K.EGGERS: On The Breakdown Of Ship Wave Ray Tracing Near The Bow.

Over a period of more than 10 years, attempts have been made to simulate the ship wave pattern by application of ray tracing methods as developed in geometrical optics.This was motivated from the fact that such model is certainly adequate far away, where rays degenerate to straight lines, and through the plausible argument that over a slightly curved flow in the far field the wave patter geometry should be determined through local features (i.e. the orientation and the magnitude of flow) rather than a parallel flow relative to the ship as observed at great distance. It turns out then that the direction of rays can be defined as that of the resultant of the basic flow with a group velocity vector.

With the dispersion relation between the components of the wave number vector depending then on the space coordinates, this vector can no longer be invariant along a ray; the construction of rays thus becomes indeterminate, unless we specify in addition some conservation principle. For rays thus defined, the problem becomes equivalent to determining characteristics for a certain partial differential equation, which can be formulated and has regular solutions in the entire part of the undisturbed free surface where the basic flow is regular. The ray geometry will then come out independent from the Froude number.

Still it is not evident that solutions to this problem are pertinent for simulating the near field wave pattern, even for the case of a submerged body, where the basic flow is only slowly varying there. Note that ray tracing upstream and downstream are mutually inverse operations. Under the linearised free surface condition $U^2\phi_{xx} + g\phi_z = 0$, we know that the wave pattern admits a formal double Fourier integral representation, which degenerates to a finite number of wave components only after twofold application of asymptotic analysis, and the near field can not be uniquely determined from such far field information, in particular if no *upstream* radiation condition is specified. This should equally hold under the "slow ship free surface condition" $u^2\phi_{xx} + 2uv\phi_{xy} + v^2\phi_{yy} + g\phi_z = 0$ underlying the ray tracing approach. Note that Lighthill [1] found that this problem is of parabolic nature (if effects of finite amplitude are disregarded; otherwise hyperbolic for transverse, elliptic for divergent waves).

The relevant analysis has been formulated (A) by J. Keller [2], who even concluded that for his class of "streamlined" ships rays *must* originate from the double body stagnation points only. Whereas Yim [3] seems to have successfully calculated rays under use of this approach, Brandsma [4] found that even through "backshooting" no rays associated with *transverse* waves (as referred to the basic flow) emanating from the bow stagnation point can be found. This is not necessarily in conflict with the calculations of Yim, who could not start his rays directly at the stagnation point, as the wave length tends to zero there and no calculations can be performed.

In a recent analysis, we evaluated analytical expressions for the flow around prismatic bi-circular vertical struts and showed that the rate of change of the wave front angle along a ray (and hence of the ray tangent via the dispersion relation) tends to infinity as the inverse distance from the stagnation point, unless the ray is starting tangentially to the water line. If so, the *finite* curvature depends of the direction of approach, being zero when approached in streamline direction, i.e. tangentially to the ship hull.Thus even if the ray should split up to a continuous bundle, it can carry only transverse waves in its initial stage. This insight may shed some light on the numerical difficulties experienced by Brandsma.

In an attempt to make ray tracing pertinent even in this domain, we modified Keller's approach taking $(A)'$ account of surface tension effects. Then we find some zone around the stagnation point where no stationary waves can exist; at its boundary, only one wave length is admitted corresponding to the minimum phase velocity under transition from gravity-dominated to capilarity-dominated waves. But even for rays originating at some finite outward distance from this boundary, we found that after a short stage of growth the wave length showed decay along the ray until the minimum of gravity-dominated waves was approached, and the ray tracing had to be terminated.

The above situation is not essentially changed if we base our analysis on an alternative free surface condition (A^+) as advocated by the author [5] and derived independently from new arguments by van Gemert [6]. It is the author's conclusion that ray theory, having its origin in consideration of wave packets under application of *asymptotic* tools (such as the principle of stationary phase), is not pertinent for the near field of a ship. We should perform a complete numerical evaluation of some solution of the slow ship free surface condition to assess the limitations of the ray approximation!

For sake of record it should be mentioned that the results of our investigations have recently been presented at the International Symposium on Numerical Ship Hydrodynamics in Hiroshima. However, no substantial discussion of above controversial issue could be provoked so far.

