A Rankine Source Approach to Forward Speed Diffraction Problems

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Rankine source methods (RSM) have been successfully applied to the steady wave problem (wave resistance problem) by several authors. Two problems have long impeded their application to free-surface flows:

1. The radiation condition:
In the steady wave problem, waves will propagate only downstream; i.e. far ahead of the ship no waves may appear. In time-harmonic problems for $\tau = U\omega/g > 0.25$, radiated or diffracted waves will also propagate only downstream.

2. The open-boundary condition:
Only a limited area of the free surface can be discretized. Waves must pass through the outer boundary of this area without significant reflection.

Both conditions must be fulfilled numerically in RSM. *Nakos and Sclavounos (1989)* used successfully a quadratic-spline scheme for steady and time-harmonic ($\tau > 0.25$) problems. *Jensen (1988)* showed the effectiveness of another numerical technique for the steady case: By shifting the Rankine sources above the free surface versus the collocation points on the surface, both radiation and open-boundary condition are fulfilled. His trial computations for a submerged dipole show excellent agreement with analytical results. *Bertram (1990)* investigated in analogy the applicability of Jensen's shifting technique for time-harmonic problems.

The first test case is a submerged point source moving steadily in an ideal fluid at a distance d under a free surface. The source strength pulsates with unit amplitude and reduced frequency $\tau = U\omega/g$. The problem is governed by Laplace's equation subject to the boundary conditions: (a) no water flows through the free surface, (b) constant (atmospheric) pressure at the free surface, (c) for $\tau > 0.25$ waves do not propagate ahead.

In a coordinate system moving with the source, the total potential Φ is approximated by the sum of a uniform-flow potential and a time-harmonic potential φ. φ includes the pulsating submerged source and a time-harmonic free-surface correction. Linearizing the usual free-surface condition with respect to ϕ and φ gives:

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2 \varphi - g\frac{\partial \varphi}{\partial z} = 0 \qquad \text{at } z = 0 \qquad (1)$$

The surface is discretized into a regular grid of $n_x \cdot n_y$ collocation points. Above the free surface a corresponding number of Rankine point sources is located. All sources are shifted uniformly in x-direction and gradually in y-direction. In numerical experiments *Bertram (1990)* determined suitable rules for shifting sources for various τ, largely confirming earlier results of *Jensen (1988)* for $\tau = 0$, $U > 0$.

For $\tau = 1$ and $d = 0.45U^2/g$ *Nakos and Sclavounos (1989)* compared their RSM results with an analytical solution. The analytical solution does not appear to be very accurate outside of the sector of waves due to the particular integral representation of the solution used in the code (Nakos, personal communication). Fig. 1. shows that the shifting technique also gives very good agreement with the analytical solution in the wave sector. Compared are contour lines of the real part of the potential.

Using the same shifting technique, the diffraction problem for a submerged ellipsoid in head seas was solved. The total potential Φ is divided as follows:

$$\Phi = \phi^s + Re((\phi_0 + \varphi)e^{i\omega t}) = \phi^s + Re(\phi^i e^{i\omega t}) \qquad (2)$$

ϕ^s is the stationary potential fulfilling the nonlinear free-surface condition as determined by *Jensen (1988)*, ϕ_0 the incident-wave potential and φ the diffraction potential. ω is the frequency of encounter. The free-surface condition is linearized with respect to ϕ^i:

$$(-\omega^2 + Bi\omega)\phi^i + ((2i\omega + B)\nabla\phi^s + \vec{a} + \vec{a}^g)\nabla\phi^i + \nabla\phi^s(\nabla\phi^s\nabla)\nabla\phi^i = 0 \qquad \text{at } z = \zeta^s \qquad (3)$$

where ζ^s is the stationary wave elevation, $\vec{a} = (\nabla\phi^s\nabla)\nabla\phi^s$ the particle acceleration in the stationary flow, $\vec{a}^g = \vec{a} - (0,0,g)$ and $B = -(\vec{a}^g\nabla\phi^s)_z/a_3^g$, where indices z and 3 mean partial derivative resp. vertical component. If the stationary flow is approximated by uniform flow, this boundary condition would reduce to the familiar condition:

$$-\omega^2\phi^i - 2i\omega U\phi^i_x - g\phi^i_z + U^2\phi^i_{xx} = 0 \qquad \text{at } z = 0 \qquad (4)$$

On the body surface the Neumann condition gives ($\vec{n}$ normal unit vector):

$$\vec{n}\nabla\phi^i = 0 \qquad (5)$$

Both body and free-surface condition are approximately satisfied by distributing Rankine sources on these surfaces of the type used by *Jensen (1988)* for the stationary problem. First the stationary problem, then the diffraction problem is solved using RSM. Test results for an ellipsoid with $L/B = 5$, $d/B = 0.75$ (d depth of body center), $F_N = 0.2$ agree well with experiments by *Ohkusu and Iwashita (1989)*, Fig. 2. Only for $\lambda/L = 0.4$ (λ wave length of incident wave), the experimental results could not be reproduced. 172 collocation points on the body and about 900 at the free surface were used. The grid on the free surface was adjusted to wave length, using finer spacing for smaller λ.

