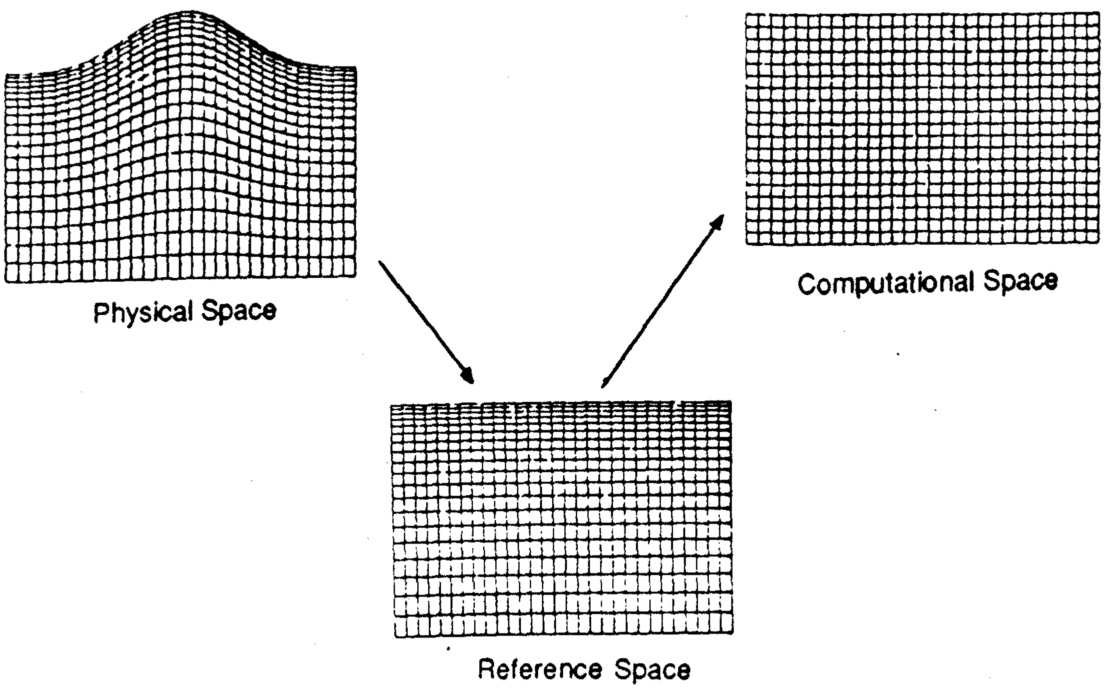

Fig.1 Grids Generated by a Variational Method

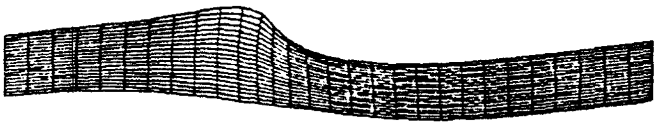

Fig 2-a Typical Grid

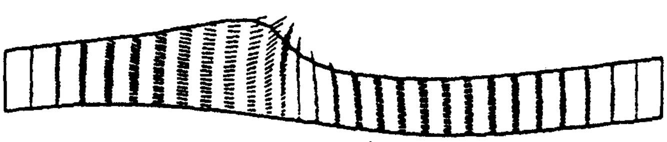

Fig. 2-b Velocity Vector Plot

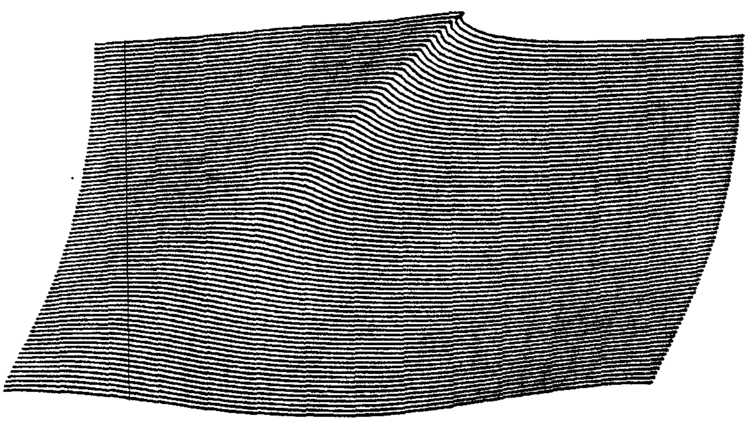