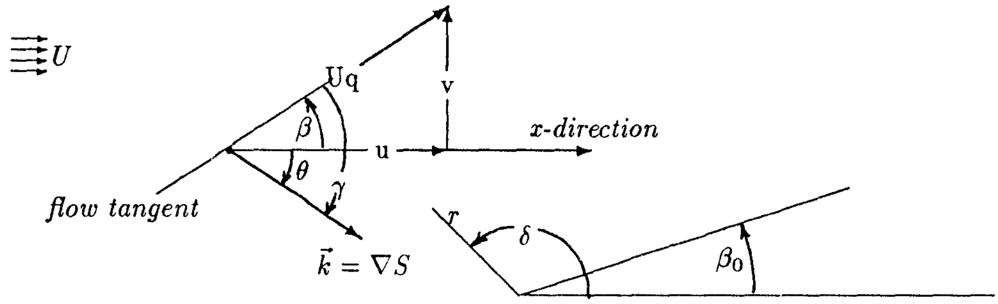

Fig.1 Sketch for flow angle β and wave angles θ and γ (both shown with negative values, typical for the starboard side of the bow).

Let us present the situation of ray tracing near a corner for the canonical case of a wedge of infinite extent, where the basic flow is 2-D, so that complex analysis can be used.
We introduce

$$Z \doteq x + iy = r \cdot e^{i\delta} \quad V \doteq u - iv = Uq \cdot e^{-i\beta}, \tag{1}$$

$$K \doteq k_1 - ik_2 = k \cdot e^{-i\theta} \tag{2}$$

$$P \doteq (g\zeta_r)_x - i(g\zeta_r)_y = -(dV/dZ)V^* \tag{3}$$

Then P stands for some gradient of the double body flow pressure.[1] The ray equations may be written as

$$\frac{dZ^*}{d\tau} = \frac{2}{kc}(V - \frac{c_g}{k}K) = \frac{2}{kc}c_{at} \cdot e^{-i(\alpha+\beta)} \tag{4}$$

With ds as differential of the arclength along the ray, this implies that $d\tau/ds = 1/\,|\,dZ/d\tau| = kc/2c_{at}$, where c_{at} stands for the action transport velocity along the ray, so that we can write the ray equation as

$$\frac{1}{K}\frac{dK}{ds} = \left(\frac{1}{k}\frac{dk}{d\tau} - i\frac{d\theta}{d\tau}\right) \cdot \frac{d\tau}{ds} = -\frac{e^{i\theta}}{c_{at}} \cdot \frac{dV}{dZ}\left(\diamond\frac{V^*}{2c} + e^{i\theta}\right) \tag{5}$$

Here $\diamond$ means 1 for (A^+) and $(A^+)'$, it means 0 for (A) and $(A)'$. The flow in the vicinity of a stagnation point due to a corner is basically the flow near a corner between infinite planes as decribed by Milne-Thomson [10], we have

$$V \approx Q \cdot e^{(i\pi\beta_0/(\pi-\beta_0)} \cdot Z^{\beta_0/(\pi-\beta_0)} \tag{6}$$

[1]Tulin [8] considered a quantity related to $|\,P\,|$ as a disturbance parameter and came to the vexing conclusion that ray theory does not apply for bow entrance angles $\beta_0 \leq \pi/3$ as otherwise P is not bounded. On the other hand, Maruo [9] disclaimed the validity of ray theory for $\beta_0 \geq \pi/3$ due to divergence of an integral representing the phase.

where Q is a real constant; this means that the range

$$\beta_0 \leq \delta \leq \pi$$

for the polar angle δ is mapped on the range

$$\beta_0 \geq \beta(\delta) = \beta_0(\pi - \delta)/(\pi - \beta_0) \geq 0$$

for the flow angle. In the special case of a bi-circular strut of opening angle $2\beta_0$ and length L, under parallel flow of strength U, we have

$$Q = U\frac{\pi}{\pi - \beta_0} \cdot L^{\frac{-\beta_0}{\pi-\beta_0}} \tag{7}$$

so that

$$q = q(r) = \pi/(\pi - \beta_0) \cdot (r/L)^{\beta_0/(\pi-\beta_0)}$$

and hence

$$\frac{dV}{dZ} = \frac{\beta_0}{\pi - \beta_0} \cdot \frac{V}{Z} \tag{8}$$

$$P = -\frac{\beta_0}{\pi - \beta_0} \cdot \frac{1}{Z} V \cdot V^{\star} \tag{9}$$

