COMPRESSED AIR BREAKWATERS

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INTRODUCTION

In a recent paper, Ikeno, Shimoda & Iwata (1988), a theoretical and experimental study was performed to determine the reflection characteristics of a breakwater consisting of a chamber placed on the fluid surface as shown in Fig 1. The equilibrium position of the free surface ($y = d$) can be changed by either increasing or decreasing the air pressure inside the chamber. The problem is closely related to models of oscillating water columns used for extracting energy from ocean waves, though here the compressibility of the air is crucial. Evans (1982) showed that close analogies exist between theories for oscillating rigid bodies and for oscillating surface pressure distributions in the context of wave-energy extraction and in this paper we develop the problem in a manner analogous to that used to describe wave reflection by oscillating rigid bodies in Evans & Linton (1989) in the context of breakwater design theory.

FORMULATION

The problem will be solved using classical linear water wave theory and all motion will be assumed to be simple harmonic in time with angular frequency ω. Thus there exists a harmonic velocity potential

$$\Phi(x,y,t) = \mathrm{Re}\{\phi(x,y)e^{-i\omega t}\} \tag{1}$$

The pressure in the chamber is

$$p_c = p_a + \rho gd + P(t) \tag{2}$$

where $P(t) = \mathrm{Re}\{pe^{-i\omega t}\}$, p_a is atmospheric pressure and ρ is the density of the fluid. A simple application of Bernoulli's equation shows that

$$K\phi + \phi_y = 0 \quad \text{on the external free surface} \tag{3}$$

and

$$K\phi + \phi_y = -\frac{i\omega p}{\rho g} \quad \text{on the internal free surface} \tag{4}$$

where $K = \omega^2/g$.

We take the incident wave to be given by

$$\phi_I = \frac{igA_I}{\omega}\,\frac{\cosh k(h-y)}{\cosh kh}\,e^{-ikx} \tag{5}$$

that is, from $x = +\infty$, where k(real) satisfies $k\tanh kh = K$. The reflection and transmission coefficients for this problem will be denoted by R_1 and T_1 respectively.

In order to solve this problem we decompose it into a scattering and a radiation problem. Thus we put

$$\phi = \phi_S - \frac{i\omega p}{\rho g}\phi_R \qquad (6)$$

where ϕ_S is the solution to the problem which has $K\phi + \phi_y = 0$ on the internal free surface having reflection and transmission coefficients R and T respectively, and ϕ_R satisfies $K\phi + \phi_y = 1$ on the internal free surface and there is no incident wave. Thus ϕ_R is the solution of a symmetric radiation problem and we assume

$$\phi_R \sim A_S \frac{\cosh k(h-y)}{\cosh kh} e^{\pm ikx} \quad \text{as } x \to \pm\infty. \qquad (7)$$

The relationship between T_1, T and A_S is

$$T_1 = T - \frac{pKA_S}{\rho g A_I}. \qquad (8)$$

Let the volume flux across the internal free surface be $\phi(t) = \mathrm{Re}\{qe^{-i\omega t}\}$. Then

$$q \equiv q_s + q_r = \int_{-a}^{a} \phi_y(x,d)\,ds \qquad (9)$$

with the obvious decomposition. The volume flux for the radiation problem can be decomposed into components in phase and in antiphase with the pressure fluctuation $P(t)$. Thus

$$q_r = -(\tilde{B} - i\omega\tilde{A})p. \qquad (10)$$

Finally we consider the gas dynamics.
Assuming isentropic flow in the gas we have

$$\frac{dP}{dt} = -\frac{\gamma p_o}{V_o}\frac{dV}{dt} = -\frac{\gamma p_o}{V_o}Q(t) \qquad (11)$$

where $V_o = 2a(b+d)$ and γ is the ratio of the specific heats.

Use of (8), (10), (11) and the equivalent of the Haskind relation for this problem leads to

$$T_1 = T\left[\frac{C+x}{C+i}\right] \qquad (12)$$

where $C = (\tilde{A} - V_o/\gamma p_o)\omega/\tilde{B}$ and $x = \mathrm{Im}(R/T)$. This is analogous to equation (28) in Evans & Linton (1989) though the signs are different due to the different time dependence used. In order to calculate T_1 therefore we must first compute $\tilde{A}$, $\tilde{B}$, T and x and this can be done using the method of matched eigenfunction expansions. This method was used by McIver (1985) to solve the scattering problem for $d = 0$. When $d \neq 0$ the method is rather more complicated due to the need to use different sets of eigenfunctions in the two regions.

