Oblique Water Entry of Spherical Shapes with Special Reference to the Ricocheting Phenomenon

TOUVIA MILOH
SCHOOL OF ENGINEERING
UNIVERSITY OF TEL AVIV
RAMAT AVIV 69987
ISRAEL

One of the fascinating problems in hydroballistics is the ricocheting, or bouncing, of a rigid body (projectile) off a free-surface. Often one strives to enhance the bouncing effect, as we have all tried to make stones skip across a water surface by throwing them at grazing angles. On the other hand, ricocheting phenomenon may be a hazard and should be avoided when possible. The interest in the ricochet problem is indeed long-standing and goes back to the turn of the 18th century when the ricochet phenomenon was used as a technique for attacking ships with cannon balls. Maybe the most well known military application of this effect was the British so-called Wallis's 'bouncing bomb', which was used by the allies during World War II for demolishing the dams in the Ruhr. An interesting historical account and a general discussions of ricochet firing including a list of some possible applications, has been given in [1].

The theorectical aspects of the ricochet problem for arbitrary shape are very involved and analytic solutions do not exist. For this reason a first attempt is made here to analyze spherical shapes. In addition to the simplications introduced by the choice of the particular geometry, the theorectical predictions may be also compared against some available experimental data on the bouncing effects of spherical projectiles [2]. However, the main motivation for the present study was to verify analytically the following rather simple empirical relation which has been suggested by Birkhoff for the critical ricocheting angle of a sphere [1];

$$\theta_c \cong 18^{o}\sqrt{\rho_f/\rho_s} \qquad (1)$$

where ρ_f and ρ_s denote the densities of the fluid and rigid body respectively. Here θ_c denotes the critical angle between the body velocity vector and the undisturbed free-surface at the instant of contact, beyond which a ricochet will not occur and the projectile will generally enter into the water.

A severe limitation of the empirical formula (1), which is often used as a design criterion, is that it does not depend on the Froude number, contrary to experimental results which demonstrate a very strong dependence on both velocity and sphere diameter. In this work, an attempt is made to account for both size and impact velocity in the analysis of the critical angle. Indeed, we have found that the critical angle varies considerably with Froude number and density ratio (see Fig.1) and that equation (1) may be considered only as an approximation for infinitely large Froude numbers. It has been also demonstrated that there is a critical value for the Froude number, (of the order of 400) below which a ricochet is not possible. Also obtained are actual sub-surface trajectories for various angles of incidence, from which important parameters, such as the maximum penetration and the total range, may be found (Fig. 2).

The analytic approach of the paper is based on a Lagrangian formulation of the early stages of water impact (see recent review of the subject [3]). During the initial stages, inertia forces are shown to dominate viscous, surface tension and compressibility effects. The free-surface is replaced, to first- order approximation, by an equipotential surface and a potential flow problem about a double spherical bowl is solved by employing conical harmonic functions [4]. Instead of integrating the pressure, which may introduce considerably large numerical inaccuracies especially at small depth, an energy approach has been employed, based on the extended Kelvin-Kirchhoff equations of motion for bodies with time varying added masses [5]. The analysis yields analytic expressions for the added-mass coefficients of a partially submerged sphere (the infinite frequency limit) for both heave and sway motions, as well as their derivatives with respect to the instantaneous submergence depth which render the slamming loads. The analytical solution is based on the application of the Mehler-Fock transform and on the asymptotic solution of the resulting integral equations [6].

The case of a vertical (axisymmetric) impact may be considered as a limiting one of the general oblique water entry problem. A comparison between our theorectical prediction of the slamming coefficient and the comprehensive experimental measurements of [7], shows excellent agreement (Fig. 3). Also calculated is the wetting-factor correction which determines the maximum free-surface rise on the sphere. Comparing the analytic wetting-factor with experimental data (Fig. 4), again yields a surprisingly good agreement, in view of the inherent complexity of the problem. In both figures, the full line represents theory, the dark points represent experimental measurements and the dotted lines are empirical curve-fitting correlations.

The paper concludes with an order of magnitude discussion of viscous, surface tension and compressibility effects, which were neglected in the present analysis and with some examples of possible extensions to non-spherical shapes.

