# HIGH FREQUENCY INTERACTIONS BETWEEN TLP LEGS

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The problem that we consider here is the calculation of (sum-frequency) second-order loads on TLP hulls, including the (dominant) contribution of the second-order diffraction potential through the so-called free surface integral [1]. The final aim being to predict the loading at the heave roll and pitch resonant frequencies.

We have first used a purely numerical model based on a source distribution technique [5]. As the wave frequencies are increased memory space and computer cost quickly become prohibitive since the meshings of both the hull and the free surface must be sufficiently refined. Moreover the numerical convergence of the obtained results remains to be assessed in some way.

Since at small wave periods only the top part of the columns can be reached by the first-order wave field, it appears that analytical methods, such as the one proposed by Linton and Evans at the last workshop [2], can be used to solve the first-order diffraction problem, the pontoons being neglected and the columns assumed to be infinitely deep. Such a geometric simplification may not be applied to derive the second-order loading since the second-order diffraction potential is known to penetrate deeper the water column, but it may nevertheless be used to determine some of the components of the second-order loading, and the right hand side of the free surface equation. Last, solving the complete second-order problem for infinitely deep columns provides a valuable benchmark to check the convergence of the purely numerical approach.

Along the same lines it may also be questioned to what extent interaction effects between the columns play a role as far as the second-order loads are concerned. For instance Kim and Yue [3] take the first and second-order loading on a 4 column structure to be equal to 4 times the loading on one isolated column.

A better approximation (at least at first-order) is to include the effect of phasing of the **incident** waves on the 4 columns. This results in corrective factors equal to $\cos k_0 D/2$ for the first-order loads and $\cos k_0 D$ for the double frequency second-order loads, where $D$ is the column to column spacing and $k_0$ the wave number.

According to this approach the drift force on a 4 column structure would still be equal to 4 times the drift force on an isolated one, which is known not to be correct. Nielsen [4], using the far field method, calculates the drift force by just correctly adding up, algebraically, the diffracted wave fields due to each of the 4 columns in the absence of the other 3. Even though it cannot be said that interaction effects are thus accounted for in any way, this approach seems to work quite well for the drift force. It may be wondered whether it would work as well for the double frequency loading.

These considerations have motivated the present study where we extend Linton and Evans' method to the calculation of the double frequency second-order loads, and where we compare the results to those obtained with simplified methods, such as Kim and Yue's or Nielsen's.

In the following we recall Linton and Evans' method for the diffraction problem (the radiation problem – which we need to solve to compute the free surface integral – being solved along the same lines).

The $(r,\theta)$ polar coordinate system and $(r_j,\theta_j)$ $j=1,\cdots,4$ subsystems are used. The first-order potentials are written as:

$$\Phi^{(1)}(r,\theta,z,t) = -\frac{ag}{\omega}\frac{\cosh k_0(z+H)}{\cosh k_0 H}\cdot\Re\{[\phi_I(r,\theta)+\phi_D(r,\theta)]\cdot e^{-i\omega t}\} \tag{1}$$

with the incident and diffraction potentials defined by:

$$\phi_I = I_{1j} \cdot e^{ik_0 r_j \cos(\theta_j - \beta)} \qquad \phi_D = \sum_{j=1}^{4} I_{1j}\phi_{Dj} = I_{1j}\phi_{Dj} + \sum_{k=1,k\neq j}^{4} I_{1k}\phi_{Dkj} \tag{2}$$

where the potential diffracted by the cylinder $j$:

$$\phi_{Dj} = \sum_{n=-\infty}^{\infty} A_n^j \cdot e^{in(\pi/2+\beta)} \frac{\mathrm{J}_n'(k_0 R_j)}{\mathrm{H}_n'(k_0 R_j)} \mathrm{H}_n(k_0 r_j) e^{-in\theta_j} \tag{3}$$

and potentials in the vicinity of cylinder $j$ diffracted by the cylinder $k$

$$\phi_{Dkj} = \sum_{n=-\infty}^{\infty} \{ \sum_{m=-\infty}^{\infty} A_m^k e^{im(\pi/2+\beta)} \frac{\mathrm{J}_m'(k_0 R_k)}{\mathrm{H}_m'(k_0 R_k)} \mathrm{H}_{m-n}(k_0 R_{kj}) e^{-i(m-n)\alpha_{kj}} \} \mathrm{J}_n(k_0 r_j) e^{-in\theta_j} \tag{4}$$