DISCUSSION

Amongst other results Ikeno et al (1988) claim that the clearance, $b + d$, has little effect on R_1 and T_1. Our results suggest that this is not the case and Fig 2 is an illustration of this. For these calculations $a/h = \frac{1}{2}$, $d = 0$, $\ell/h = \frac{1}{2}$ were used. The solid curve shows the transmission coefficient when there is no top on the chamber and then the effect of having different chamber heights is shown. From (12) we see that as $b \to \infty$, $C \to -\infty$ and thus $|T_1| \to |T|$ and thus for large values of b we expect little effect. However from the figure it can be seen that values of b of around 50m are required before this limit becomes valid. It is also clear that small values of b cause relatively little change in the transmission coefficient $|T_1|$. However for $b \simeq 15$m the transmission coefficient is greatly reduced over the whole range of wave frequencies. ($Kh = 0.2$ corresponds to $\lambda/h \simeq 14$ and $Kh = 2$ corresponds to $\lambda/h \simeq 3$). Results showing the effect of varying the other parameters in the problem including the motion of the internal free surface will also be presented.

REFERENCES

EVANS, D V (1982) Wave-power absorption by systems of oscillating surface pressure distributions. *J Fluid Mech* 114, 481-499.

EVANS, D V & LINTON, C M (1989) Active devices for the reduction of wave intensity. *Applied Ocean Research* 11(1), 26-32.

IKENO, M , SHIMODA, N & IWATA, K (1988) A new type of breakwater utilizing air compressibility. 21st Intern Conf on Coastal Eng, Malaga.

McIVER, P (1985) Scattering of water waves by two surface-piercing vertical barriers. IMA *J of Appl Maths* 35.

