# A discussion of the $m_j$-terms in the wave-current-body interaction problem

By

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In the linear theory describing ship advancing in waves (Ogilvie & Tuck(1969), Newman(1978)) and in the wave-current-body interaction problem (Zhao & Faltinsen(1988)), the body boundary condition for the unsteady potential due to the forced motions (in surge,sway,heave,roll,pitch and yaw) is given by

$$\partial\phi_j/\partial n = i\omega n_j + U m_j \quad j = 1,2,\ldots 6 \text{ on } S_B \tag{1}$$

Here $S_B$ is the mean oscillating position of the wetted body surface, $n_j$ and $m_j$ are defined as

$$[n_1, n_2, n_3] = \vec{n} \tag{2}$$
$$[n_4, n_5, n_6] = \vec{x} \times \vec{n} \tag{3}$$
$$[m_1, m_2, m_3] = -(\vec{n}\cdot\nabla)\nabla\phi_s \tag{4}$$
$$[m_4, m_5, m_6] = -(\vec{n}\cdot\nabla)(\vec{x}\times\nabla\phi_s) \tag{5}$$

where $\vec{n}$ is the normal vector of the body surface, $\vec{x}$ is the position vector, $\phi_s$ is the steady potential due to ship forward motion or current past the body.

The $m_j$-terms will often lead to difficulties in solving the boundary value problem. We will here discuss these difficulties.

Boundary-integral methods are often used to solve the boundary value problem described above. The boundary is then divided into panels. One possibility is to use plane panels with constant singularity density over each panel. Another possibility is to vary the singularity density over the curved panel. The first approximation is called a low order panel method, and the second one a high order panel method. The low order method is mostly used. In the following text we will give some simple examples to study the detail of the behaviour of the first and second order derivatives of the velocity potential at the body boundary by using the low order method. These are needed in the calculation of the $m_j$-terms.

By using Green's second identity, the potential in the flow can be represented by an integral around a closed boundary for two- or three-dimensional case

$$2\pi\phi_s = \int\int_S (logr\partial\phi_s/\partial n - \phi_s\partial logr/\partial n)ds \quad \text{for 2-D} \tag{6}$$

$$-4\pi\phi_s = \int\int_S (r^{-1}\partial\phi_s/\partial n - \phi_s\partial(1/r)/\partial n)ds \quad \text{for 3-D} \tag{7}$$

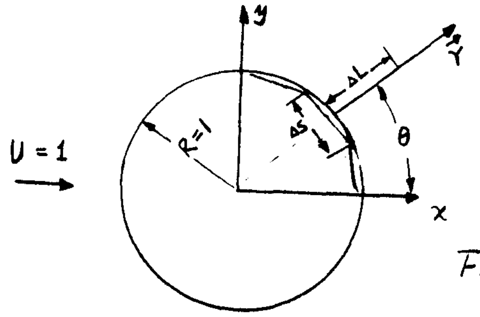


Fig. 1

If we know the potential $\phi_s$ and it's normal derivative $\partial\phi_s/\partial n$ around a closed boundary, we can obtain the potential at any point in the flow by eq.(6) and eq.(7). We will choose a simple case with uniform current past a two-dimensional circular cylinder with radius 1 in an infinite fluid domain.(see fig.1)

The potential due to the body is cos$\theta$/r and the normal derivative $\partial\phi_s/\partial n$ is -cos$\theta/r^2$ at the body boundary. We can then divide the boundary into line elements and for each element assume $\phi_s$ and $\partial\phi_s/\partial n$ are constant with values which are equal to the correct values at the mid-point of the element. The potential and its derivatives outside the body boundary can be obtained by eq.(6) and derivatives of eq.(6). Fig.2 shows the results of $\phi_s$ , $\partial\phi_s/\partial n$, $r^{-1}\partial\phi_s/\partial\theta$, $\partial^2\phi_s/\partial n^2$ as a function of the distance along the normal vector to the body boundary at the mid-point of the element. The results are for $\theta = 45^0$ (see fig.1). The effect of different number of elements NB is investigated. The solid line is the analytical solution, the triangle is the numerical results of $\phi_s$ , the square sign corresponds to $\partial\phi_s/\partial r$,the cross sign corresponds to $r^{-1}\partial\phi_s/\partial\theta$ and the plus sign is the second derivative $\partial^2\phi_s/\partial r^2$. The horizontal axis is the ratio between the distance $\Delta$l from the boundary and the length $\Delta$s of the elements. The results show that we get convergence and correct results of $\phi_s$ and $\partial\phi_s/\partial r$ at the boundary. However, for $r^{-1}\partial\phi_s/\partial\theta$ and $\partial^2\phi_s/\partial r^2$ we cannot obtain correct results at the boundary. The reason is that we are not integrating with correct curvature and with correct variation of $\phi_s$ and $\partial\phi_s/\partial r$ over the elements. If we integrate with correct variation of $\phi_s$ and $\partial\phi_s/\partial r$ and along the exact boundary of the closest elements , we can obtain correct results. For practical problems this is difficult to do. From fig.2 we can see that $r^{-1}\partial\phi_s/\partial\theta$ , $\partial^2\phi_s/\partial r^2$ are satisfactorily estimated at a distance of O($\Delta$s) along the normal vector of the element. That means we may use an extrapolation method to calculate the velocity along the body and the second order derivatives. After some tests with a circular cylinder , a sphere and an ellipsoid we found that the velocity along the body , the second order derivatives and the $m_j$-terms are in good agreement with the analytical solutions. The other way to integrate the term $m_j logr(or1/r)$ over the body boundary is to apply the formula given by Ogilvie and Tuck(1969).