Fig. 2-C Free Surface Evolution

SPILLING BREAKER

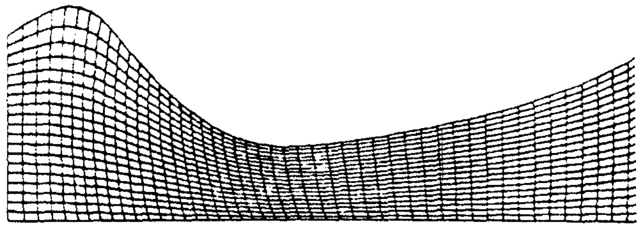

Fig. 3-a Typical Grid

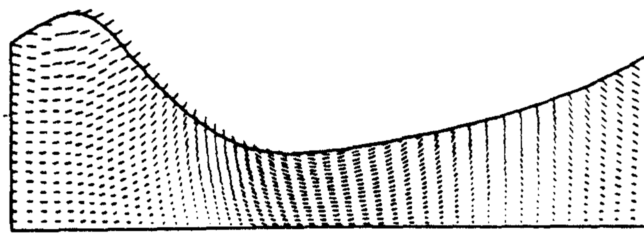

Fig. 3-b Velocity Vector Plot

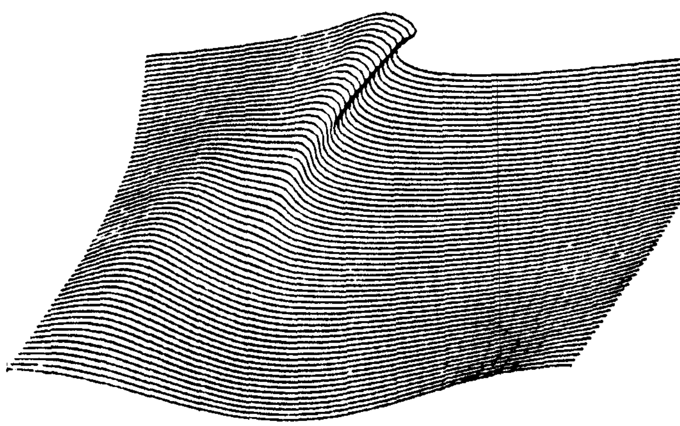

Fig. 3-C Free Surface Evolution

PLUNGING BREAKER