The coefficients $A_n^j$ in (3) and (4) are determined by the linear system:

$$A_n^j + \sum_{k=1,k\neq j}^{4} I_{1kj} \sum_{m=-N}^{N} A_m^k \frac{\mathrm{J}_m'(k_0 R_k)}{\mathrm{H}_m'(k_0 R_k)} \mathrm{H}_{m-n}(k_0 R_{kj}) e^{i(m-n)(\pi/2+\beta-\alpha_{kj})} = -1 \tag{5}$$

where the phase factors are written as:

$$I_{1j} = e^{ik_0(x_{oj}\cos\beta + y_{oj}\sin\beta)} \qquad I_{1kj} = e^{ik_0[(x_{ok}-x_{oj})\cos\beta + (y_{ok}-y_{oj})\sin\beta]} \tag{6}$$

Note that if the coefficients are taken as $A_n^j = -1$ ($j = 1, \cdots, 4$ $\;n = -N, \cdots, 0, \cdots, N$) instead of being determined by equations (5), expression (2) represents the superposition of 4 single-cylinder solutions (i.e. Nielsen's approach).

The exciting forces are obtained by direct integration of the hydrodynamic pressure on the cylinders. The second-order double frequency forces consist of one component which corresponds to the quadratic term in Bernoulli's equation plus a corrective term on the waterline, and another component which depends on the second-order diffraction potential. The first component is easily obtained. The second one is transformed into the free surface integral :

$$F_{22}(2\omega) = 2i\omega\frac{\rho}{g} \int\int_{z=0} a_D \cdot \phi_R(2\omega) dS \tag{7}$$

where $a_D$ is the right-hand side of the free-surface equation and $\phi_R$ is the radiation potential at the double frequency $2\omega$. The free surface integral is evaluated numerically in an inner domain to some radial distance $R_d$ and analytically from that distance to infinity.

According to Kim and Yue's approach one just multiplies by 4 the results corresponding to one isolated cylinder :

$$F_1|_{4cyl} = 4 * F_1|_{1cyl} \qquad F_{21}|_{4cyl} = 4 * F_{21}|_{1cyl} \qquad F_{22}|_{4cyl} = 4 * F_{22}|_{1cyl} \tag{8}$$

($F_{21}$ refers to the first component of the second order loads, and $F_{22}$ to the free surface integral).

In the second approach one only takes account of the phase differences in the **incident** wave field :

$$F_1|_{4cyl} = \sum_{j=1}^{4} I_{1j} \cdot F_{1j}|_{1cyl} = F_1|_{1cyl} \cdot 4\cos k_0 D/2 \tag{9}$$

$$F_{21}|_{4cyl} = \sum_{j=1}^{4} I_{2j} \cdot F_{21j}|_{1cyl} = F_{21}|_{1cyl} \cdot 4\cos k_0 D \qquad F_{22}|_{4cyl} = \sum_{j=1}^{4} I_{2j} \cdot F_{22j}|_{1cyl} = F_{22}|_{1cyl} \cdot 4\cos k_0 D \tag{10}$$

where D is the horizontal distance between cylinders.

Last, Nielsen's method yields the following :

$$F_1|_{4cyl} = \sum_{j=1}^{4} I_{1j} \cdot F_{1j}|_{1cyl} + \sum_{j=1}^{4} \sum_{k=1,k\neq j}^{4} I_{1k} \cdot F_{1kj}|_{1cyl} \tag{11}$$

$$F_{21}|_{4cyl} = \sum_{j=1}^{4} I_{2j} \cdot F_{21j}|_{1cyl} + \sum_{j=1}^{4} \sum_{k=1,k\neq j}^{4} (I_{2k} \cdot F_{21kj}|_{1cyl} + I_{1j} I_{1k} \cdot F_{21jkj}|_{1cyl}) \tag{12}$$

where the phase factors $I_{2j}$ are defined by:

$$I_{2j} = e^{2ik_0(x_{oj}\cos\beta + y_{oj}\sin\beta)} \tag{13}$$

In the numerical calculations the column to column spacing $D$ has been taken equal to 8 times the column radius, and the draft equal to 4 times the radius. Figure 1 shows the modulus of the first-order diffraction load, according to each of the 4 methods. A fair agreement is obtained between methods 2 (incident phasing), 3 (Nielsen) and 4 (Linton and Evans).