Then the rate of change of θ, β, and k along the ray is found from

$$\frac{1}{K}\frac{dK}{ds} = -\frac{e^{i\theta}}{c_{at}} \cdot \frac{\beta_0}{\pi - \beta_o} \cdot \frac{V}{Z}\left(\diamond\frac{V^{\star}}{2c} + e^{i\theta}\right) \tag{10}$$

Observing (34) and (5), separating real and imaginary part in (43), we find

$$\frac{d\theta}{ds} = -\frac{1}{r}\frac{c}{c_{at}} \cdot \frac{\beta_0}{\pi - \beta_0}\left(\diamond\frac{\sin(\theta - \delta)}{2\cos\gamma} + \sin(\gamma + \theta - \delta)\right) \tag{11}$$

$$\frac{d\beta}{ds} = -Im\frac{dV}{dZ}\frac{1}{V}\frac{dZ}{ds} = -Im\frac{\beta_0}{\pi - \beta_0}\frac{1}{Z} \cdot e^{i(\alpha+\beta)} = -\frac{1}{r} \cdot \frac{\beta_0}{\pi - \beta_0} \cdot \sin(\alpha + \beta - \delta) \tag{12}$$

and thus

$$\frac{d\gamma}{ds} = \frac{d(\theta - \beta)}{ds}$$

$$= \frac{1}{r} \cdot \frac{\beta_0}{\pi - \beta_0}\left(\sin(\alpha + \beta - \delta) - \frac{c}{c_{at}}\left(\diamond\frac{\sin(\gamma + \beta - \delta)}{2\cos\gamma} + \sin(2\gamma + \beta - \delta)\right)\right) \tag{13}$$

In the vicinity of a stagnation point, for rays emanating from there, we have $\delta = \alpha + \beta$, hence $d\beta/ds = 0$, thus $d\alpha/ds = 1/r \cdot \beta_0/(\pi - \beta_0) \cdot 2\sin(2\gamma - \alpha) \cdot d\alpha/d\gamma$ with $\diamond = 0$ under (A). For $d\alpha/d\gamma$ non zero, this can tend to a finite limit without invalidation of the dispersion relation

$$\tan\alpha = \frac{\sin 2\gamma}{1 + \cos 2\gamma - 2c/c_g} \tag{14}$$

only if $\gamma = \alpha = 0$. Hence, unless showing infinite curvature, all rays must emanate tangentially to the hull from the stagnation point, with wave front normal in ray direction. The finite limit should depend on δ i.e. on the direction of approach, with zero curvature if approached tangentially to the hull. This explains the numerical difficulties as experienced by Brandsma.

A ray can not coincide with a streamline (or with the waterline in particuar) if there is curvature. We would have to require $\alpha \equiv 0$, i.e.$\gamma \equiv 0$ hence $d\alpha/ds$ i.e. $d\gamma/ds \equiv 0$; with $c_{at} = c - c_g$ then, this implies

$$\frac{d\gamma}{ds} = Im\frac{dV}{dZ}\left(\frac{1}{c} - \frac{\diamond/2 + 1}{c - c_g}\right)e^{2i\beta} \tag{15}$$

valid even away from stagnation points; however, as $V = u - iv = Uqe^{-i\beta}$, this means that a change of the flow occurs in flow direction only, thus a ray can coincide with a streamline only if the rate of change of V is in the flow direction, i.e. that the streamline has no curvature!

For the rate of change of the wave number k we find

$$\frac{1}{k}\frac{dk}{ds} = -\frac{1}{r}\frac{c}{c_{at}}\frac{\beta_0}{\pi - \beta_0}\left(\diamond\frac{\cos(\gamma+\beta-\delta)}{2\cos\gamma} + \cos(2\gamma+\beta-\delta)\right) \tag{16}$$

If the value of Λ along a ray should equal the critical value $p^2/2$, this would correspond to the minimum for gravity waves; hence k then *must* decrease along the ray. However, due to the rapid increase of $|\gamma|$ near a stagnation point, the sum of cosine terms may change sign, so that k increases (in particular for $(A)'$ where the first cosine term is deleted) and Λ approaches $p^2/2$ again. Here the ray *must* terminate, as for $\Lambda = p^2/2$, even off the waveless zone boundary, the partial derivatives of Λ both regarding q^2 and γ vanish simultaneously in conflict with the ray equations, q^2 can not be varied independent from γ. This explains the previously mentioned occurence of short life rays. Hence the choice of initial points for rays is moot, quite apart from the ambiguity of assigning initial values there for amplitude and phase.