References

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CRITICAL CONSTANT ANGLE VS. DENSITY

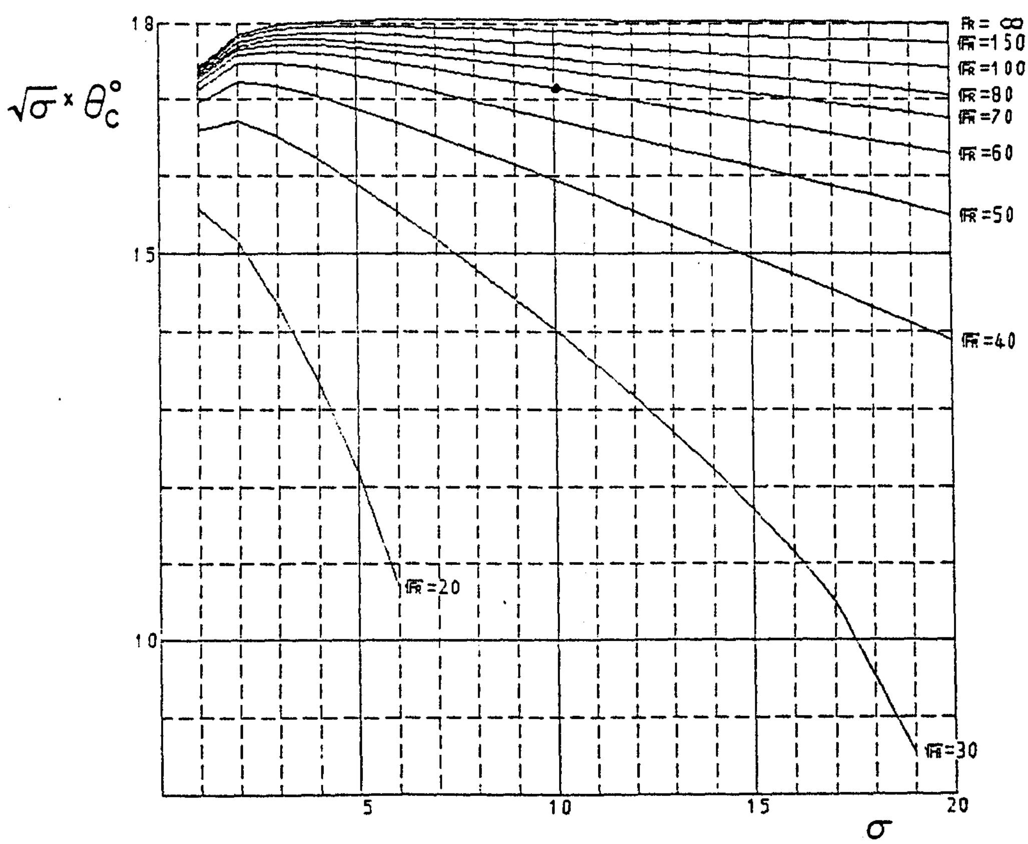
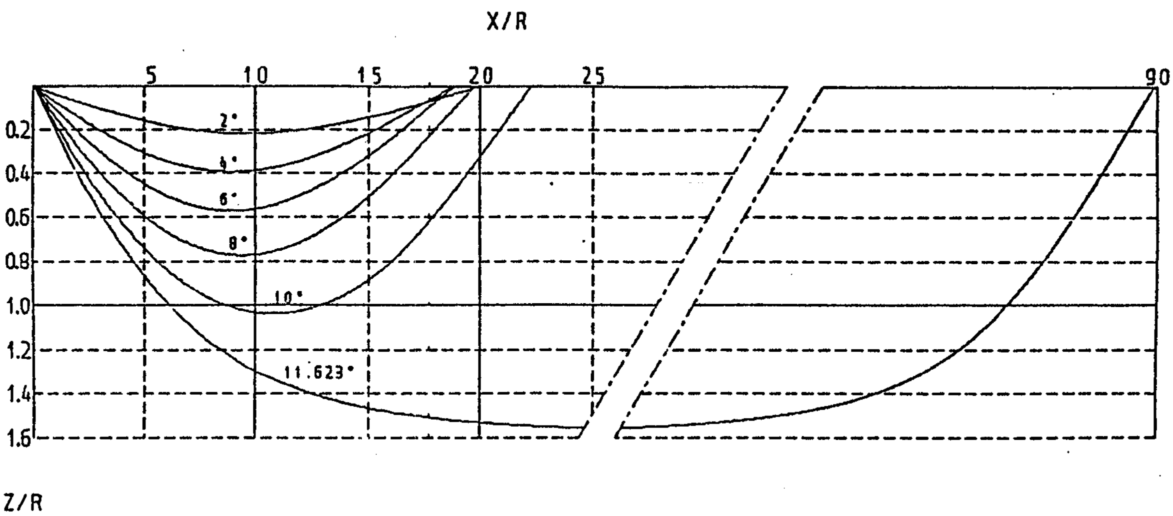