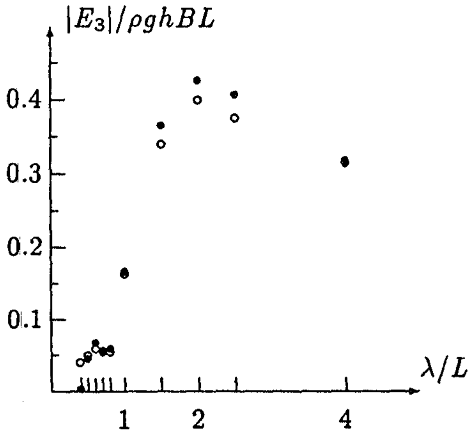

Fig. 2. Vertical wave force for a submerged ellipsoid at $F_N = 0.2$,
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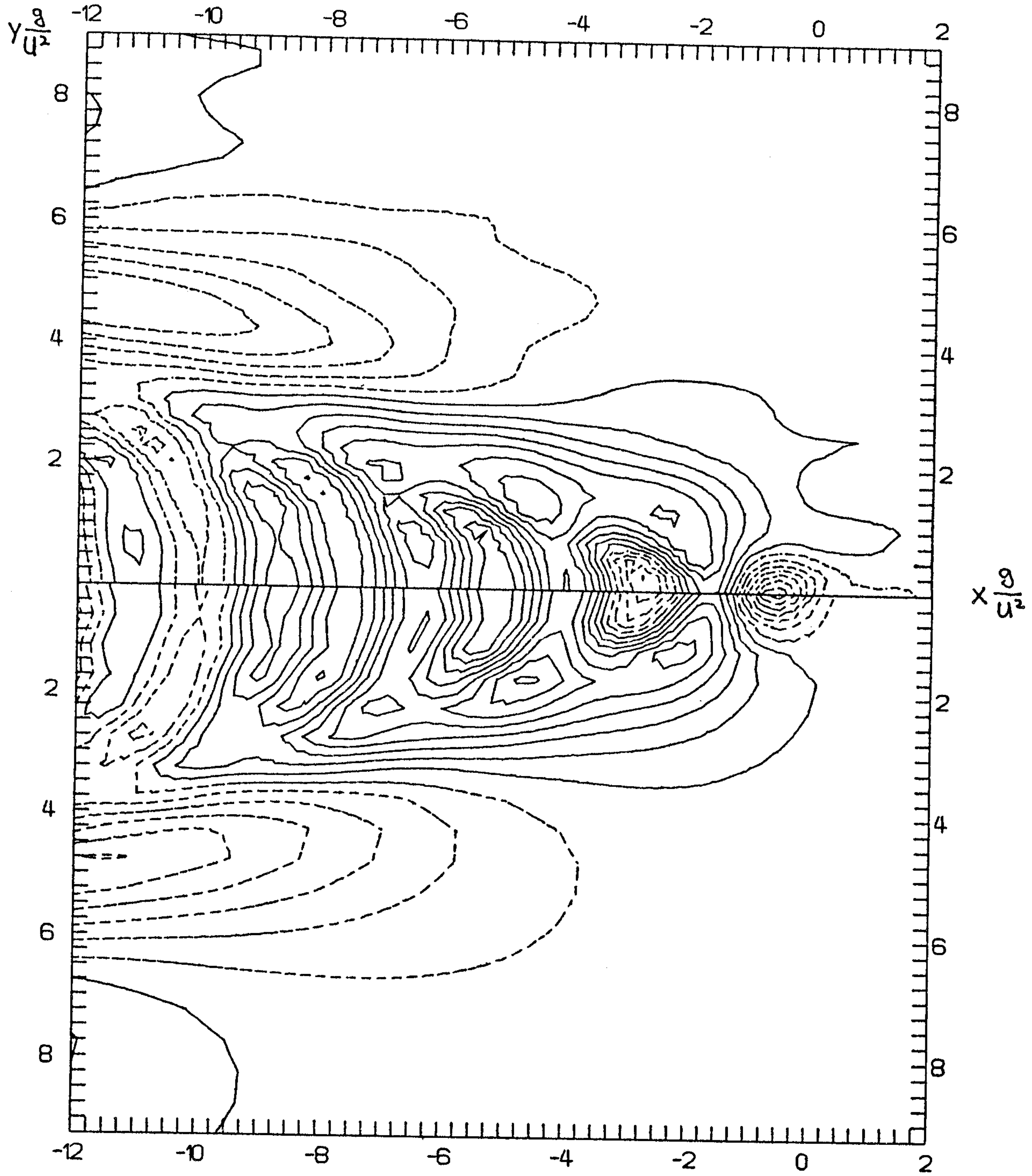

Fig. 1. Contour lines of the real part of the velocity potential on the mean free surface due to a submerged time-harmonic source with depth of submergence $d = 0.45U^2/g$. Numerical solution (bottom half) and analytical solution of Nakos (top half).