<u>References:</u>

1 Lighthill, M.J.:"Special cases treated by the Whitham theory". Proc. Roy. Soc. A, vol.229 pp.28-53 (1967).

2 Keller, J.B.:"The ray theory of ship waves and the class of streamlined ships". J.Fluid Mech. vol.91 pp.465-487 (1979)

3 Yim, B.:" A ray theory for non-linear ship waves and wave resistance". Proc. Third Intern. Conf. on Num. Ship Hydrodynamics Paris pp.55-70 (1981)

4 Brandsma, F.J.:"Low Froude number expansions for the wave pattern and the wave resistance of general ship forms." Thesis T.U. Delft 111pp.(1987).

5 Eggers, K.:"Non-Kelvin dispersive waves around non-slender ships". Schiffstechnik vol.28 pp.223-252 (1981).

6 Hermans, A.J. and van Gemert, P.H.: "A linearized surface condition on low speed hydrodynamics". Ship Technology Research Schiffstechnik vol.36 pp.181-196 (1989).

7 Eggers, K. and Huang, De-bo:"Ship wave ray tracing including surface tension". Bericht Nr. 501 Institut für Schiffbau der Universität Hamburg, Febr.1990 14pp.(This is a refined version of the Hiroshima lecture 1989).

8 Tulin,M.P.:"Surface waves from the ray point of view" Proc. 14th. Symp. Naval Hydr. Hamburg pp.9-19 (1984).

9 Maruo, H. and Ikehata, M.: "Some discussion on the free surface flow around the bow". Proc. 16th. Symp. Naval Hydr. Berkeley pp.65-77 (1986).

10. Milne-Thomson, L.M.: "Theoretical Hydrodynamics" Macmillan (1967).

DISCUSSION

Raven: Surely ray tracing is based on the assumption that the wavelength is small compared to the length-scale of the base flow? Thus, it must break down near the bow, where the length-scale goes to zero, in general. This is likely to inhibit the convergence of *any* discretisation of the slow-ship condition, say by a panel method, for vanishing panel size. The results obtained with such methods, however, suggest that this near-bow behaviour has only a local effect on the solution. Could you comment on this please?

Peregrine: Ray theory is not consistent near the ship, since it assumes a length-scale much greater than a wavelength *and* waves like a wavetrain. The next approximation which is generally used is the parabolic approximation. For waves generated by a wedge, see Yue & Mei [1], where nonlinear effects are also included.

Kleinman: What is unacceptable about having rays with different curvature emanate from one point, provided a canonical problem can be solved which gives the launching coefficients or amplitudes of the different rays?

Eggers: As under Keller's approach the ray geometry does not depend on the Froude number, the shortness of waves is not felt when rays are constructed. Just as streamlines from a general three-dimensional stagnation point emanate along two mutually orthogonal directions only (along one of them under continuous variation of initial curvature), this may certainly occur with rays. Still, the physical relevance of such lines is linked to the possibility of assigning amplitudes, phases and wavenumber vector to these different rays with common initial direction. Even if this could be achieved in a canonical example, this does not remove the need for a *proof* of the pertinence of the ray approach in the near field of a ship. In the spirit of matched asymptotic expansions, we must expect the existence of some boundary layer around the ship where the ray approach is not appropriate, even without showing singular features, and even for the case of a submerged body. The near field is just not determined from downstream information.

I do not feel that this dilemma can be reduced through a parabolic approximation. The problem treated by Yue & Mei [1] is wave *diffraction* near a sharp bow, and finite water depth is obviously required for the application of their method.

Reference

[1] D.K.P. Yue & C.C. Mei, 'Forward diffraction of Stokes waves by a thin wedge', *J. Fluid Mech.* **99** (1980) 33–52.