FIG. 1

PENETRATION DEPTH VS. HORIZONTAL RANGE



$\sigma = 2.7$ $\sigma = \rho_s / \rho_f$

FIG. 2

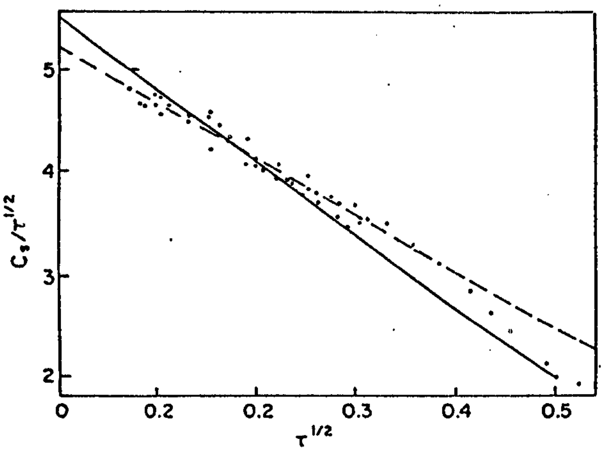


FIG. 3

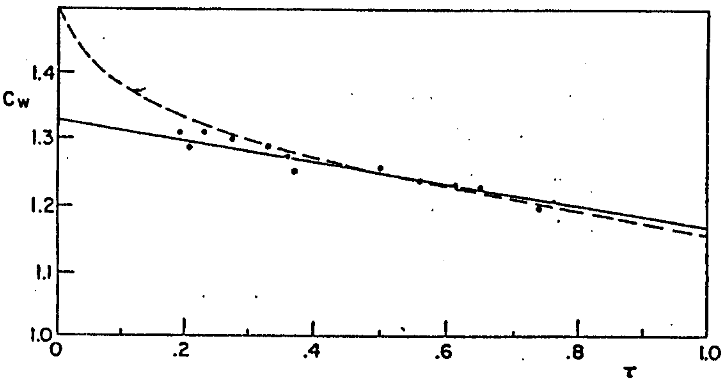


FIG. 4

DISCUSSION

Korsmeyer: I am curious to know if you mean that $\frac{d}{dt}$(Added mass) is *difficult* to compute numerically or *impossible* to compute numerically?

Miloh: It is not easy (standard methods are not very accurate) to compute the time derivatives of the added-mass coefficients for relatively small submergences. In this limit, I think that one should use the proposed asymptotic expansion.

Tuck: I thought that the horizontal (sway) problem might have more of a lateral asymmetry (like planing surfaces) with a detachment point that is determined by the solution.

Miloh: The rear detachment point in the solution is simply determined, to first order, by the condition that the limiting streamline is tangential to the body at the detachment point, i.e. that it is parallel to the instantaneous velocity there. This assumption has been verified experimentally by Birkhoff *et al.*; see [1].

McGregor: Some years ago, during slamming studies at the University of Glasgow ⟦1⟧, we found it necessary to accept that, immediately before a vertical entry, the air is compressed and, consequently, the water surface is depressed. In addition, immediately after impact, a spray root develops. Are these mechanisms likely to be significant for an oblique impact? Incidentally, we thought that we were computing the initial rate of change of added mass adequately, using the VOF method.

Miloh: Fluid compressibility effects and those of compressed air may become significant during the initial stages of impact of bluff and flat-bottomed bodies. These effects may be neglected for the sphere-impact problem; see [3]. The spray root which develops immediately after impact will clearly alter the velocity field and the pressure distribution in the neighbourhood of the confluence of the fluid-solid-air contour. This effect is more pronounced for two-dimensional shapes. However, for spherical shapes, it will have only a secondary effect on the total hydrodynamic force experienced by the body ⟦2⟧.

References

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