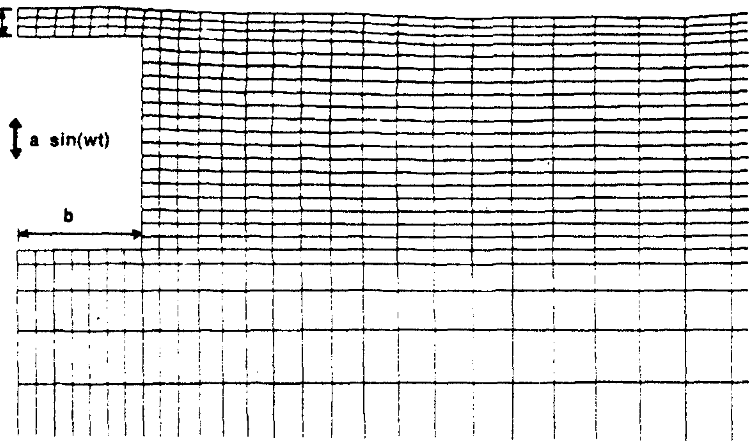

Fig.4-a: Typical Grid

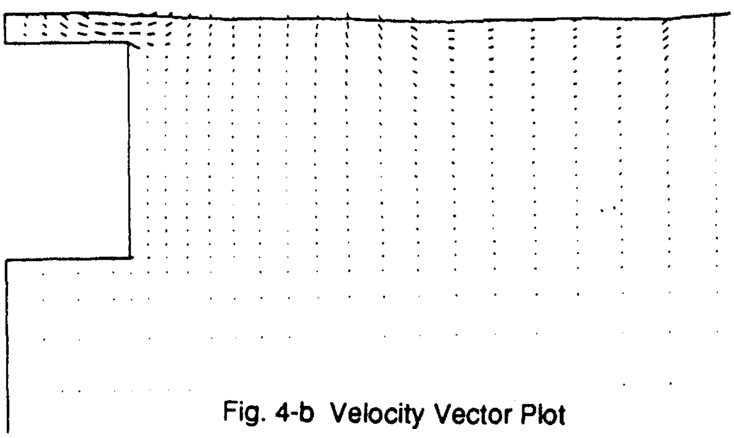

Fig. 4-b Velocity Vector Plot

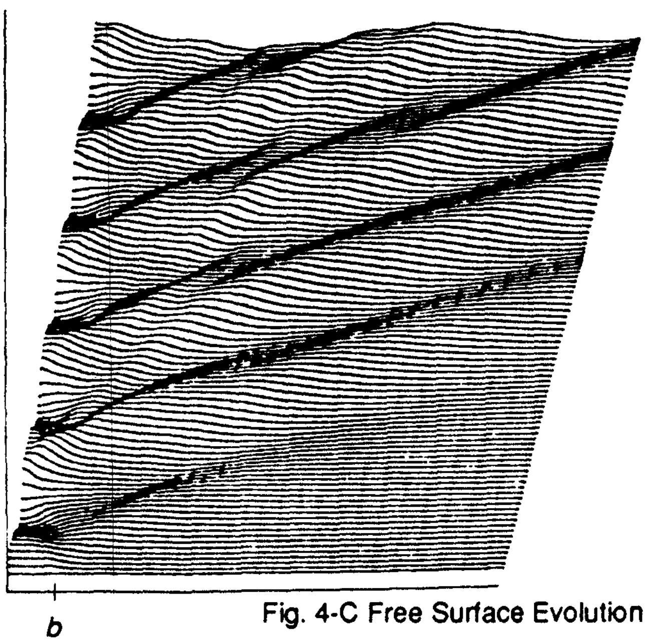

Fig. 4-C Free Surface Evolution

Flow about submerged Cylinder for the case of d/b=0.25; a/b=0.05 and N=w*w*b/g =0.813