Figure 2 shows the drift force, where it can be seen that Nielsen's method is appropriate only at low values of the non-dimensional wave number (still the cases of practical interest).

Figure 3 shows the modulus of the first component of the double frequency second-order force. Again a reasonable agreement is offered by methods 2, 3 and 4. Figure 4 shows the modulus of the second component according to methods 1, 2 and 4 (Nielsen' s method results are missing), which appears to be around 3 times larger than the first component. Method 4 predicts results appreciably larger than those provided by the approximate methods. When both components are added up (Figure 5) this trend becomes even more visible, because of cancellation effects between both components as predicted by the approximate methods. It may therefore be concluded that interaction effects do play a dominant role for the double frequency loading.

**References:**


[1] **B. Molin** 1979 "Second order diffraction loads upon three-dimensional bodies" Applied Ocean Reseach. Vol:1 No:4 pp197-202

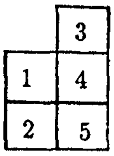
[2] **C.M. Linton** and **D.V. Evans** 1989 "The scattering of waves by vertical cylinders" The Fifth International Workshop on Water Waves and Floating Bodies. pp133-137.

[3] **M.H. Kim** and **Dick K.P. Yue** 1988 "The Nonlinear Sum-Frequency Wave Excitation and Response of a Tension-Leg Platform" Proceeding of BOSS. pp687-703.

[4] **F.G. Nielsen** 1988 "An approximate method to compute the drift force on an array of vertical cylinders" Applied Ocean Research, Vol:10 No:1 pp35-42.

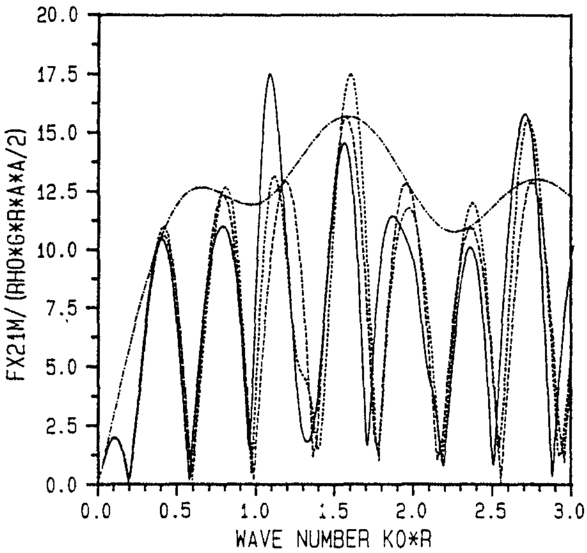
[5] **X.B. Chen** 1988 "Etude des réponses du second-ordre d'une structure soumise à une houle aléatoire" Thèse de Doctorat de l'ENSM. pp1-225.