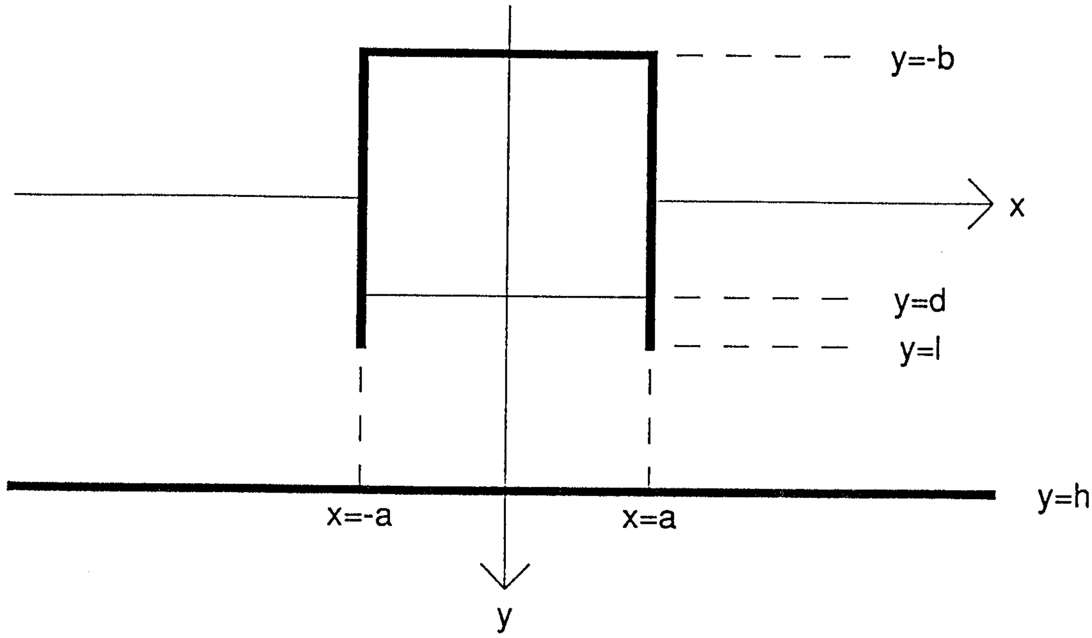

Figure 1. Definition Sketch

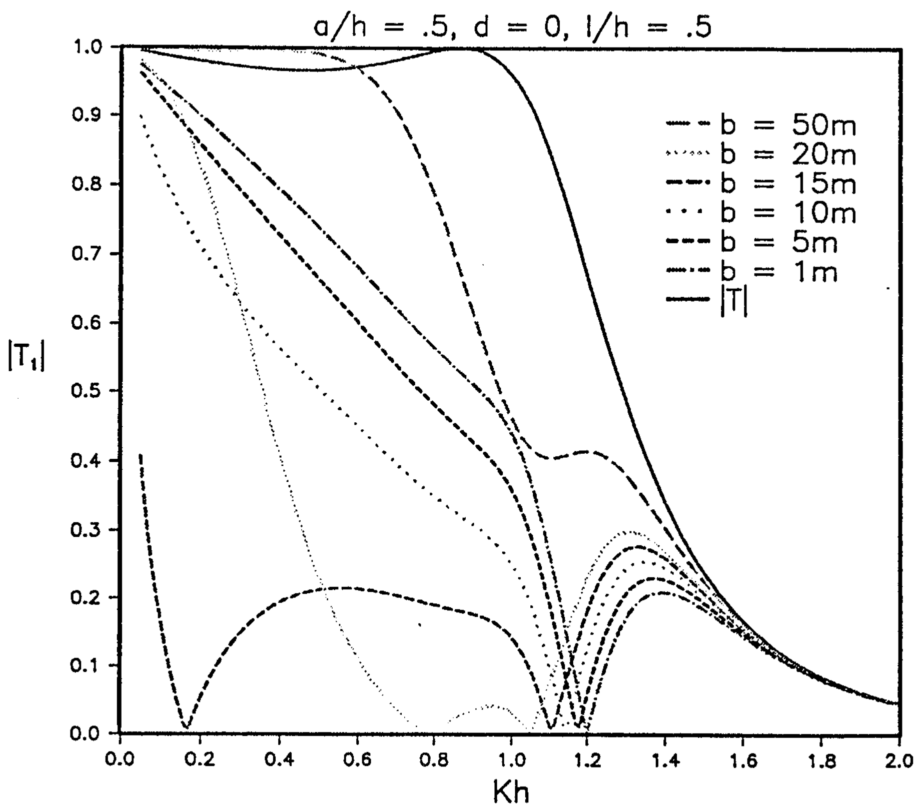

Figure 2

DISCUSSION

King: In the isentropic flow, have you linearized the effect of the water on the gas to (acoustically) small disturbances?

Linton: Yes.

King: In acoustics, work is done compressing the gas. How is this energy loss related to

$$L \equiv |R_1|^2 + |T_1|^2 = 1 \tag{A}$$

in the waves?

Linton: The form of the gas law used implies that the net work done over a cycle is zero, and so (A) is satisfied. There is no mechanism for any dissipation to occur.

Grue: What is the result for $|T_1|$ if one places a fan on top of the open breakwater, with a damping mechanism? In this case, how large may the total damping be, i.e. how small will L be?

Linton: With a fan on top, we have a model of an oscillating water column wave-power device. Because of the symmetry of the plates, the smallest value for L is $\frac{1}{2}$, with one quarter of the energy being reflected and one quarter transmitted. Depending on what is required, it ought to be possible to achieve a desired transmission by varying the fan characteristics.

Eatock Taylor: For an appropriate range of parameters, the internal free surface acts as a rigid plate. Have you considered using one of the diffraction-radiation programs to solve this problem, i.e. by assuming a radiation mode in which the rigid plate oscillates while the rest of the structure is fixed, and vice versa?

Linton: Yes, but we have not yet done it. It would be interesting to determine the range of parameters for which the rigid piston model provides a good approximation to our more accurate solution.

Greenhow: How big is this device when $Kh \sim 1.0$?

Linton: $Kh = 1$ might correspond to waves of period $\simeq 7$ seconds on water of depth 10m, which would give a body dimension of $\simeq$ 5m.

Greenhow: Have you considered oblique waves with internal walls, or vortex shedding as in [1]?

Linton: Both oblique wave incidence and vortex shedding would have to be considered in any application, but here we were simply concerned with seeing whether the idea *could* work and so we have kept things as simple as possible.

Reference

[1] M. Stiassnie, E. Naheer & I. Boguslavsky, 'Energy losses due to vortex shedding from the lower edge of a vertical plate attacked by surface waves', *Proc. Roy. Soc.* A**396** (1984) 131–142.