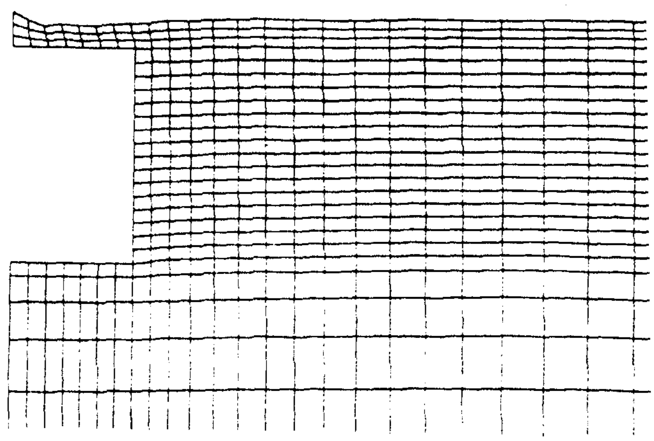

Fig.5-a: Typical Grid

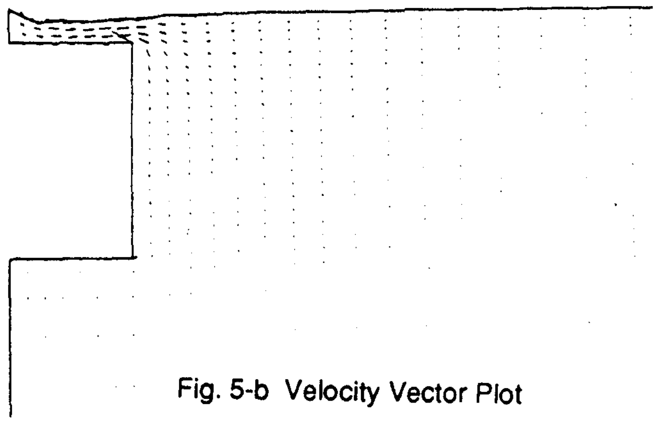

Fig. 5-b Velocity Vector Plot

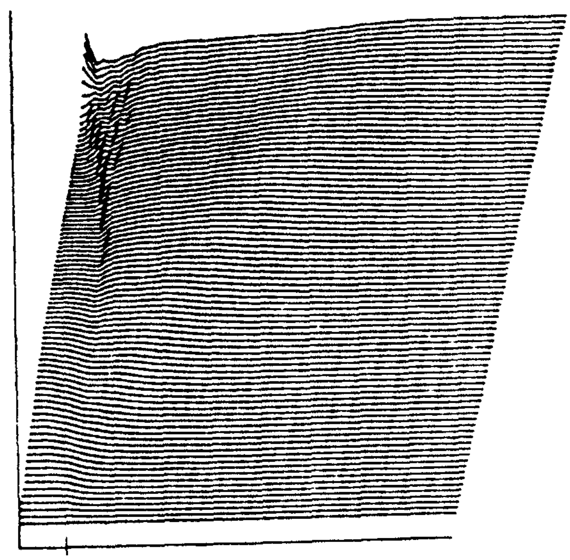

Fig. 5-C Free Surface Evolution

Flow about submerged Cylinder for the case of d/b=0.25; a/b=0.10 and N=w*w*b/g =0.169

DISCUSSION

Grilli: 1) Are there motivations in your choice of a finite-difference scheme other than the possibility to later introduce viscosity in your computations? Do you think you can address non-viscous problems that cannot be addressed by the methods based on BIE?
2) How do you technically impose the boundary conditions in your FD scheme?

Yeung & Ananthakrishnan: In principle all free-surface potential-flow problems that could be solved by finite-difference methods can also be tackled by boundary integral methods. The method presented here is based on field-discretization procedure and hence the problems arising from evaluation of the time-dependent Green function in certain boundary integral methods can be avoided and problems that require solution of Euler's equations can also be handled.
For implementation of boundary conditions and solution of field equation, please refer to Yeung and Wu (ref. [6] in the abstract).

Cointe: Your results are very encouraging in order to account for viscous effects in the nonlinear simulation of free surface flows. However, your time-stepping procedure is based on Longuet-Higgins' & Cokelet's method. I wonder if there is a straightforward extension of this method to deal with the Navier-Stokes equations for which you cannot use the Bernoulli equation and have to account for zero shear stress at the free surface.

Yeung & Ananthakrishnan: Using the Lagrangian kinematic conditions, the free surface can be advanced in time. However when solving the viscous free-surface flow problems, both the continuity of shear and normal stresses have to be satisfied on the free-surface. These conditions need to be implemented in a different form in the case of viscous flow but do not cause any apparent difficulties. Works in this direction are underway.

Pawlowski: Could you please tell us if your discretization scheme can deal with cases when the tip of the breaking wave touches the free surface?

Yeung & Ananthakrishnan: By mapping the physical domain onto a rectangular computational domain, it was not possible to advance the free surface any further than what is shown in the figure corresponding to a plunging breaker. However, we believe that by mapping the physical domain to a different type of computational domain at later times, it might be possible to advance the free surface further. When the tip of the breaking wave touches the free surface, the mapping would no longer be proper (i.e. one-to-one) and different formulation of the problem and mapping procedure are necessary.