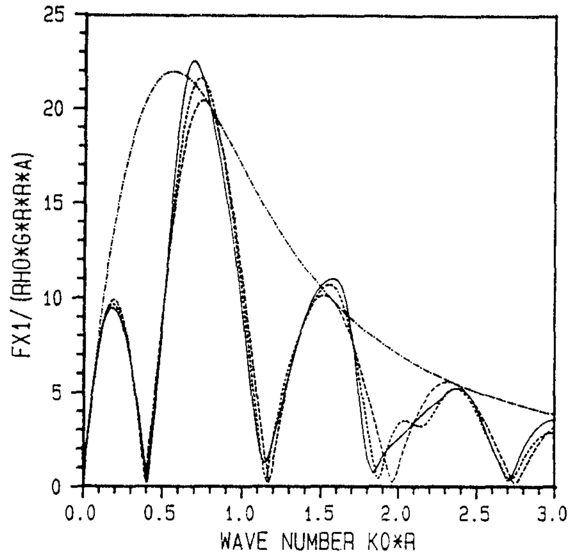
**Figures :**

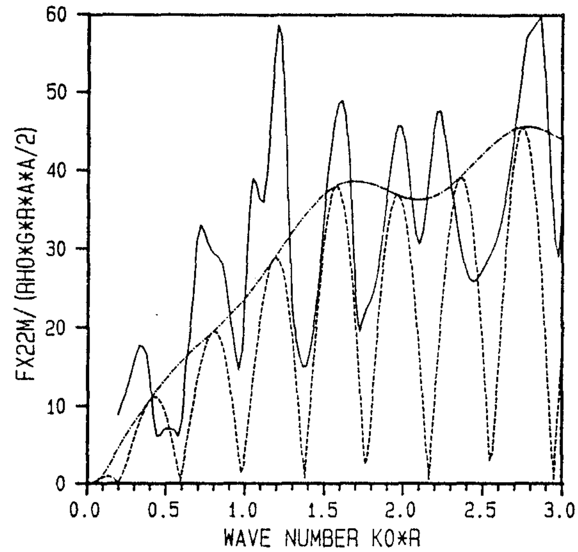


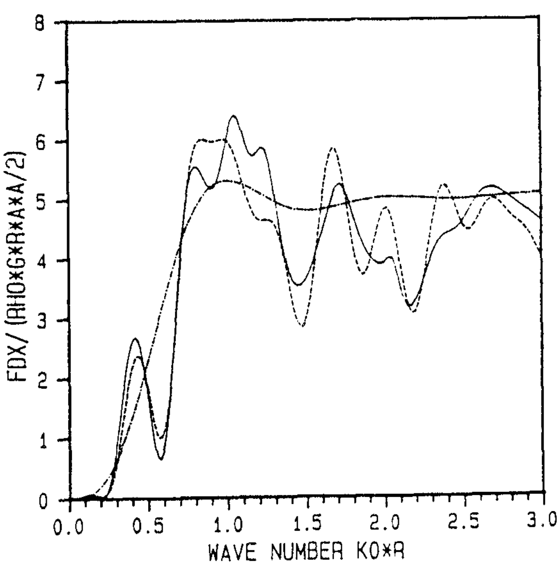

**Legends:**

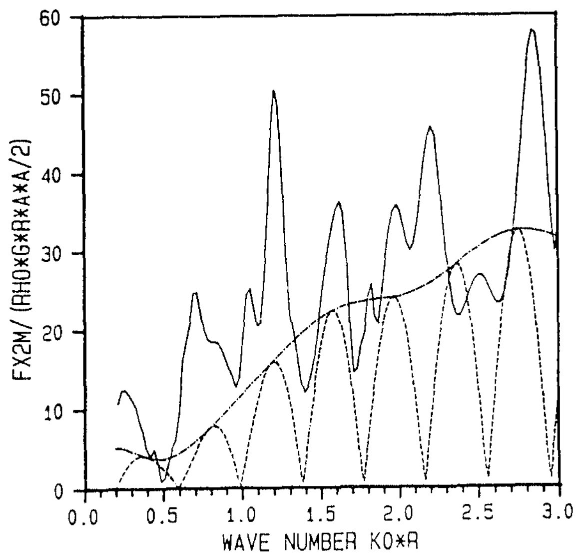
- –·–·– Kim and Yue's approach
- - - - - Incident phasing
- ······ Nielsen's method
- —— Linton and Evans' method



## DISCUSSION

**Rainey**: To me, this is a very impressive piece of work, because it improves our insight into why TLP's behave as they do. Perhaps the key to this success is in the use of a range of different approximations, and the careful comparison of the different results.

I hope that my own 'wavy lid' approximation [*J. Fluid Mech.* **204** (1989) 295–324] might one day help this process on another TLP problem, which is nonlinear wave loads in 'survival' conditions. Here, the waves are, of course, much larger than the resonant conditions considered above, so the important nonlinearities may be quite different. The effects of the quadratic Bernoulli pressure term at the cylinder ends, and of the structural motion, would, I believe, be handled very neatly by my method.