$$\int\int_{S_B} m_j logr(or1/r)ds = -\int\int_{S_B} \nabla\phi_s\nabla logr(or1/r)n_j ds \tag{8}$$

This formula is valid for a body without sharp corners , wall-sided at the free surface and when $\phi_s$ satisfies the rigid free surface condition. From numerical point of view this formula is more simple to calculate because it only includes first order derivatives of the steady potential. It is expected to give more accurate numerical results than by direct integration of the $m_j$-terms.

An alternative way to solve the $\phi_j$ problem will be outlined in the following text. In this procedure the numerical difficulties with the $m_j$-terms are taken care of.

We divide the velocity potential $\phi_j$ into two parts

$$\phi_j = \phi_j^a + \phi_j^b \tag{9}$$

where $\phi_j^a$ and $\phi_j^b$ satisfy the following body boundary conditions.

$$\partial\phi_j^a/\partial n = Um_j \tag{10}$$

and

$$\partial\phi_j^b/\partial n = i\omega n_j \tag{11}$$

The following solutions of $\phi_j^a$ satisfy the body boundary conditions and Laplace equation.

$$\phi_1^a = -\partial\phi_s/\partial x \tag{12}$$
$$\phi_2^a = -\partial\phi_s/\partial y \tag{13}$$
$$\phi_3^a = -\partial\phi_s/\partial z \tag{14}$$
$$\phi_4^a = -y\partial\phi_s/\partial z + z\partial\phi_s/\partial y \tag{15}$$
$$\phi_5^a = x\partial\phi_s/\partial z - z\partial\phi_s/\partial x \tag{16}$$
$$\phi_6^a = -x\partial\phi_s/\partial y + y\partial\phi_s/\partial x \tag{17}$$

By using Green's second identity we obtain the following expressions for $\phi_j^a$ and $\phi_j$

$$4\pi\phi_j^a = \int\int_{S=S_B+S_F^I+S_{INF}^I} [-r^{-1}\partial\phi_j^a/\partial n + \phi_j^a\partial(1/r)/\partial n]ds \tag{18}$$
$$4\pi\phi_j = \int\int_{S=S_B+S_F+S_{INF}} [-r^{-1}\partial\phi_j/\partial n + \phi_j\partial(1/r)/\partial n]ds \tag{19}$$

The integration surfaces are closed (see fig.3). $S_F^I$ is part of the free surface and does not need to coincide with $S_F$. By subtracting these two equation we obtain

$$\begin{aligned} 4\pi(\phi_j - \phi_j^a) &= \int\int_{S_B} [-r^{-1}\partial(\phi_j - \phi_j^a)/\partial n + (\phi_j - \phi_j^a)\partial(1/r)/\partial n]ds \\ &+ \int\int_{S_F+S_{INF}} [-r^{-1}\partial\phi_j/\partial n + \phi_j\partial(1/r)/\partial n]ds \\ &- \int\int_{S_F^I+S_{INF}^I} [-r^{-1}\partial\phi_j^a/\partial n + \phi_j^a\partial(1/r)/\partial n]ds \end{aligned} \tag{20}$$

The last integral is known quantity. The unknowns are $\phi_j - \phi_j^a$ on the body and $\phi_j$ on $S_F$,$S_{INF}$ and so. By writing the integral over $S_B$ like it is shown in last equation the integrand of the integral over $S_B$ is integrable. This procedure is also valid when ship motions at forward speed is evaluated. More detail about solving this problem and discussions are given by Zhao & Faltinsen (1989).

References:

Ogilvie,T.F. ,Tuck,E.O.(1969) "A rational strip theory of ship motion: Part I" Report no.013. The department of navel architecture and main engineering, the university of Michigan, college of engineering.

Newman,J.N.(1978) :"The theory of ship motions", advances in applied mechanics, volume 18.

Zhao,R., Faltinsen,O.M.(1988) "Interaction between waves and current on a two-dimensional body in the free surface" Applied ocean research, vol.10, no.2

Zhao,R., Faltinsen,O.M.(1989) "Interaction between waves and current on marine structures" 5th international conference on numerical ship hydrodynamics ,Hiroshima.

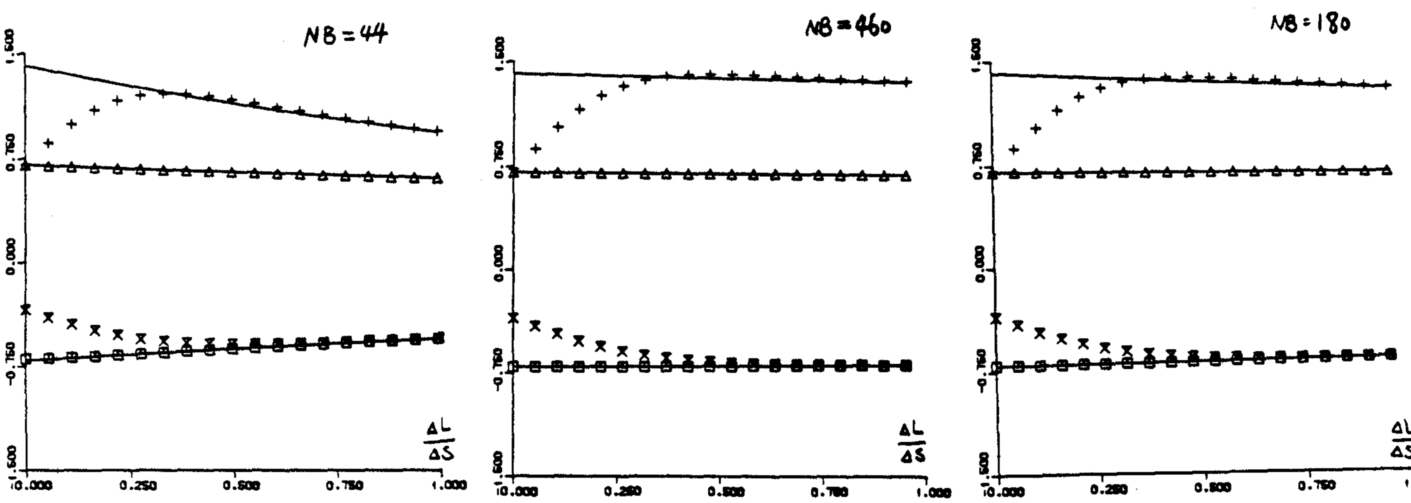


Fig.2

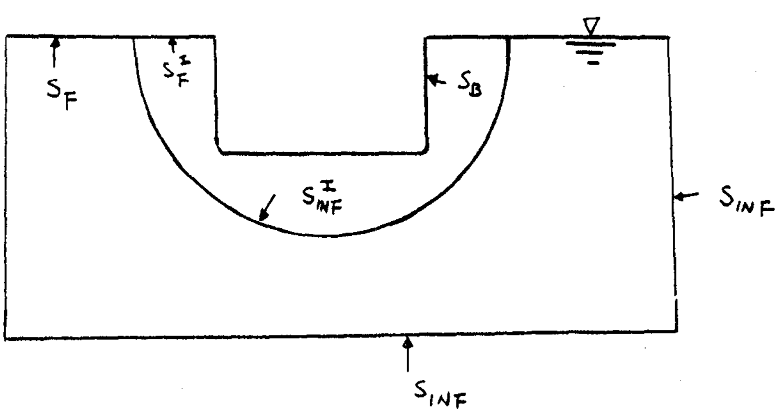


Fig. 3

## DISCUSSION

Newman: The corner-flow sigularity discussed here also comes up in the second-order (zero-speed) solution for an oscillating body.

Zhao & Faltinsen: The same procedure can also be used to solve the second order potential problem (U=0). The body condition for the second order potential is given by $\partial\varphi_2/\partial n = -\partial(F(\varphi_1))/\partial n$, where $F(\varphi_1)$ satisfies Laplace equation and includes the first order derivatives of $\varphi_1$. ($\varphi_1$ - first order potential, $\varphi_2$ - second order potential). We can then assume $\varphi_2=\varphi_2{}^a+\varphi_2{}^b$ where $\varphi_2{}^a$ is equal $-F(\varphi_1)$ which satisfies the body boundary condition.

Kleinman: Do the singularities in $m_j$ result from differentiation of lower order singularities in the potential (e.g. eqn (4) in abstract), and if so, have you tried separating out the singular behaviour by adding a singular element (instead of piecewise constants near the singularity) which has proven useful in other problems using boundary elements to solve integral equations with singular solutions?

Zhao & Faltinsen: For the body with sharp corners it is not possible to apply this method since the singularity is not integrable in the integral equation. If one introduces a bilge radius R and let $R \to 0$, it may be possible to apply the method.

Tuck: Is it possible that the bounded integral on the right of eq. (8) is correct, but that (for sharp corners) the integral on the left of eq. (8) is not the correct one?

Zhao & Faltinsen: If we introduce a bilge radius R and let $R \to 0$, the integrals both on the left and on the right of eq. (8) will be correct.